Further Differentiation (8/9 pers)

Mathematics 2 Outcome 1

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M2	TeeJay Publishers
1	Go over the proofs of the derivatives on $\sin^{-1}x$ and $\cos^{-1}x$. [use $y = \sin^{-1}x \Rightarrow x = \sin y$ etc.] * Ask students to find derivative of $\cos^{-1}x$. * Ask why $\frac{d}{dx}(\sin^{-1}x) = -\frac{d}{dx}(\cos^{-1}x)$ see graphs. Go over Chain Rule => $\frac{d}{dx}(\tan^{-1}\left(\frac{x-1}{x+1}\right))$ etc	Page 32 Ex 2 Qu 1 a, b, e, 2 b, c, d 3a, d Page 33 Ex 3A Qu 2 and 3	Page 3 Ex 1 As many as possible in the period/h'work
2	Go over the idea of Implicit Differentiation and show that $2x^2 - 3xy - y^2 = 1 \Rightarrow \frac{dy}{dx} = \frac{4x-3y}{3x+2y}$ etc + more examples	Page 36 Ex 4A Qu 1, 2	page 6 Ex 2 Qu 1
3	Go over (i) equation of a tangent to an explicit fn (ii) $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$ or by finding the **[A/B]** derivative of the original un arranged fn.	Page 36 Ex 4A Qu 5, 9, 4 Page 38 Ex 5 Qu 1 a, d, f, k, ,6	Page 6 Ex 2 Qu 2 - 6
4	Introduce Logarithmic Differentiation via $y = x^x$, $y = 2^{\sin x}$, $y = (\sin x)^x$ etc. Possibly $y = \frac{(3x+5)^6(2x-1)^4}{(4x+3)^5}$?? **[A/B]**	Page 40, Ex 6 Qu 1, 2, (7)	page 10 Ex 4
5	Explain idea of Parametric Equations of Curves e.g. $x = r\cos\theta$, $y = r\sin\theta$, => $x^2 + y^2 = r^2$ (circle) $x = a\cos\theta$, $y = b\sin\theta$, => $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (ellipse) $x = a\sec\theta$, $y = b\tan\theta$, => $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ (hyperbola) $x = ct$, $y = \frac{c}{t}$ => $xy = c^2$ (hyperbola also) Explain :- $\frac{dy}{dx} = \frac{dy}{dt} / \frac{dx}{dt}$ and find equation of tangent to a curve given in parametric form.	Page 42 Ex 7A Qu 1 a - c Page 44 Ex 8A Qu 1 ($\frac{dy}{dx}$ only)	page 13 Ex 5
6	Find 2nd derivative => $\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{dy}{dt}\left(\frac{dy}{dx}\right) / \frac{dx}{dt}$ and use this to check the nature of turning points	Page 44 Ex 8A Qu 1 ($\frac{d^2y}{dx^2}$) only, 2, 3	Page 15 Ex 6
7	Apply parametric and implicit diffn to related problems e.g. rates of change of volumes, surface areas	Page 53 Ex 2A Qu 1 - 6	page 8 Ex 3
8	Use parametric differentiation to find 1st and 2nd derivatives and apply this to motion in a plane. $x = f(t)$, $y = g(t)$ => speed = $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ velocity = $\frac{dx}{dt}$ and $\frac{dy}{dt}$ in x and y directions	Page 50 Ex 1 Qu 1, 2, 5, 6, 7	Page 17 Ex 7
9	Review		
	Cumulative total = 64 periods		

Further Integration (10/11 periods)

Mathematics 2 Outcome 2

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M2	TeeJay Publishers
1	Know $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x$, $\int \frac{1}{1+x^2}dx = \tan^{-1}x$ $\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a}$, $\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}$	Page 61 Ex 1A Qu 1 a, b, e, f 2 a, c, f,h Page 62 Ex 1B Qu 1 a, c, e, h	Page 22 Ex 1
2	Integrate using Partial Fractions (1) e.g. $\int \frac{x+16}{2x^2+x-6}dx$ and $\int \frac{2x^3+7x^2-2x-2}{2x^2+x-6}dx$	Page 64 Ex 2 Qu 5 a, b, d, e 6 a, c	Page 25 Ex 2
3	Integrate using Partial Fractions (2) e.g. $\int \frac{-8x^2+14x-15}{(2x-1)^2(x+2)}dx = \ln(2x-1) + \frac{2}{2x-1} - 3\ln(x+2) + c$	Page 66 Ex 3A Qu 1, 3 a, d, e	Page 25 Ex 3
4	Integrate using Partial Fractions (3) e.g. $\int \frac{x-1}{(x+1)(x^2+1)}dx = -\ln(x+1) + \frac{1}{2}\ln(x^2+2) + c$ *and* $\int \frac{x^2+2x+1}{x(x^2+1)}dx = \ln x + 2\tan^{-1}x + c$	Page 66 Ex 3A Qu 2, 3 b, c, f 7	Page 27 Ex 4
5	Integration by Parts. Show that $\int \left(u\frac{dv}{dx}\right)dx = uv - \int \left(v\frac{du}{dx}\right)dx$	Page 69 Ex 4 Qu 1, 2	Page 29 Ex 5
6	Integration by Parts twice (or more) **[A/B]**	Page 71 Ex 5A Qu 1 a - d, g, m, n	Page 29 Ex 6
7	Integration by Parts using a "dummy" function e.g. $\int \ln x dx = \int 1\ln x dx$, $\int \sin^{-1}x dx = \int 1\sin^{-1}x dx$		Page 30 Ex 7
	Extension :- Intn by Parts returning to original function e.g. $\int e^x \sin x dx$ etc	Page 71 Ex 5A Qu 1 e, f, q, s	Page 31 Ex 8
8	First Order Differential Equations (Variable separable)	Page 77 Ex 8 Qu 1	Page 33 Ex 9
9	Particular solution of a First Order Differential Equation	Page 77 Ex 8 Qu 2, 3, 5	Page 35 Ex 10
10	Solving Real Problems with First Order Differential Equations including "rate of change" e.g. growth of bacteria (x) is given by $\frac{dx}{dt} = \frac{kx(n-x)}{n}$ (chemical reactions, Newton's Laws of cooling etc)	Page 81 Ex 9A Qu 2 and 4 -9	Page 38 - 39 Ex 11
11	Review		
	Cumulative total = 75 periods		

Complex Numbers (9/10 pers)

Mathematics 2 Outcome 3

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M2	TeeJay Publishers
1/2	Introduce Imaginary number $i = \sqrt{(-1)}$ and Complex Number $z = a + ib$ Show that $i^2 = -1$, $i^3 = -i$, $i^4 = 1$ etc. Show how to add, subtract, multiply and introduce the complex conjugate $\bar{z} = a - ib$ to divide Show also how to find simple powers $(2 + i)^4$ (binomial).	Page 90 Ex 1 Qu 1, 2, 3, 6, 7, 8 Page 91 Ex 2 Qu 1 a, b, c, 2 c, e 3 a, b, f 5 a, b	Pages 49 - 50 Ex 2 Ex 1 (some) if needed
3/4	Show how to represent a complex number pn an Argand Diagram (treat addition etc as "vector" addition etc) Define modulus and argument and show how to find r and θ. Show $z_1 \times z_2 = r(\cos\theta + i\sin\theta) \times s(\cos\phi + i\sin\phi)$ $= rs(\cos(\theta + \phi) + i\sin(\theta + \phi))$ Same for division	Page 94 Ex Qu 3 a, b, d, e, i Qu 6 a, b, f 7 a, b, c Page 98 Ex 5 Qu 1 a, b, f, g	Page 53 - 54 Ex 3 Page 55 Ex 4
5	Show De Moivre's Theorem $z^n = r^n(\cos n\theta + i\sin n\theta)$ (Introduce $z = r\text{cis}\theta$ and/or $z = re^{i\theta}$ as shortened form)	page 101 Ex 6 Qu 1, 2, 3a, 4 g, h, i, j	Page 57 - 58 Ex 5
6	Introduce Fundamental Theorem of Algebra to find <u>all</u> the roots (factors) of a polynomial equation (function)	Page 108/109 Ex 8 Qu 2a, d 3 a, b, 4, 5, 6a, b	Page 61 Ex 6
7	Show the Geometric Interpretation of equations and inequalities in the Complex Plane	Page 96 Ex 4 Qu 1 a, b, d, f, j 3 a, b, 4 a, c	Page 63 Ex 7
8	Apply De Moivre's Theorem to solve Multiple Trig formulae e.g. Express $\sin 5\theta$ in terms of $\sin\theta$ only	Page 102 Ex 6 Qu 5, 6, 7	Own Examples
9	use De Moivre's Theorem to find the Roots of Unity (extension :- find roots of other complex numbers)	Page 106 Ex 7 Qu 2 + selection from 1	own Examples
10	Review		
	Cumulative total = 85 periods		

Sequences and Series (7/8 pers)

Mathematics 2 Outcome 4

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M2	TeeJay Publishers
1	Revise Fifth Year definitions of Sequences and Series Define finite and infinite sequences & series Define notation $u_n = n^2$ **Introduce idea of Arithmetic Sequence** $u_{n+1} = u_n + d$ **Build up to** $u_n = a + (n-1)d$ **- general term of A.S.**	Page 117 Ex 2A Qu 1 a, c, d, 2 a-e, 3, 4, 6	Page 72 Ex 1 Qu 1 - 3
2	**Show how to find sum of *n* terms of an Arithmetic Series** **=>** $S_n = \frac{n}{2}(2a + (n-1)d)$ **Show two types e.g. find** S_{15} **= 2 + 5 + 8 + ...** **and** S **= 7 + 11 + 15 + ...+ 75** **Show how to use simultaneous equations in :-** $u_2 = 9$, $u_5 = 27$, **find *a*, *d*, and** S_{10}	Page 120 Ex 3A Qu 1, 3, 4, 5, 8	Page 72 Ex 1 Qu 4 - 9
3	**Introduce the Geometric Sequence :-** $u_{n+1} = ru_n$ **Build up to** $u_n = ar^{n-1}$ **- the general term of a G.S.** **Find** u_{10} **= 180 + 90 + 45 + 22·5 + ...**	Page 123 Ex 4A Qu 1 a - e, 2, 3, 5, 7	Page 75 Ex 2A Qu 1 - 4
4	**Show how to find sum of *n* terms of a Geometric Series** **Build up to** $S_n = \frac{a(r^n-1)}{r-1}$ **(or** $\frac{a(1-r^n)}{1-r}$**)** **Find** S_{19} **= 3 – 6 + 12 – 24 + ...** **Use sim equ^{n}s to solve** u_3 **= 32,** u_6 **= 4 => find** S_8	Page 127 Ex 5A Qu 1, 2, 3, 4	Page 75 Ex 2A Qu 5 - 7
5	**Go over** S_∞ **of a G.S. and define its existence** **(Graphics Calculators are useful here)**	Page 131 Ex 6A Qu 1 - 4 and 7	Page 77 Ex 2B
6	**Expand** $\frac{1}{1-r}$ **as a G.S. with common ratio *r*, first term 1.** **as** $\frac{1}{1-r} = 1 + r + r^2 + r^3 + r^4 + ...$ **show alternative using binomial on (1 – *r*)$^{-1}$ or** **by dividing => (1 – *r*)** $\overline{)\,1}$ **etc.** **Expand** $\frac{1}{a+b}$ **as** $\frac{1}{a}\left(\frac{1}{1+\frac{b}{a}}\right)$ **etc.** [A/B]	Page 134 Ex 7A Qu 2, 4 Page 135 Ex 7B Qu 2, 5	Page 79 Ex 3
7	Show that $\lim_{n\to\infty}\left(1 + \frac{1}{n}\right) = e$ (calculator) Reintroduce the $\sum$ notation to show that $\sum_{k=1}^{k=n} k = 1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$ (use A.S.) and show how to find $\sum(k+2)$, $\sum(4k-1)$ etc.	Page 137 Ex 8 Qu 1 a - d, 2 a, b, 4	Page 81 Ex 4 page 82 Ex 5
8	Review		
	Cumulative total = 93 periods		

Elementary Number Theory & Proofs (10/11 periods)

Mathematics 2 Outcome 5

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M2	TeeJay Publishers
1	Go over idea of "statement", true/false, "implications (=>, <=, <=>) and "equivalence" if $a \Rightarrow b$ and $b \Rightarrow a$. Converse Statements. Direct Proof and "Disproof by Counter example"	Page 3 Ex 1A A selection	Page 85-86 Ex 1, 2 & 3
2	Proof by "Contradiction". e.g. "set of primes is infinite" "$\sqrt{2}$ is irrational" etc.	Page 14 Ex 3A Qu 1, 2, 3, 6, 11, 12	Page 88 Ex 4
3/4	"Simple" proof by "Induction" (At this stage, dependent on time, you may wish to do <u>ALL</u> the proofs by induction by combining the work of this Unit with that of the Induction Chapter in Maths 3 e.g. (a) Prove that $8 \mid 3^{2n} - 1 \quad \forall n \in W$ (b) Prove that $\sum_{r=1}^{k=n} r^2 = \frac{1}{6} n(n+1)(2n+1)$ Prove Binomial and De Moivre's Theorem by Induction	Page 20 Ex 4 Qu 6, 7, 8, 9, 11, 12, 13	Page 89 Ex 5 + whole sheet of $\sum$ questions
5	Review		
	For Session 2001-2002 **End of Mathematics 2** **Total = 43 periods for Mathematics 2** **Cumulative Total for M1 & M2 = 98 periods** **Assuming Maths 2 started around 1st Nov, this unit should end around Friday 18th Jan.** **=>3 periods revision (including specimen NAB3) + 1 period for test = 47 periods** **<u>Cumulative total for M1 & M2 = 102 periods</u>** **TEST around Thurs 24th January** **PRELIMS start Wed 20th Feb for 2 weeks-** **Suggest doing at least two weeks work on Maths 3 before beginning revision for Prelims (possibly Matrices could be done or Vectors if a little more time is available)**	**Actual Test Date**	**=**