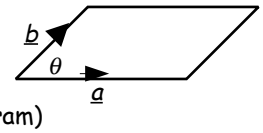
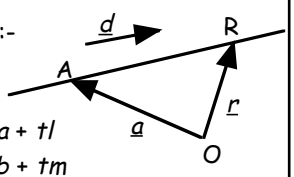
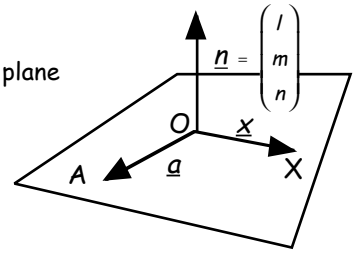
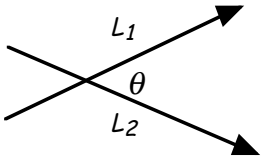
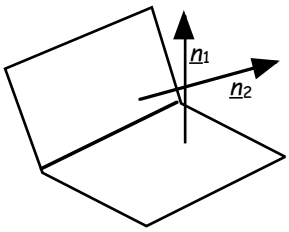
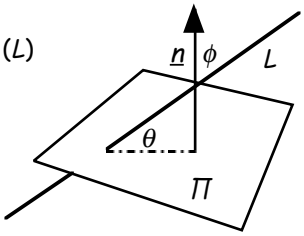
Vectors (9/10 pers)

Mathematics 3 Outcome 1

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M3	TeeJay Publishers
1/2	Revise position vector, $\overrightarrow{PQ} = \underline{q} - \underline{p}$, communicative, associative, zero vector, multiplication by a scalar k, components, magnitude, unit vector, ($\underline{i}$, $\underline{j}$, and $\underline{k}$) as well as direction vectors and direction cosines. Revise "section formula" (not compulsory in S5) Revise "scalar product" and its usage in calculating angles => $\cos\theta = \dfrac{\underline{a}.\underline{b}}{\lvert\underline{a}\rvert\,\lvert\underline{b}\rvert}$ Go over the fact that scalar product is distributive => $\underline{a} . (\underline{b} + \underline{c}) = \underline{a} . \underline{b} + \underline{a} . \underline{c}$ and the fact that if $\underline{a} . \underline{b} = 0$ => $\underline{a}$ is perpendicular to $\underline{b}$ Go over example of finding a unit vector $\underline{u}$ which is at 45° to vector $\underline{a}$ and is 60° to vector $\underline{b}$.	Page 44 Ex 1 Qu 6 - 8 Page 46 Ex 2 Qu 1, 2, 9	Page 4 Ex 1
3	Introduce the Right Handed Vector system (and L.H.) Define $\underline{a} \times \underline{b}$ as a new vector $\underline{c}$, i.e. $\underline{a} \times \underline{b} = \underline{c}$ where $\lvert\underline{a} \times \underline{b}\rvert = \lvert\underline{a}\rvert\lvert\underline{b}\rvert \sin\theta$ = (area of parallelogram)  and where [$\underline{a}$, $\underline{b}$, ($\underline{a} \times \underline{b}$)] forms a right handed system Show how to calculate $\underline{a} \times \underline{b}$ using :- $\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & j & \underline{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$ => $\underline{a} \times \underline{b} = (a_2b_3 - b_2a_3)\underline{i} - (a_1b_3 - b_1a_3)\underline{j} + (a_1b_2 - b_1a_2)\underline{k}$ Do a few examples	Page 52 Ex 4 Qu 1, 2 a, b, 5, 8, 9 a, b, 12	3 - 4 examples on finding $\underline{a} \times \underline{b}$
4	Show that ($\underline{a} \times \underline{b}$) = -($\underline{b} \times \underline{a}$) Show that $\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a}.\underline{c})\underline{b} - (\underline{a}.\underline{b})\underline{c}$ (vect. triple product) Calculate the area of triangle ABC where A(1,3,-2), B(4,3,0) and C(2,1,1) using vector product (and area = $\lvert \overrightarrow{AB} \times \overrightarrow{AC} \rvert$) be able to find a unit vector perpendicular to vectors $\underline{a}$ & $\underline{b}$	Page 52 Ex 4 Qu 3, 4, 6, 7, 14, a, b	Page 9 Ex 2
5	Equation(s) of a line in 3 dimensions:- (i) Vector form $\underline{r} = \underline{a} + t\underline{d}$  (ii) Parametric form $x = a + tl$, $y = b + tm$, $z = c + tn$ (iii) Symmetric form $\dfrac{x-a}{l} = \dfrac{y-b}{m} = \dfrac{z-c}{n} = t$	Page 66 Ex 9A Qu 1, a, b, 2, a, 3, a, c, e, 5	Page 12 Ex 3
	cont'd		

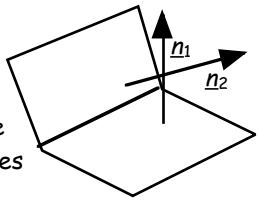
Vectors (9/10 pers) (cont'd ....)

Mathematics 3 Outcome 1

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M3	TeeJay Publishers
5	... cont'd Point out that to find the equation(s) of a line, you require two things: a point on the line and the direction of the line. Example:- Find line joining A(1,0,2) to B(2,1,0) Show that equation of a line is not unique in how it appears.	Page 67 Ex 9B Qu 2	
6	Equation of a plane (Π) Discuss the NORMAL ($\underline{n}$) to a plane If A is a known point on the plane (Π), $\underline{n}$ the normal and X is any other point on the plane, then :- $\underline{n} \cdot \underline{x} = \underline{n} \cdot \underline{a}$ => $lx + my + nz = la + mb + nc$ or $lx + my + nz = k$ (Cartesian form) Show also parametric form $\underline{r} = \underline{a} + \lambda\underline{u} + \mu\underline{v}$ Establish that to find the equation of a plane you require two facts: a point A on the plane and the normal ($\underline{n}$) to it.	 Page 57 Ex 6 Qu 1, a, b, 2, a, 3 4, a, c, 5, a, 9, 10	Page 15/16 Ex 4
7/8	(i) Angles between two lines = angle between the two direction ratios => use the scalar product	 Page 70 Ex 11 Qu 1, 2	page 22 Ex 5 Qu 1
	(ii) Angle between two planes [A/B] = angle between the two normals. => use $\underline{n_1} \cdot \underline{n_2}$ (etc.)	 Page 59 Ex 7A Qu 1, 2, 3	Page 22 Ex 5 Qu 2
	(iii) Angle (θ) between a line (L) and a plane (Π) => $\theta = (90 - \phi)$ where ϕ is the angle between the line L and the normal $\underline{n}$ to the plane => simply find ϕ from $\underline{l} \cdot \underline{n}$	 Page 68 Ex 10 Qu 1 (ii) a, b, c Qu 2 a, b, 3, 4, a	Page 22 Ex 5 Qu 3
9/10	cont'd		

Vectors (9/10 pers) (cont'd ....)

Mathematics 3 Outcome 1

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M3	TeeJay Publishers
9/10	(i) Intersection of two lines Study the set of 3 simultaneous equations, solve any two and check the solution in the third.	Page 70 Ex 11 Qu 1, 2	Page 22 Ex 5 Qu 4
	(ii) Line of intersection of two planes (a) use $\underline{n}_1 \times \underline{n}_2$ to get direction of the line of intersection (b) set x (or y or z) = any val;ue and use this to find the values of the other two variables, giving the coordinates of a point on the line. etc. 	Page 72 Ex 12 Qu 1, 2	Page 22 Ex 5 Qu 5. a/b
	(iii) Intersection of three planes (a) single point (b) meet on a line (c) don't meet at all } (use Gaussian elimination)	Page 78 Ex 15 Qu 1, a, c, 2, a, c	Page 22 Ex 5 Qu 5. c
	(iv) Intersection of a line and a plane $x = a + tl,\ y = b + tm,\ z = c + tn$ (1) and $px + qy + rz = k$ (2) => substitute equations (1) into (2) and solve	Page 68 Ex 10 Qu 1 (i) d, e, f Qu 2, c, 4, b, c	Page 22 Ex 5 Qu 6-12
	Cumulative total = 112 periods (+ two weeks	**for prelims ≅ 122)**	

Matrices (7 pers) (or condensed to 5/6)*

Mathematics 3 Outcome 2

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M3	TeeJay Publishers
1	Explain Matrix, term/element, row, column, order, equating matrices and transpose, square, zero matrix.	Page 4 Ex 1, Qu 1, 2, 3a 4 a, c, e, 10	Page 27, Ex 1
2	Go over adding, subtracting, and multiplying by a real number and establish that $A + B = B + A$, $(A')' = A$ $(A + B)' = A' + B'$	Page 5 Ex 1 Qu 6 g, i, p, r, t, 7a, f, 9, 10	Page 28, Ex 2, Page 29 Ex 3 & page 30 Ex 4
3/4	Show how to multiply two matrices which are "conformable" and find A^2 etc and establish rules $AB \neq BA$, $(AB)C = A(BC)$, $A(B + C) = AB + AC$, $(AB)' = B'A'$	page 10 Ex 3 Qu 1 a, c, 2, a, c, k, m, o 3 a, 4, 5, a, c Page 12 Ex 4A, 6, 7, 8	Page 33, Ex 5
5	Define det $A = \|A\|$ for 2 x 2 and 3 x 3 matrices and show that det (AB) = detA x detB	Page 16 Ex 5 Qu 1, b, d, h Page 25, Ex 7, Qu 4, 5a, b	Page 35, Ex 6
6	Find inverse A^{-1} of a 2 x 2 matrix and show that A^{-1} exists iff det $A \neq 0$. Show that $(AB)^{-1} = B^{-1} A^{-1}$	Page 19 Ex 6A, Qu 1, 2, 4, , 8, 9 (some)	Page 38, Ex 8
7.	Find the inverse of a 3 x 3 matrix using elementary row operations (possibly covered in Maths 1)	Page 28 Ex 8, Qu 1	Page 41 Ex 9
8	Solve simultaneous equations in 2 and 3 unknown using matrix inverses.	Page 28 Ex 8 Qu 3	Page 41 Ex 10 page 42 Ex 11
9	Use matrices to represent simple geometric transformations such as reflection, rotation, and dilatation (dilation)	Page 32 ex 9a Qu 1, 2, 5 (some) 6	Page 44 Ex 12
	* note if the work on Matrices in Maths 1 has been extended, this can be shortened to 5-6 periods		
	Cumulative total = 120 periods (+ two weeks	**for prelims ≅ 130)**	

Further Sequences and Series (4 periods)

Mathematics 3 Outcome 3

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M3	TeeJay Publishers
1	Define a "Power Series" and the Maclaurin series:- $f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + ...$ Show that $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \quad \forall x$ $\ln(1+x) = x - \frac{x^2}{2!} + \frac{x^3}{3!} + ... \quad -1 < x < 1$ $\sin x = \frac{x}{1!} - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + ... \quad \forall x$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ... \quad \forall x$ $\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + ... \quad -1 < x < 1$ $(1+x)^n = \binom{n}{0} + \binom{n}{1}x + \binom{n}{2}x^2 + \binom{n}{3}x^3 + ...$	Page 91 Ex 4 Qu 1 a, f Page 95 Ex 6 Qu 1 a, f, 2, a, d Qu 3 a, 5 a, 6 a (i)	Page 53 Ex 1
2	Combine expansions e.g. show how to expand $e^{-2x}\sin 3x$, ln(cos$x$), etc	Page 96 Ex 6 Qu 7, 8	Page 54 Ex 2
3/4	Iteration:- Remind of simple Iteration techniques learned in S4/5 Show how to rearrange the function, $x^2 - 4x - 8 = 0$ to $x = \frac{1}{4}x^2 - 2 \Rightarrow x_n = \frac{1}{4}x_{n-1}^{\ 2} - 2, \quad (g(x) = \frac{1}{4}x^2 - 2)$ $x = \frac{8}{x-4} \Rightarrow x_n = \frac{8}{x_{n-1}-4}, \quad (g(x) = \frac{8}{x-4})$ $x = 2(x+2)^{\frac{1}{2}} \Rightarrow x_n = 2(x_{n-1}+2)^{\frac{1}{2}}, \ (g(x) = 2(x+2)^{\frac{1}{2}})$ and use a calculator (computer) to find solutions to the above recurrence relations. Go over idea of "staircase" and "cobweb" diagrams and the test for convergence (if a root at α, then will converge to iff $\lvert g'(\alpha)\rvert < 1$) Go over 2-3 examples of testing for convergence and homing in on solutions.	Page 99 ex 8 page 102 Ex 9 Qu 1, 2, 4 (hard)	Page 61 Ex 3
	Cumulative total = 124 periods (+ two weeks for prelims ≅ 134)		

Further Differential Equations (5/6 periods)

Mathematics 3 Outcome 4

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M3	TeeJay Publishers
1	First Order Linear Differential Equations (FOLDE's) Go over where integrating factor comes from & its usage in solving:- $\frac{dy}{dx} + P(x)y = Q(x)$ => $\mu(x) = e^{\int P(x)dx}$ (integrating factor) => $\frac{d}{dx}(\mu(x)y) = \mu(x)Q(x)$ => $y = \frac{1}{\mu(x)}\int \mu(x)Q(x)dx$ Go over 1 example such as :- $x\frac{dy}{dx} + (x-2)y = x^3$		
2/3	Go over 2-3 more examples like (a) $\frac{dy}{dx} - 2y = 6e^{-x}$ (b) $(1+x^2)\frac{dy}{dx} - xy = x(1+x^2)$ (c) $\frac{dy}{dx} + y = 2x + 4$ including General and Particular solutions	Page 114 ex 1 Qu 2 (+ some of Qu 3) page 116 Ex 2 Qu 1	page 67 Ex 1
4/5	Second order Linear Differential Equations :- $a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$ Start with homogeneous case where $f(x) = 0$ Define "big D" method => Auxiliary Equation :- => $aD^2 + bD + c = 0$ and go over types of soln^s (i) 2 solutions $x = \alpha, \beta$ => $y = Ae^{\alpha x} + Be^{\beta x}$ (ii) 1 solution $x = \alpha$ => $y = (Ax + B)e^{\alpha x}$ (iii) 2 complex solutions => $y = e^{\alpha x}(A\sin\beta x + B\cos\beta x)$ ($x = \alpha \pm \beta i$) (define this as the Complementary Function (C.F.) Go over 1 example of each type)	Page 119 Ex 3 Qu 1 a, b, 2 a, b Page 120 Ex 4 Qu 1 a, b, 2 a, b Page 122 Ex 5A Qu 1 a, b, 2 a, b [A/B] [A/B]	Page 69 Ex 2
5/6	Go over non-homogeneous case where $f(x) \neq 0$. ($f(x)$ is simple polyl or $a\sin x$ or $b\cos x$) ** could extend here to include exponential functions) ** Solution to this called the Particular Integral (P.I.) Be able to find the General Solution y = P.I. + C.F. and particular Solutions given boundary conditions. Go over 2-3 examples	Page 126 Ex 7A As many as possible [A/B]	Page 74 Ex2 * assesses beyond poly^l and trig work
	Cumulative total = 130 periods (+ two weeks for prelims ≅ 140)		

Further Number Theory & Proofs (4 periods)

Mathematics 3 Outcome 5

Pd	Lesson, Outline, Approach etc.	Nelson MIA - AH M3	TeeJay Publishers
1	Much may already have been covered in Maths 2. Revise method of Direct proof, by Contradiction (Contrapositive as well ??)	A mixture from Ex 1, 2A and 2B	Page 82 Ex 4
2/3	Expand Induction Prove that $\sum_{r=1}^{r=n} r^3 = \frac{n^2(n+1)^2}{4} = \left(\sum_{r=1}^{r=n} r\right)^2$	Page 141 Ex 3A As much as possible	Page 85 Ex 5 (+ further examples)
4	Go over Division Algorithm. Introduce the Euclidian Algorithm Show that if the g.c.d $(a,b) = d$ the $d = xa + yb$ for unique integers x and y. [A/B] (Possibly some "base" work if time)	Page 145 Ex 4 Qu 1 a, c, e, g, i Page 147 Ex 5 Qu 1 - 4 (page 151 Ex 7 - some)	page 87 Ex 7 Page 88 Ex 8 (page 89 Ex 9)
5	Review		
	For Session 2001-2002 **End of Mathematics 3** **Total = 32 periods for Mathematics 3** **Cumulative Total for M1, M2 & M3 = 134 p's** **Assuming Maths 3 started around Mar 4th, and 8 period done before prelims, this unit should end about Thursday 11th April** **=>3 periods revision (including specimen NAB3) + 1 period for test = 36 periods** **TEST around Wed 17th April** **FINAL Exam in May** **TeeJay Revision Booklet (handed out Earlier) along with TeeJay Specimen papers and Last year's paper to be used in run-up to Exam**	**Actual Test Date**	**=**