MATHEMATICS

H

Applications

Unit Assessment Practice

FORMULAE LIST

Circle: The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a,b) and radius r .

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + c$

Outcome 1

1. (a) A straight line has the equation $5x + y - 3 = 0$.

   Write down the equation of the line parallel to the given line, which passes through the point (4, –8)

   (b) A straight line has the equation $y = -4x + 7$.

   Write down the equation of the line parallel to the given line, which passes through the point (3, –12)

   (c) A straight line has the equation $3x + y - 1 = 0$.

   Write down the equation of the line parallel to the given line, which passes through the point (6, –4)

   (d) A straight line has the equation $y = -5x + 2$.

   Write down the equation of the line parallel to the given line, which passes through the point (3, –7)

2. (a) A straight line has the equation $y = 4x + 1$

Write down the equation of the line perpendicular to the given line, which passes through the point (2, –3)

(b) A straight line has the equation $2x + 5y = 10$

Write down the equation of the line perpendicular to the given line, which passes through the point (-3, –1)

(c) A straight line has the equation $5x + 5y – 1 = 0$

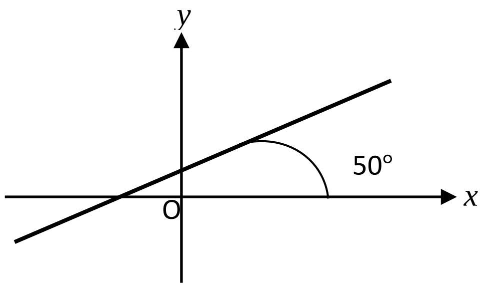
Write down the equation of the line perpendicular to the given line, which passes through the point (2, –5)

(d) A straight line has the equation $4x + 2y = 6$

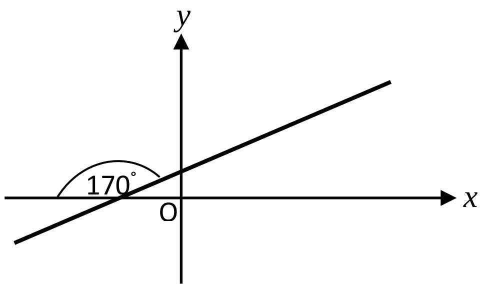
Write down the equation of the line perpendicular to the given line, which passes through the point (-3, 8)

3. Find the gradient of the following lines:

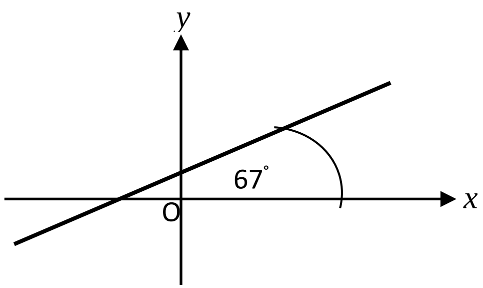
(a)



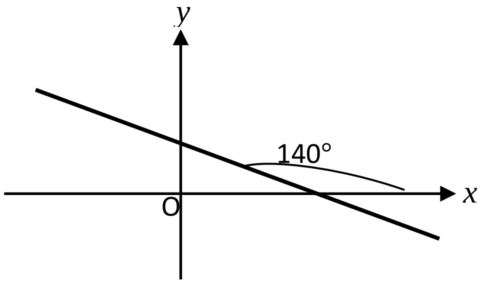
(b)



(c)



(d)



4. (a) Determine algebraically if the line $y = x - 1$ is a tangent to the circle $(x + 4)^2 + (y - 2)^2 = 49$

(b) Determine algebraically if the line $y = x + 1$ is a tangent to the circle $x^2 + y^2 + 2x - 4y - 15 = 0$

(c) Determine algebraically if the line $y = 3x + 10$ is a tangent to the circle $(x - 4)^2 + (y - 2)^2 = 40$

(d) Determine algebraically if the line $y = x + 3$ is a tangent to the circle $x^2 + y^2 + 4x - 8y + 11 = 0$

5. Get the equation of the following circles:

(a) Centre (1, -2) and radius 4

(b) Centre (5, 6) and radius 5

(c) Centre (-3, -7) and radius 10

6. Do the following points lie on the circle given?

(a) (3, 6) $x^2 + y^2 - 6x - 2y - 15 = 0$

(b) (-2, 3) $x^2 + y^2 = 20$

(c) (3, -1) $(x - 2)^2 + (y + 1)^2 = 2$

(d) (2, 3) $(x - 2)^2 + (y - 6)^2 = 9$

7. (a) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 2, u_2 = 4$ and $u_3 = 14$.

Find the recurrence relation described by the sequence and use it to find the value of u_6.

(b) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 10, u_2 = 35$ and $u_3 = 47 \cdot 5$.

Find the recurrence relation described by the sequence and use it to find the value of u_6.

(c) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 5, u_2 = 9 \cdot 5$ and $u_3 = 20 \cdot 75$

Find the recurrence relation described by the sequence and use it to find the value of u_6.

(d) A sequence is defined by the recurrence relation $u_{n+1} = mu_n + c$

Where m and c are constants.

It is known that $u_1 = 12, u_2 = 10$ and $u_3 = 8$.

Find the recurrence relation described by the sequence and use it to find the value of u_6.

8. (a) On a particular day at 07:00, a doctor injects a first dose of 300mg of medicine into a patient's bloodstream. The doctor then continues to administer the medicine in this way at 07:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 20% of what it was at the start.

(i) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 390mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

(b) On a particular day at 06:00, a doctor injects a first dose of 150mg of medicine into a patient's bloodstream. The doctor then continues to administer the medicine in this way at 06:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 10% of what it was at the start.

(i) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 170mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

(c) On a particular day at 09:00, a doctor injects a first dose of 50mg of medicine into a patient's bloodstream. The doctor then continues to administer the medicine in this way at 09:00 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be 25% of what it was at the start.

(i) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 70mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

(d) On a particular day at 08:30, a doctor injects a first dose of 225mg of medicine into a patient's bloodstream. The doctor then continues to administer the medicine in this way at 08:30 each day.

The doctor knows that at the end of the 24-hour period after an injection, the amount of medicine in the bloodstream will only be17% of what it was at the start.

(i) Set up a recurrence relation which shows the amount of medicine in the bloodstream immediately after an injection.

The patient will overdose if the amount of medicine in their bloodstream exceeds 275mg.

(ii) In the long term, if a patient continues with this treatment, is there a danger they will overdose?
Explain your answer.

9. (a) A box with a square base and open top has a surface area of $192cm^{2}$. The volume of the box can be represented by the formula:

$$V(x) = 48x - \frac{1}{4}x^3 \text{ where } x > 0$$

Find the value of x which maximises the volume of the box.

(b) A box with a square base and open top has a surface area of $972cm^{2}$. The volume of the box can be represented by the formula:

$$V(x) = 243x - \frac{1}{4}x^3 \text{ where } x > 0$$

Find the value of x which maximises the volume of the box.

(c) A box with a square base and open top has a surface area of $432cm^{2}$. The volume of the box can be represented by the formula:

$$V(x) = 108x - \frac{1}{4}x^3 \text{ where } x > 0$$

Find the value of x which maximises the volume of the box.

(d) A box with a square base and open top has a surface area of $484cm^{2}$. The volume of the box can be represented by the formula:

$$V(x) = 121x - \frac{1}{4}x^3 \text{ where } x > 0$$

Find the value of x which maximises the volume of the box.

10. (a) The curve with equation $y = x^2(3 - x)$ is shown below.

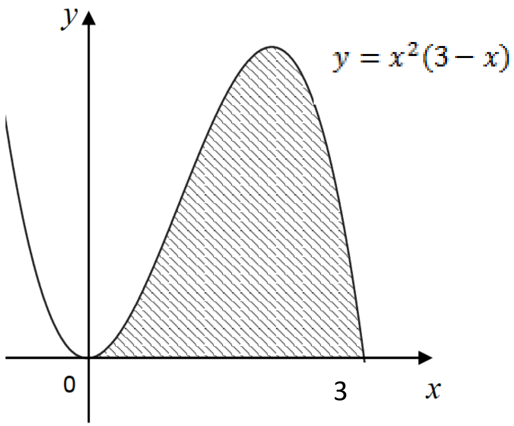


Diagram 1

Calculate the shaded area.

(b) The curve with equation $y = x^2(5 - x)$ is shown below.

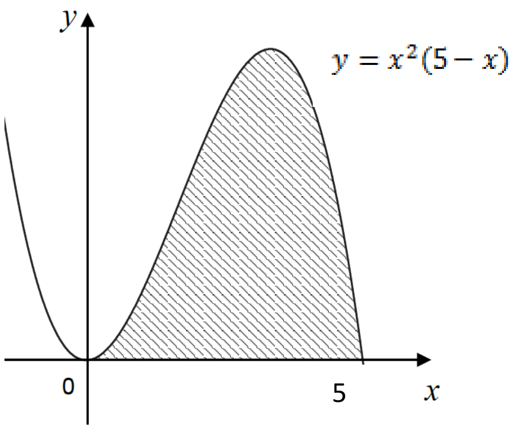


Diagram 1

Calculate the shaded area.

(c) The curve with equation $y = x^2(6 - x)$ is shown below.

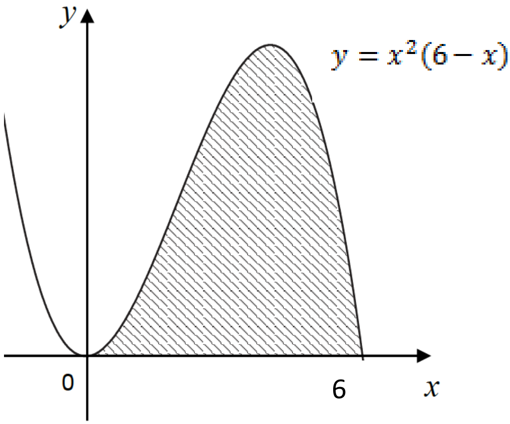


Diagram 1

Calculate the shaded area.

(d) The curve with equation $y = x^2(10 - x)$ is shown below

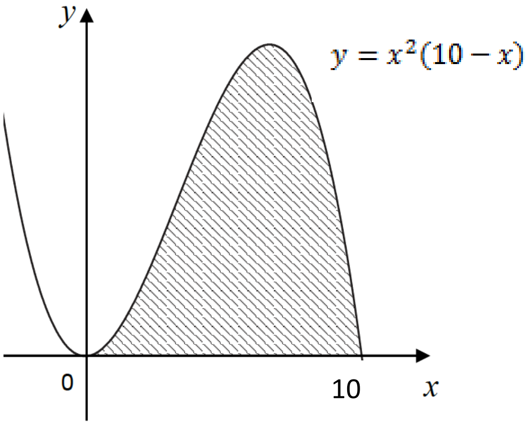
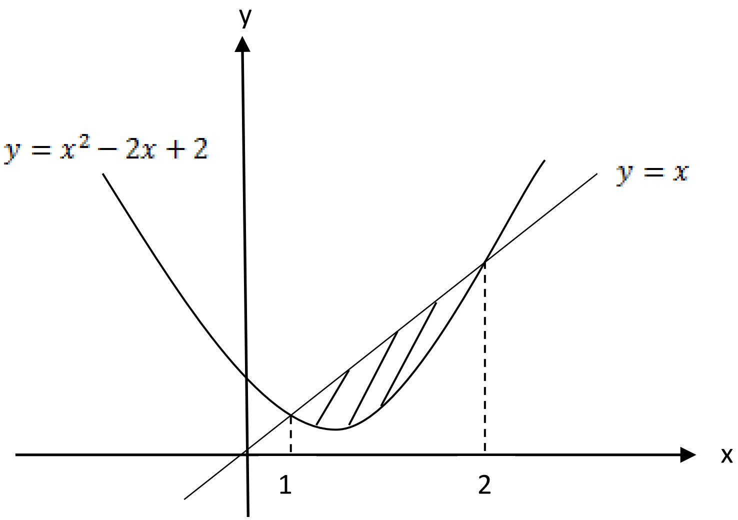


Diagram 1

Calculate the shaded area.

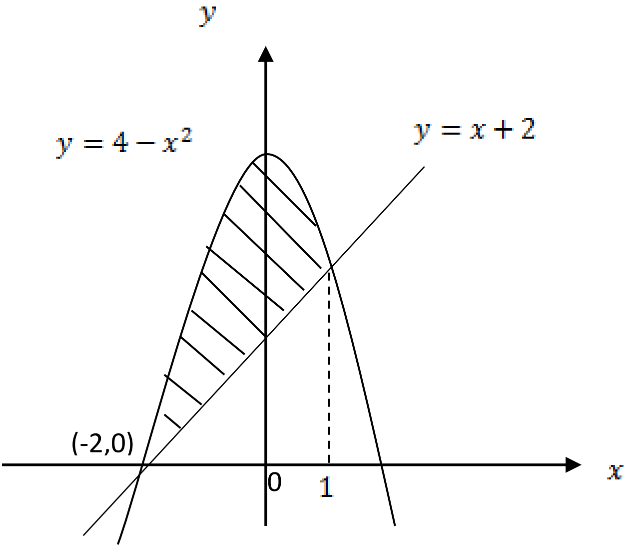
11. (a) The line with equation $y = x$ and the curve with equation $y = x^2 - 2x + 2$ are shown below



The line and the curve meet at the points where $x = 1$ and $x = 2$.

Calculate the shaded area.

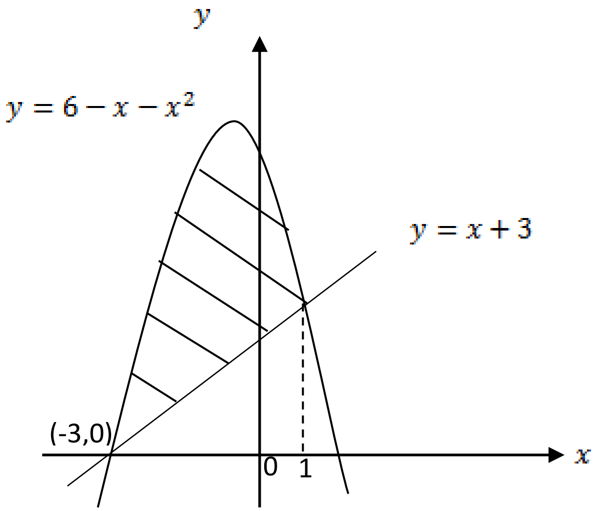
(b) The line with equation $y = x + 2$ and the curve with the equation $y = 4 - x^2$ are shown below.



The line and the curve meet at the points where $x = -2$ and $x = 1$.

Calculate the shaded area.

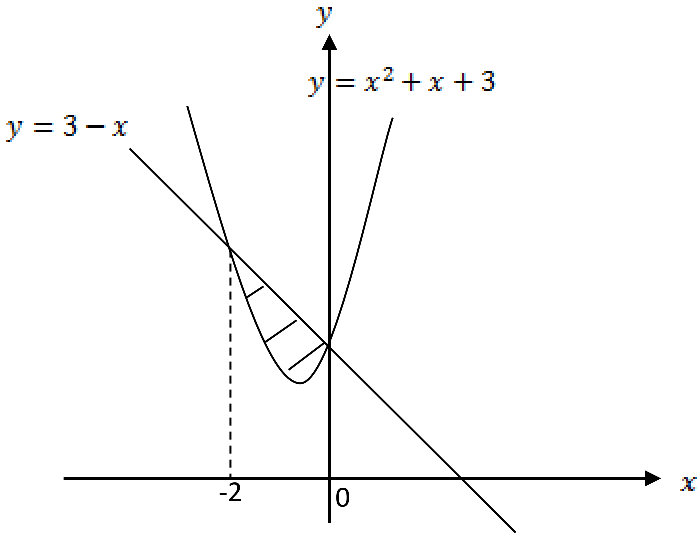
(c) The line with equation $y = x + 3$ and the curve with equation $y = 6 - x - x^2$ are shown below.



The line and the curve meet at the points where $x = -3$ and $x = 1$.

Calculate the shaded area.

(d) The line with equation $y = 3 - x$ and the curve with equation $y = x^2 + x + 3$ are shown below.



The line and the curve meet at the points where $x = -2$ and $x = 0$.

Calculate the shaded area.

ANSWERS

1 (a) $y = -5x + 12$ (b) $y = -4x$ (c) $y = -3x + 14$ (d) $y = -5x + 8$

2 (a) $4y = -x - 10$ (b) $2y = 5x + 13$ (c) $y = x + 7$ (d) $2y = x + 19$

3 (a) 1.2 (b) 0.18 (c) 2.4 (d) -0.84

4 (a) Line meets at $x = -4,3$ => Not a tangent.

(b) Line meets at $x = -3,3$ => Not a tangent.

(c) Line meets at $x = -2(twice)$ => A tangent.

(d) Line meets at $x = -2,1$ => Not a tangent.

5 (a) $(x - 1)^2 + (y + 2)^2 = 16$

(b) $(x - 5)^2 + (y - 6)^2 = 25$

(c) $(x + 3)^2 + (y + 7)^2 = 100$

6 (a) Yes (b) No (c) No (d) Yes

7 (a) $U_{n+1} = 5U_n - 6$ (ii) $U_6 = 1564$

(b) $U_{n+1} = 0 \cdot 5U_n + 30$ (ii) $U_6 = 58 \cdot 44$

(c) $U_{n+1} = 2 \cdot 5U_n - 3$ (ii) $U_6 = 294 \cdot 97$

(d) $U_{n+1} = U_n - 2$ (ii) $U_6 = 2$

8 (a) $U_{n+1} = 0 \cdot 2U_n + 300$ (ii) $L = 375$ ∴ No danger

(b) $U_{n+1} = 0 \cdot 1U_n + 150$ (ii) $L = 166.67$ ∴ No danger

(c) $U_{n+1} = 0 \cdot 25U_n + 50$ (ii) $L = 66.67$ ∴ No danger

(d) $U_{n+1} = 0 \cdot 17U_n + 225$ (ii) $L = 271\frac{7}{83}$ ∴ No danger

9 (a) Max at $x = 8$

(b) Max at $x = 18$

(c) Max at $x = 12$

(d) Max at $x = 12 \cdot 7$

10 (a) $6\frac{3}{4}$ (b) $52\frac{1}{12}$ (c) 108 (d) $833\frac{1}{3}$

11 (a) $\frac{1}{6}$ (b) $4\frac{1}{2}$ (c) $10\frac{2}{3}$ (d) $\frac{4}{3}$