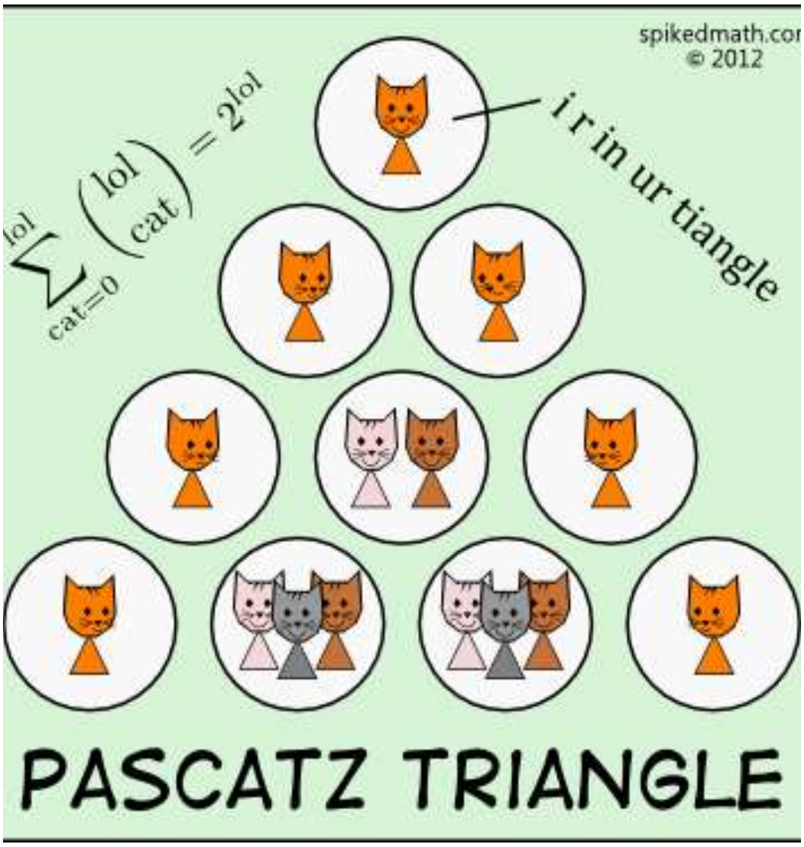
Advanced Higher Mathematics

UCS

Binomial Theorem



The factorial function

Example 1 Six cyclists enter a race. All six finish at *different times. There is a 1st and a 2nd prize.*

a How many different ways can the cyclists finish the race?
b How many different ways can the prizes be awarded?

a There are 6 different possible winners.
For each possibility there are 5 possible *seconds*: $6 \times 5 = 30$ ways of getting a 1st and a 2nd.
For each of these, there are 4 possible *thirds*: $6 \times 5 \times 4 = 120$ ways of getting a 1st, 2nd and 3rd.
For each of these, there are 3 possible *fourths*: $6 \times 5 \times 4 \times 3 = 360$.
For each of these, there are 2 possible *fifths*: $6 \times 5 \times 4 \times 3 \times 2 = 720$.
For each of these, there is only 1 possible *last*: $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$.
Therefore there are 720 different ways the cyclists can finish the race.

b Looking at the working above we see that there are $6 \times 5 = 30$ ways of getting a 1st and a 2nd prize.

Calculations similar to $6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$ appear often enough for there to be a special function created, to represent them. $n!$, read as *n factorial*, is defined as $n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$. For example, $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$

Most calculators carry this function. $\boxed{n!}$

Part **b** of Problem 1 can be answered using factorials when you note that

$$6 \times 5 = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1} = \frac{6!}{4!}$$

This calculation is a special function ${}^nP_r$ and is also supported by most calculators: look for $\boxed{{}^nP_r}$. It calculates the number of ways of arranging r objects when they are first to be selected from a pool of n objects.

Example **1b** required the number of ways of arranging 2 cyclists when they are first to be selected from a pool of 6 cyclists.

$${}^6P_2 = 6! \div (6-2)! = 30$$

$$\boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

P refers to the word *Permutation* which means *arrangement.*

Example 2 From a palette of 7 colours you can pick 4 to blend. How many ways can this be done?

The order in which the colours are blended does not matter: *red, blue, yellow, green* is the same choice as *red, blue, green, yellow.*

So, in this example, we are selecting 4 colours from a pool of 7 but the arrangement (order) of the 4 does not matter.

There are $4 \times 3 \times 2 \times 1 = 4! = 24$ ways of arranging 4 colours, so ${}^7P_4$, which includes all of these ways, will be 24 times too big.

We get our desired answer by performing the calculation ${}^7P_4 \div 4! = 35$

This also is a very common type of problem and this calculation has a special function ${}^nC_r$, supported by most calculators: look for $\boxed{{}^nC_r}$.

$$\boxed{{}^nC_r = \frac{n!}{r!(n-r)!}}$$

C refers to the word *Combination* which is *selection without arrangement.*

Example 3 *Hobson's Choice.* Hobson hired out horses. You paid your money and got the horse of your choice – as long as you chose the horse he offered you. How many ways can you pick one horse when there is only one horse from which to pick?

$${}^1C_1 = \frac{1!}{1!(1-1)!} = \frac{1!}{1! \times 0!}$$

We know the answer is that there is only one way, so $\frac{1!}{1! \times 0!} = 1$

For this to make sense we must give a value of 1 to 0!.

As a definition, we have: $0! = 1$.

Note

- the domain of the factorial function is W, the set of whole numbers.
- $n!$, ${}^nP_r$ and ${}^nC_r$ on a graphics calculator are generally found in the MATHS menu.

EXERCISE 2A

1 a Use your calculator to obtain values for **(i)** 4! **(ii)** 6! **(iii)** 0!

b What happens when you attempt to get **(i)** –4! **(ii)** (–4)! **(iii)** 4.2!

2 a The combination on the lock on my case uses the four digits 1, 2, 3 and 4. I've forgotton the order in which they appear. How many different ways can they be arranged?

b In a game of *Scrabble* a player has 7 different letters. He rearranges them, looking for words. How many different arrangements can he make of the 7 letters?

c A pack of cards has 52 different cards. Calculate the number of ways these can be arranged, giving your answer correct to three significant figures.

3 a Evaluate **(i)** $\frac{7!}{3!}$ **(ii)** $\frac{10!}{6!}$ **(iii)** $\frac{12!}{11!}$

b Can you account for the fact that the values of these divisions are all exact?

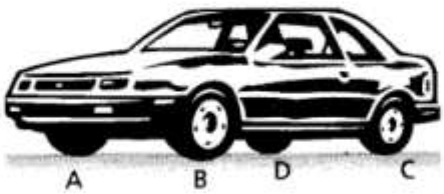
c Use the ${}^nP_r$ facility on your calculator to evaluate the same expressions.

Hint
The first one will require ${}^7P_4$.

4 a From a class of 23 students, 3 have to be selected to be class representative, secretary and treasurer of the newly formed student committee. In how many different ways can this be done?

b At the bank, a customer invents her own personal identification number (PIN) by choosing four different digits. Given that there are 10 different digits, how many arrangements of 4 different digits can be made? An arrangement may start with zero.

c A driver has 5 tyres on his car: the 4 on the road and 1 in the boot. He rotates them regularly so that they wear evenly.



(i) How many different arrangements of 4 tyres on the road can he make, assuming position matters?

(ii) How many different arrangements of one tyre in the boot can he make?

(iii) Comment on your answers.

5 a Evaluate **(i)** $\frac{7!}{4!3!}$ **(ii)** $\frac{10!}{6!4!}$ **(iii)** $\frac{12!}{11!1!}$ **(iv)** $\frac{7!}{3!4!}$ **(v)** $\frac{10!}{4!6!}$

b The answers are all integers. Can you explain this?

c Each expression in **a** is the expansion of a function of the form ${}^nC_r$. Express each expansion in this form.

d Explain why ${}^nC_r = {}^nC_{n-r}$.

6 a (i) In the card game *Brag*, each player is given 3 cards. Assuming the pack is made up of 52 different cards, how many different hands are possible?

(ii) In the game *Bridge*, each player is given 13 cards. How many different hands are possible? Give your answer to three significant figures.

(iii) In the game *Solitaire*, each player is given 52 cards. How many different hands are possible?

Using a special notation, *sigma notation*, the theorem can be quoted in a very compact form

$$(x + y)^n = \sum_{r=0}^{n} \binom{n}{r} x^{n-r} y^r$$

Σ stands for *sum* and is pronounced *sigma*. It acts like an instruction set:

1 create terms using the formula given to the right of Σ by replacing r with each of the integers from 0 to n in turn;
2 add all these terms together.

Example 1 Expand $(1 + x)^5$ using the binomial theorem.

$$\binom{5}{0}1^5x^0 + \binom{5}{1}1^4x^1 + \binom{5}{2}1^3x^2 + \binom{5}{3}1^2x^3 + \binom{5}{4}1^1x^4 + \binom{5}{5}1^0x^5$$

Using the ${}^nC_r$ button on a calculator, or otherwise, we get

$$1 + 5x + 10x^2 + 10x^3 + 5x^4 + x^5$$

Example 2 Expand $(1 - 3p)^3$ using the binomial theorem.

$$\binom{3}{0}1^3(-3p)^0 + \binom{3}{1}1^2(-3p)^1 + \binom{3}{2}1^1(-3p)^2 + \binom{3}{3}1^0(-3p)^3$$
$$= 1 + 3(-3p) + 3(9p^2) + (-27p^3)$$
$$= 1 - 9p + 27p^2 - 27p^3$$

EXERCISE 3A

1 Use the binomial theorem to expand the following.

a $(a + b)^5$ **b** $(1 + 2x)^3$ **c** $(2 + 3b)^4$ **d** $(3a + 2b)^3$
e $(a - b)^4$ **f** $(1 - p)^3$ **g** $(3 - x)^4$ **h** $(2a - 3b)^3$

2 a Expand the following expressing your answer as positive powers of x.

(i) $\left(x + \frac{1}{x}\right)^3$ **(ii)** $\left(x + \frac{1}{x}\right)^4$ **(iii)** $\left(x - \frac{1}{x}\right)^5$ **(iv)** $\left(x - \frac{1}{x}\right)^6$

b Which of these expressions produced a term independent of x?

3 Work out

a the third term in the expansion of $(x + y)^{12}$, i.e. the term containing x^{10}
b the fourth term in the expansion of $(3 + a)^8$
c the seventh term in the expansion of $(2x + 3y)^9$
d the second term in the expansion of $(2x + 5)^7$
e the eighth term in the expansion of $(x - y)^9$
f the fifth term in the expansion of $(3x - 4y)^5$

4 Calculate the term

a containing x^4 in the expansion of $(x + y)^8$
b containing a^3 in the expansion of $(3 + 2a)^5$
c whose coefficient is 64 in the expansion of $(2 + x)^6$

d containing x^3 in the expansion of $(x - 7)^5$

e containing a^4 term in the expansion of $(1 - 3a)^6$

f independent of x in the expansion of $\left(x + \frac{1}{x}\right)^4$

5 **a** Expand $(1 + x + y)^3$ by first expressing it as $[(1 + x) + y]^3$.

b In a similar fashion expand **(i)** $(2 + a + 2b)^3$ **(ii)** $(1 - x + y)^5$ **(iii)** $(1 + x - y)^4$

6 By considering $(1 + x)^n$, prove that $2^n = {}^nC_0 + {}^nC_1 + {}^nC_2 + \cdots + {}^nC_n$.

7 Every quadratic expression can be written in the form $a(x + b)^2 + c$ by a process known as *completing the square*.
By considering a similar process write $x^3 + 6x^2 + 10x + 4$ in the form $(x + a)^3 + bx + c$

8 *Remember, Pascal used the binomial theorem for probability theory. In what way?*
If p is the probability of being stopped at any one set of traffic lights and q is the probability of not being stopped, then the terms of the expansion $(p + q)^3$ provide formulae for the probability of being stopped by 3, 2, 1, 0 sets of lights.

$$(p + q)^3 = p^3 + 3p^2q + 3pq^2 + q^3$$

$P(3) = p^3$, $P(2) = 3p^2q$, $P(1) = 3pq^2$, $P(0) = q^3$

a If $p = 0.8$ and $q = 0.2$ calculate the probability of being stopped at
(i) two sets of lights **(ii)** all three sets of lights

b By expanding $(p + q)^4$, find the probability of being stopped at 2 out of 4 sets of lights.

9 The probability that it will rain on any day is 0.4, and that it won't rain is 0.6.
By considering the expansion $(p + q)^7$, calculate the probability that

a it will rain twice in a 7 day week.

b it won't rain in the week.

Harder examples

Example 1 What is the coefficient of x^5 in the expansion of $(1 + x)^4(1 - 2x)^3$?

Terms in x^5 are obtained by multiplying certain terms together, namely:
the term containing x^2 in the first expansion with the term containing x^3 in the second,
the term containing x^3 in the first expansion with the term containing x^2 in the second,
the term containing x^4 in the first expansion with the term containing x in the second.

$$\binom{4}{2}x^2\binom{3}{0}x^3 + \binom{4}{1}x^3\binom{3}{1}x^2 + \binom{4}{0}x^4\binom{3}{2}x$$

$$= 6x^2.x^3 + 4x^3.3x^2 + x^4.3x$$

$$= 21x^5$$

The required coefficient is therefore 21.

Example 2 Expand $(x+1)^2(1+2x+x^2)^3$.

Consider each set of brackets separately:

$$(x+1)^2 = \binom{2}{0}x^2 + \binom{2}{1}x + \binom{2}{2}1 = x^2 + 2x + 1$$

$$\begin{aligned}(1+2x+x^2)^3 &= ((1+2x)+x^2)^3 \\ &= \binom{3}{0}(1+2x)^3 + \binom{3}{1}(1+2x)^2(x^2) + \binom{3}{2}(1+2x)(x^2)^2 + \binom{3}{3}(x^2)^3 \\ &= (1+6x+12x^2+8x^3) + 3(1+4x+4x^2)x^2 + 3(1+2x)x^4 + x^6 \\ &= 1+6x+15x^2+20x^3+15x^4+6x^5+x^6\end{aligned}$$

$$(x+1)^2(1+2x+x^2)^3 = (x^2+2x+1)(1+6x+15x^2+20x^3+15x^4+6x^5+x^6)$$

The expansion can be made clearer by a tabular layout.

multiplying by x^2:	$x^2 + 6x^3 + 15x^4 + 20x^5 + 15x^6 + 6x^7 + x^8$
multiplying by $2x$:	$2x + 12x^2 + 30x^3 + 40x^4 + 30x^5 + 12x^6 + 2x^7$
multiplying by 1:	$1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$
Total:	$1 + 8x + 28x^2 + 56x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + x^8$

Therefore

$$(x+1)^2(1+2x+x^2)^3 = 1 + 8x + 28x^2 + 56x^3 + 70x^4 + 56x^5 + 28x^6 + 8x^7 + x^8$$

EXERCISE 3B

1 What is the coefficient of

a x^4 in the expansion $(1+x)^2(1+2x)^3$

b x^5 in the expansion $(1-x)^3(2+x)^4$

c x^7 in the expansion $(1+2x)^4(1-2x)^6$?

2 Find the coefficients of x^3 and x^5 in the expansion of $(1+x+x^2)^5$.

3 What are the terms in x^3 and x^{10} in $(1+x)^5(1-x+x^2)^4$?

4 Expand

a $(1+x)^2(1+x+x^2)^3$

b $(1-3x)^3(1+2x+x^2)^2$

c $(3+2x)^2(1-x+x^2)^4$

d $(1-2x)^4(1+2x+4x^2)^3$

5 **a** Expand $\left(x+\frac{1}{x}\right)^2\left(x-\frac{1}{x}\right)^3$.

b Expand $\left(x+\frac{1}{x}\right)^2\left(x-\frac{1}{x}\right)^4$.

c Under what circumstances do you get terms independent of x?

d Find the term independent of x in the expansion of $\left(x+\frac{1}{x}\right)^6\left(x-\frac{1}{x}\right)^8$.

6 Find the terms in a^5 and a^6 in $\left(3a^2-\frac{1}{a}\right)^6\left(a+\frac{1}{a}\right)^4$.

7 Find the term independent of a in $\left(\frac{3}{2}a^2 - \frac{1}{3a}\right)^9$.

8 **a** Use the binomial theorem to help you write down expressions for the coefficients of x^r and x^{r+1} in the expansion of $(3x + 2)^{19}$.

b Find the value of r if these coefficients are equal.

9 If $x = \frac{1}{4}$, find the ratio of the 8th and 7th terms in the expansion of $(1 + 2x)^{15}$.

10 Which are the greatest terms in the following expansions?

a $(1 + 3x)^{18}$ when $x = \frac{1}{4}$

b $\left(1 + \frac{1}{2}x\right)^{12}$ when $x = \frac{1}{2}$

c $(4 + x)^8$ when $x = 3$

d $(x + y)^n$ when $n = 14$, $x = 2$, $y = \frac{1}{2}$

11 Find the numerically greatest terms in the following expansions.

a $(2 - x)^{12}$ when $x = \frac{2}{3}$

b $(3a + 2b)^n$ when $n = 16$, $a = 1$, $b = \frac{1}{2}$

12 Find which terms have the greatest coefficients in:

a $(1 + x)^{10}$

b $(2 + x)^{11}$

c $(1 + x)^{2n+1}$

13 In the following expansion, show that there are two greatest terms and find their values.

$$(a + x)^n \text{ when } n = 9,\ a = \tfrac{1}{2},\ x = \tfrac{1}{3}$$

14 Find the coefficients of x^2 and x^3 in the expansion of $(2 + 2x + x^2)^n$

15 Expand $(1 - x + x^2)^n$ in ascending powers of x as far as the term in x^3.

Approximation

For $-1 < a < 1$, as $n \to \infty$ then $a^n \to 0$.
Using this fact allows us to make useful approximations.

Example 1 Calculate 1.02^3 correct to three decimal places.

$$\begin{aligned} 1.02^3 = (1 + 0.02)^3 &= 1 + 3 \times 0.02 + 3 \times 0.02^2 + 0.02^3 \\ &= 1 + 0.06 + 0.0012 + 0.000\,008 \\ &= 1.061 \text{ to 3 d.p.} \end{aligned}$$

Note that the term 0.02^3 does not contribute to the rounded result.

Example 2 Calculate 0.9^7 correct to two decimal places.

$$\begin{aligned} 0.9^7 = (1 - 0.1)^7 &= 1 - 7 \times 0.1 + 21 \times 0.1^2 - 35 \times 0.1^3 + 35 \times 0.1^4 \ldots \\ &= 1 - 0.7 + 0.21 - 0.035 + 0.0035 \ldots \\ &= 0.48 \text{ to 2 d.p.} \end{aligned}$$

Note that the terms in 0.1^4 and higher do not contribute to the rounded result.

Answers

CHAPTER 1.1

Exercise 1 (page 1)

1 **a** Freq. 1, 3, 3, 1 **b** Freq. 1, 4, 6, 4, 1

2 **a** **(i)** $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
(ii) $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
(iii) $x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6$

3 **a** **(i)** $u_n = \frac{1}{2}n(n-1)$
(ii) $u_n = \frac{1}{6}n(n-1)(n-2)$
b **(i)** $x^8 + 8x^7y + 28x^6y^2 + 56x^5y^3 + 70x^4y^4 + 56x^3y^5 + 28x^2y^6 + 8xy^7 + y^8$
(ii) $x^9 + 9x^8y + 36x^7y^2 + 84x^6y^3 + 126x^5y^4 + 126x^4y^5 + 84x^3y^6 + 36x^2y^7 + 9xy^8 + y^9$
(iii) $x^{10} + 10x^9y + 45x^8y^2 + 120x^7y^3 + 210x^6y^4 + 252x^5y^5 + 210x^4y^6 + 120x^3y^7 + 45x^2y^8 + 10xy^9 + y^{10}$
c **(i)** 1, 5, 10, 10, 5, 1; $(n, r) = (n, n-r)$
(ii) No; (a, b) is only defined for $b \le a$.
d **(i)** $(n, 2) + (n+1, 2)$
$= \frac{1}{2}n(n-1) + \frac{1}{2}n(n+1) = n^2$

Exercise 2A (page 5)

1 **a** **(i)** 24 **(ii)** 720 **(iii)** 1
b **(i)** −24 **(ii)** error **(iii)** error

2 **a** 24 **b** 5040 **c** 8.07×10^{67}

3 **a** **(i)** 840 **(ii)** 5040 **(iii)** 12
b since denominator is shorter than the numerator, all its terms are included in the numerator and so divides.

4 **a** 10 626 **b** 5040
c **(i)** 120 **(ii)** 1

5 **a** **(i)** 35 **(ii)** 210 **(iii)** 12 **(iv)** 35 **(v)** 210
b proof
c ${}^7C_4$, ${}^{10}C_6$, ${}^{12}C_{11}$, ${}^7C_3$, ${}^{10}C_4$
d ${}^nC_r = \frac{n!}{r!(n-r)!}$;
${}^nC_{n-r} = \frac{n!}{(n-r)!(n-(n-r))!} = \frac{n!}{r!(n-r)!}$

6 **a** **(i)** 22 100 **(ii)** 635 000 000 000 **(iii)** 1
b 13 983 816
c $\frac{1}{66}$

Exercise 2B (page 7)

1 **a** 210 **b** 45 **c** 120

2 **a** 55 **b** 55 **c** ${}^{11}C_2 = {}^{11}C_9$

3 **a** **(i)** 10 **(ii)** 5 **(iii)** 5
b n sides means n vertices. Number of joins $= {}^nC_2$. Number of sides $= n$.
So number of diagonals $= {}^nC_2 - n$
c $n = 6$

4 **a** 4 **b** 10 **c** 8 **d** 16
e 3 **f** 5 **g** 6 **h** 12

5 **a** $n = 4$ **b** $n = 5$ **c** $n = 7$
d $n = 10$

6 **a** $n = 6$ **b** $n = 11$ **c** $n = 9$

7 **a** $n = 7$ **b** $n = 10$ **c** $n = 1$ or 4
d $n = 9$

Exercise 3A (page 9)

1 **a** $a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$
b $1 + 6x + 12x^2 + 8x^3$
c $16 + 96b + 216b^2 + 216b^3 + 81b^4$
d $27a^3 + 54a^2b + 36ab^2 + 8b^3$
e $a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$
f $1 - 3p + 3p^2 - p^3$
g $81 - 108x + 54x^2 - 12x^3 + x^4$
h $8a^5 - 36a^2b + 54ab^2 - 27b^3$

2 **a** **(i)** $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$
(ii) $x^4 + 4x^2 + 6 + \frac{4}{x^2} + \frac{1}{x^4}$
(iii) $x^5 - 5x^3 + 10x - \frac{10}{x} + \frac{5}{x^3} - \frac{1}{x^5}$
(iv) $x^6 - 6x^4 + 15x^2 - 20 + \frac{15}{x^2} - \frac{6}{x^4} + \frac{1}{x^6}$
b expansions with even powers

3 **a** $66x^{10}y^2$ **b** $13\,608a^3$ **c** $489\,888x^3y^6$
d $2240x^6$ **e** $-36x^2y^7$ **f** $3840xy^4$

4 **a** $70x^4y^4$ **b** $720a^3$ **c** second term
d $490x^3$ **e** $1215a^4$ **f** 6

5 **a** $1 + 3x + 3x^2 + x^3 + 3y + 6xy + 3x^2y + 3y^2 + 3xy^2 + y^3$
b **(i)** $8 + 12a + 6a^2 + a^3 + 24b + 24ab + 6a^2b + 24b^2 + 12ab^2 + 8b^3$
(ii) $1 - 5x + 10x^2 - 10x^3 + 5x^4 - x^5 + 5y - 20xy + 30x^2y - 20x^3y + 5x^4y + 10y^2 - 30xy^2 + 30x^2y^2 - 10x^3y + 10y^3 - 20xy^3 + 10x^2y^3 + 5y^4 - 5xy^4 + y^5$
(iii) $1 + 4x - 4y + 6x^2 - 12xy + 6y^2 + 4x^3 - 12x^2y + 12xy^2 - 4y^3 + x^4 - 4x^3y + 6x^2y^2 + y^4$

6 Let $x = 1$

7 $4 + 10x + 6x^2 + x^3 = (x+2)^3 - 2x - 4$

8 **a** **(i)** 0.384 **(ii)** 0.512 **b** 0.1536

9 **a** 0.261 2736 **b** 0.027 993 6

Exercise 3B (page 11)

1 a 28 b 3 c 1024

2 $30x^3$; $51x^5$

3 $4x^3$; $4x^{10}$

4 a $1 + 5x + 13x^2 + 22x^3 + 26x^4 + 22x^5 + 13x^6 + 5x^7 + x^8$

b $1 - 5x - 3x^2 + 31x^3 + 19x^4 - 63x^5 - 81x^6 - 27x^7$

c $9 - 24x + 46x^2 - 40x^3 + 19x^4 + 20x^5 - 26x^6 + 20x^7 + x^8 - 4x^9 + 4x^{10}$

d $1 - 2x - 24x^3 + 48x^4 + 192x^6 - 384x^7 - 512x^9 + 1024x^{10}$

5 a $x^5 - x^3 - 2x + \frac{2}{x} + \frac{1}{x^3} - \frac{1}{x^5}$

b $x^6 - 2x^4 - x^2 + 4 - \frac{1}{x^2} - \frac{2}{x^4} + \frac{1}{x^6}$

c when the powers have the same parity

d −40

6 $-3618a^5$, $-7290a^6$

7 $\frac{7}{18}$

8 a ${}^{19}C_r 3^r 2^{19-r}$ ${}^{19}C_{r+1} 3^{r+1} 2^{18-r}$

b $r = 11$

9 $u_8 : u_7 = 9 : 14$

10 a 4380.740 937 b 11.125

c 1 548 288

d term in x and x^2 are both 372 736 when $x = 2$; term in x^{12} and x^{11} are both 93 184 when $y = \frac{1}{2}$

11 a ${}^{12}C_3 2^9\left(-\frac{2}{3}\right)^3 = -33\,374.81$ (2dp)

b ${}^{12}C_4 3^{12} = 967\,222\,620$

12 a ${}^{10}C_5 x^5 = 252x^5$

b ${}^{11}C_4 2^7 x^4 = 42\,240x^4 = {}^{11}C_3 2^8 x^3$

c the terms in x^n and x^{n+1} will have the same coefficients

13 ${}^9C_3\left(\frac{1}{2}\right)^6\left(\frac{1}{3}\right)^3 = {}^9C_4\left(\frac{1}{2}\right)^5\left(\frac{1}{3}\right)^4 = \frac{7}{144}$

14 $x^2 \to n^2 2^{n-1}$; $x^3 \to n(n^2 - 1)2^n/6$

15 $1 - nx + \frac{1}{2}n(n+1)x^2 - \frac{1}{6}n(n-1)(n+4)x^3$

Exercise 4 (page 13)

1 a 1.05 b 1.26 c 0.648

d 250 000 e 2980 f 3020

g 0.0156 h 1 570 000 000

2 a $2x\delta x$ b $3x^2\delta x$

3 a $1 + 6a + 15a^2$ b $-1 + 14a - 84a^2$

c $256 + 1024a^2$

4 Expand brackets separately up to terms in x^2, then multiply.

5 $f'(x) \approx n.x^{n-1}$

6 a $V_n \approx 10\,000\,(1 - 0.1n)$

b $V_n \approx 5000\,(1 + 0.05n)$

Review (page 14)

1

1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	
1	6	15	20	15	6	1

2 a $p = 20$; $q = 5$; $r = 15$

b $s = 14$; $t = 6$

c (i) 38 760 (ii) 167 960

3 a $n = 5$ b $p = 8$; $q = 4$

4 a $x^5 - 20x^4 + 160x^3 - 640x^2 + 1280x - 1024$

b $8y^3 - 36y^2 + 54y - 27$

5 a $120x^7$ b $525y^3$

6 ${}^{10}C_5 x^5\left(\frac{-2}{x}\right)^5 = -8064$

7 a $2^{11} = 2048$

b (i) 1024 (ii) 1024

CHAPTER 1.2

Exercise 1 (page 17)

1 $x + 1 + \frac{3}{x+2}$

2 $x - 5 + \frac{19}{x+3}$

3 $x + 5 + \frac{5}{x-2}$

4 $3x + 7 + \frac{29}{x-4}$

5 $3 + \frac{2-7x}{x^2+x+1}$

6 $1 + \frac{3-2x}{x^2+x-2}$

7 $1 + \frac{x-2}{x^2-x+2}$

8 $x + 3 + \frac{3x-8}{x^2+1}$

9 $x^2 - 2x + 4 - \frac{7}{x+2}$

10 $3x^2 + 12x + 46 + \frac{188}{x-4}$

11 $1 + \frac{7}{x^2-4}$

12 $x^2 + x - \frac{3}{x^2+2x}$

13 $x^2 + 1 + \frac{x-x^2}{x^3-x+1}$

14 $3x - \frac{2x+1}{x^2+3}$

Exercise 2 (page 18)

1 $\frac{1}{x-1} + \frac{1}{x+1}$

2 $\frac{2}{x-2} + \frac{6}{x+3}$

3 $\frac{-1}{x+1} + \frac{5}{x+5}$

4 $\frac{4}{x-3} - \frac{4}{x+2}$

5 $\frac{1}{x-1} - \frac{1}{x+2}$

6 $\frac{5}{x+2} - \frac{5}{x+3}$

7 $\frac{4}{x-2} + \frac{1}{x-3}$

8 $\frac{2}{x+2} + \frac{1}{x-1}$