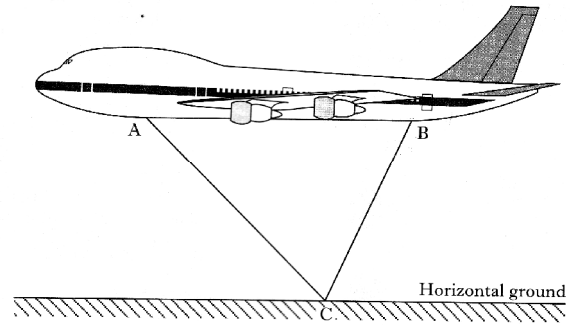


19. An aeroplane is flying parallel to the ground.

Lights have been fitted at A and B as shown in the diagram.

When the aeroplane is flying at a certain height, the beams from these lights meet exactly on the ground at C.



The angle of depression of the beam of light from A to C is 50° .

The angle of depression of the beam of light from B to C is 70° .

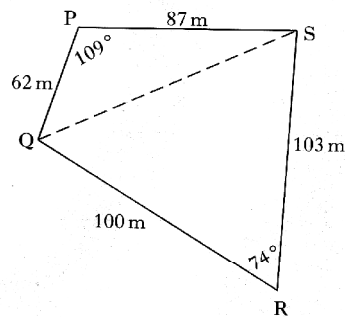
The distance AB is 20 metres.

Find the height of the aeroplane above C.

6 RE

20. The sketch shows a plot of ground, PQRS, split into two triangles.

Calculate the area of the plot of ground.



4 KU

21. The diagram shows the position of three airports, A, E and G.

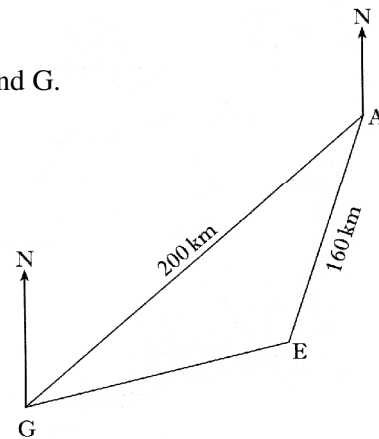
G is 200 kilometres from A

E is 160 kilometres from A

From G the bearing of A is 052°

From A the bearing of E is 216°

How far apart are airports G and E ?



6 RE

22. The side wall of a house, with measurements as shown in the diagram, requires painting.

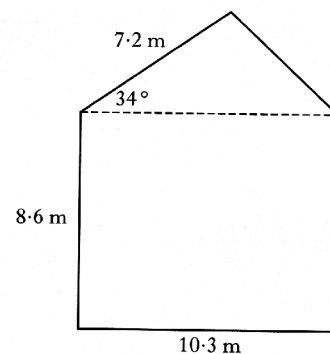
The wall is in the shape of a rectangle and a triangle.

On average, a litre of paint will cover 8 square metres.

A painter estimates that he will require 12 litres of paint.

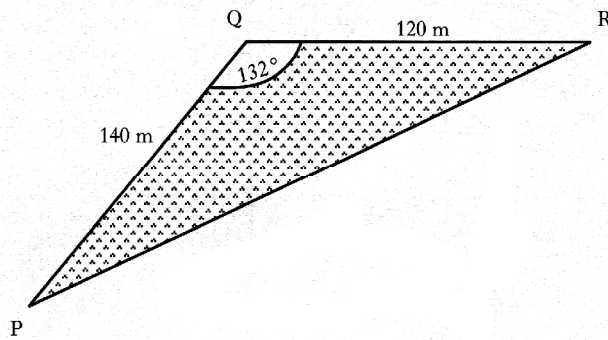
Will this be enough paint?

Justify your answer.



4 RE

23. A triangular field, PQR is shown in the diagram.
PQ = 140 metres,
QR = 120 metres
and angle PQR = 132°



Calculate the length of PR.

Do not use a scale drawing.

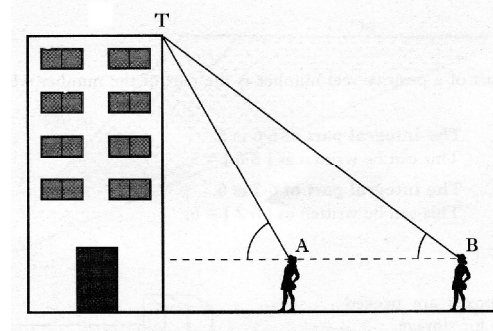
4 KU

24. The diagram shows two positions of a student as she views the top of a tower.
From position B, the angle of elevation to T at the top of the tower is 64° .

From position A, the angle of elevation to T at the top of the tower is 69° .

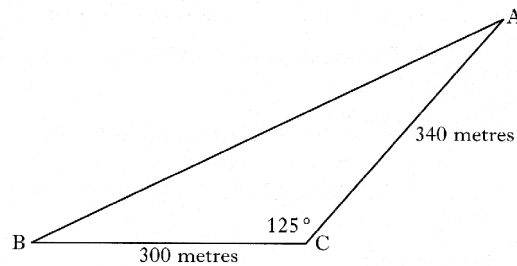
The distance AB is 4.8 metres
and the height of the student to eye level is 1.5 metres.

Find the height of the tower.



6 RE

25. A field, ABC, is shown in the diagram.



Find the area of the field.

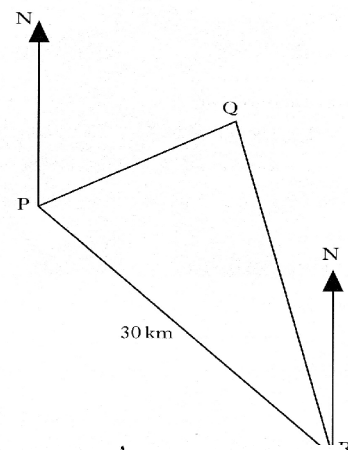
2 KU

26. A ship, at position P, observes a lighthouse at position Q on a bearing of 040° .

The ship travels 30 kilometres
on a bearing of 125° to position R.

From position R, the ship observes the lighthouse on a bearing of 340° .

When the ship is at position R,
how far is it from the lighthouse?

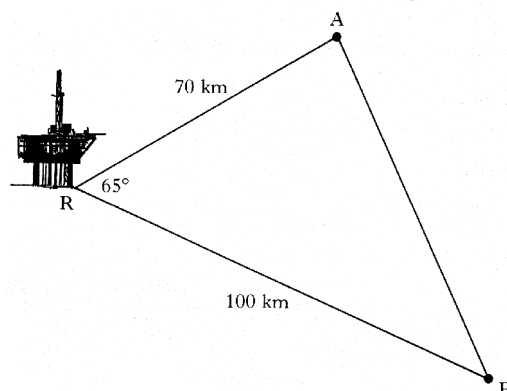


6 RE

27. The diagram shows the positions of an oilrig and two ships.
The oilrig at R is 70 kilometres
from a ship at A and 100 kilometres
from a ship at B. Angle ARB = 65° .

Calculate the distance AB.

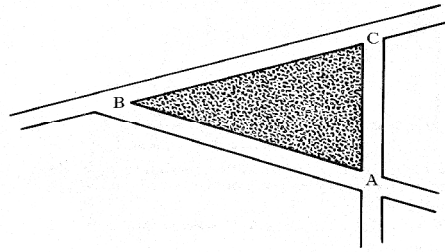
Do not use a scale drawing.



4 KU

28. A traffic island, ABC, is shown in the diagram.

Find the area of the traffic island if $AB = 12.6$ metres, $AC = 10$ metres and angle $BAC = 72^\circ$



2 KU

29. The diagram shows the goalposts on a rugby field.

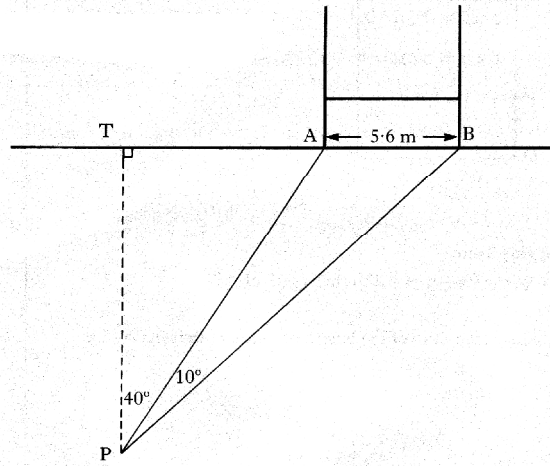
To take a kick at goal, a player moves from T to position P.

TP is perpendicular to TB.

Angle $TPA = 40^\circ$ and angle $APB = 10^\circ$

The distance AB between the goal posts is 5.6 metres.

Find the distance from T to P.

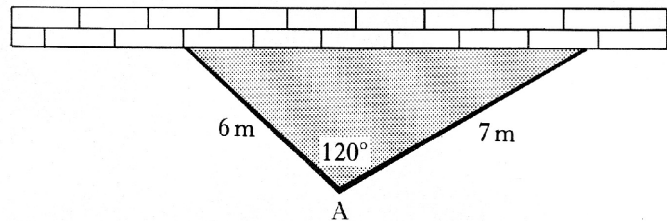


6 RE

30. A family wants to fence off a triangular part of their garden for their pet rabbits.

They have a long straight wall available and two straight pieces of fencing 6 metres and 7 metres in length.

They first erect the fencing as shown in the diagram.



- a) Find the area of garden enclosed by the wall and the two pieces of fencing.

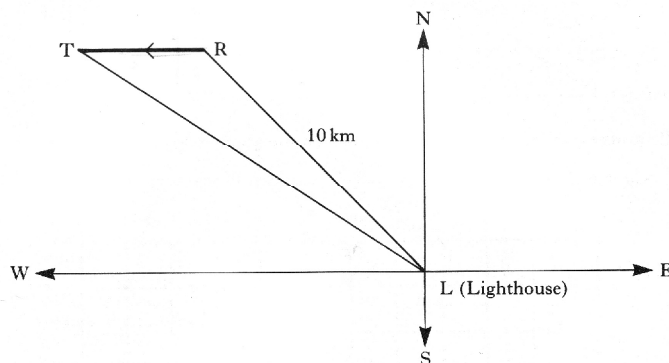
2 KU

- b) What size should they make the angle at A so that the greatest area of garden is enclosed?

Give a reason for your answer.

2 KU

31. A ship is first spotted at position R, which is on a bearing of 315° from a lighthouse, L. The distance between R and L is 10 kilometres. After the ship has travelled due West to position T, its bearing from the lighthouse is 300° .



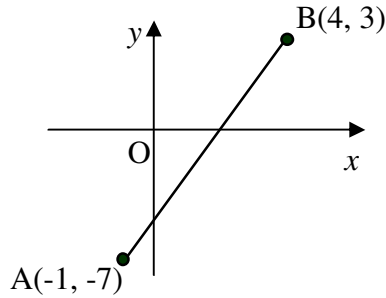
How far has the ship travelled from R to T?

5 RE

10. Gradients & The Straight Line

Finding Equations

1. In the diagram, A is the point $(-1, -7)$ and B is the point $(4, 3)$.
- Find the gradient of the line AB.
 - AB cuts the y-axis at the point $(0, -5)$. Write down the equation of the line AB.
 - The point $(3k, k)$ lies on AB. Find the value of k .



1 KU

1 KU

2 RE

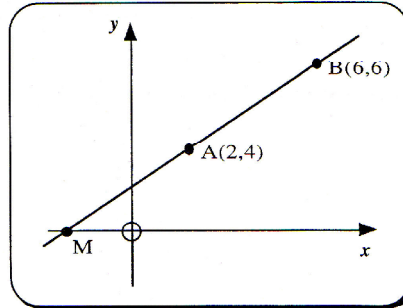
2. A is the point (a^2, a)
 T is the point (t^2, t) $a \neq t$
 Find the gradient of the line AT
 Give your answer in its simplest form.

3 KU

3. The straight line through the points $A(2, 4)$ and $B(6, 6)$ is shown in the diagram.

The point M is where the line AB cuts the x-axis.

- Find the equation of the straight line AB.
- Use this equation to find the coordinates of the point M.



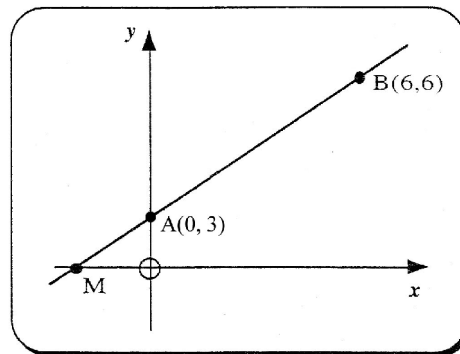
4 KU

2 RE

4. The straight line through the points $A(0, 3)$ and $B(6, 6)$ is shown in the diagram.

The point M is where the line AB cuts the x-axis.

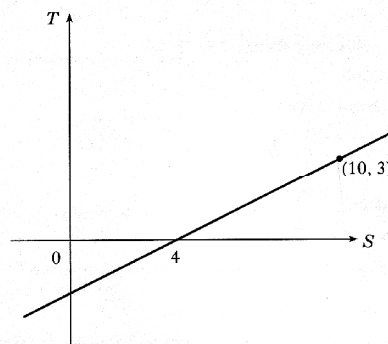
- Find the equation of the straight line AB.
- Use this equation to find the coordinates of the point M.



4 KU

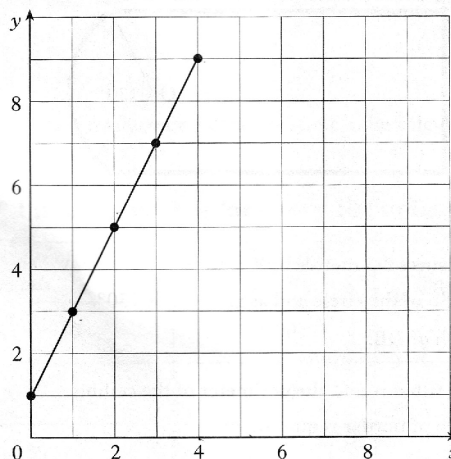
2 RE

5. Find the equation of the given straight line in terms of S and T.



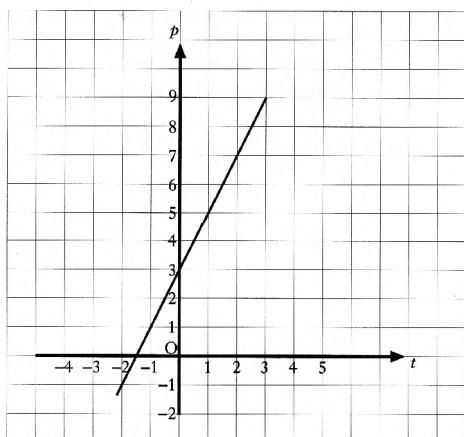
4 KU

6. Find the equation of the straight line.



3 KU

- 7.

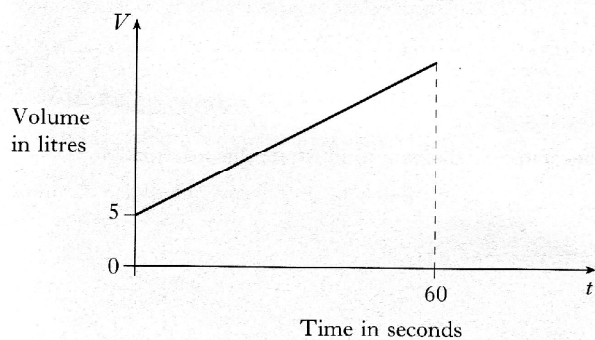


Find the equation of the straight line in terms of p and t .

4 KU

8. The tank of a car contains 5 litres of petrol.

The graph below shows how the volume of petrol in this tank changes as a further 45 litres of petrol is pumped in at a steady rate for 60 seconds.

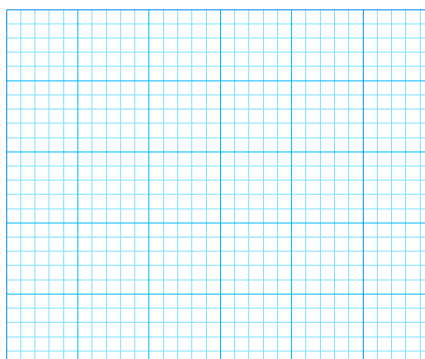


Find the equation of the straight line in terms of V and t .

4 KU

9. A tank contains 10 litres of water.
A further 30 litres of water is poured into the tank at a steady rate of 5 litres per minute.

a) On the 2mm square ruled graph paper provided, draw a graph of the volume, V litres, of water in the tank against the time, t minutes.



4 KU

b) Write down an equation connecting V and t .

2 KU

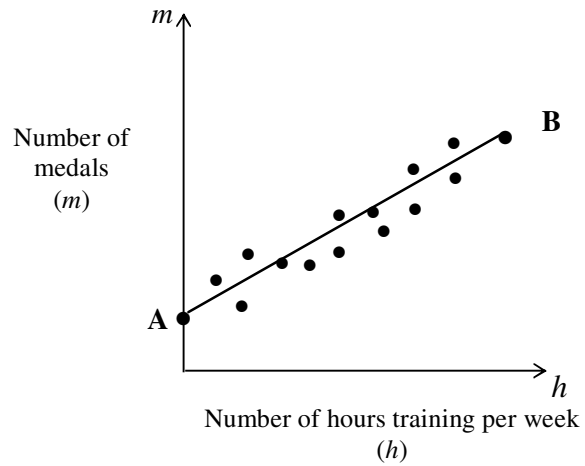
Applications of the Equation of a Straight Line

1. The graph shows the relationship between the number of hours (h) an athlete trains per week and the number of Championship medals (m) they have won.

A best fitting straight line AB has been drawn.

Athlete A does not train but has won 4 medals this year.

Athlete B who trains for 12 hours per week has won 40 medals this year.



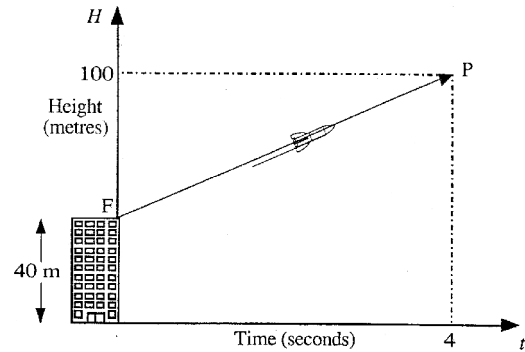
- (a) Find the equation of the straight line AB in terms of m and h . 4 RE
- (b) How many medals would you expect an athlete who trains 8 hours per week to have won? 1 KU

2. A boy sets off a rocket from the top of a 40 metre high block of flats.

The diagram shows the path of the rocket over the first 4 seconds.

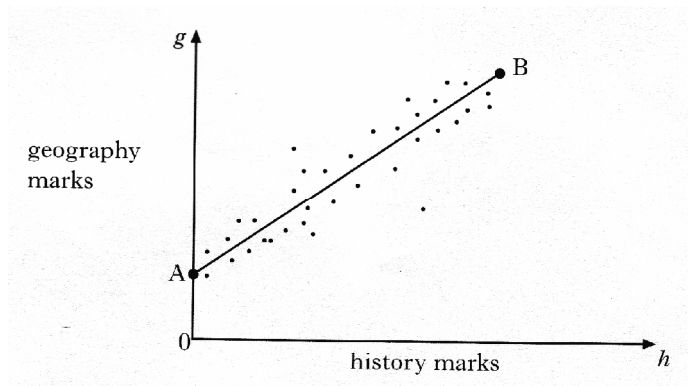
It is represented by the straight line in the graph.

After 4 seconds, the rocket has reached a point 100 metres above the ground.



Find the equation of the straight line FP in terms of H and t . 4 RE

3. The graph below shows the relationship between the history and geography marks of a class of students



A best fitting straight line, AB has been drawn.

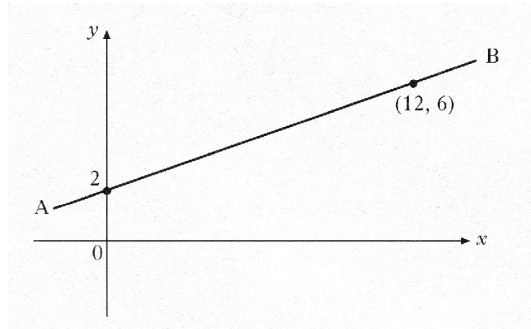
Point A represents 0 marks for history and 12 marks for geography.

Point B represents 90 marks for history and 82 marks for geography.

Find the equation of the straight line AB in terms of h and g .

4 RE

4. A water pipe runs between two buildings. These are represented by the points A and B in the diagram below.



- a) Using the information in the diagram, show that the equation of the line AB is $3y - x = 6$.
- b) An emergency outlet pipe has to be built across the main pipe. The line representing this outlet pipe has equation $4y + 5x = 46$

3 KU

Calculate the coordinates of the point on the diagram at which the outlet pipe will cut across the main water pipe.

4 RE

5. When a patient's blood pressure (B.P.), is taken, two measurements are made.

For example, in "160 over 70" (or $\frac{160}{70}$),

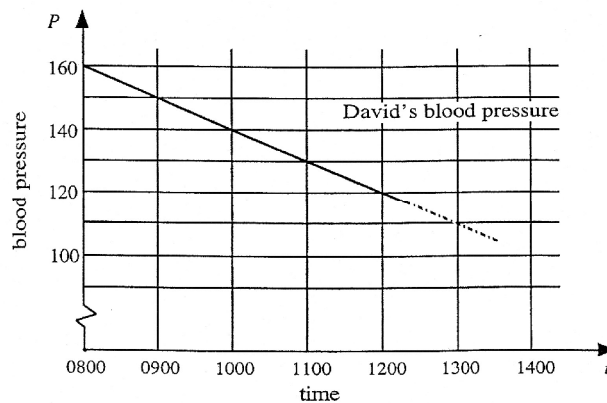
⇒ the 160 is the reading when the heart is pumping.

⇒ the 70 is the reading when the heart is at rest.



David has a heart problem, and has his blood pressure taken every hour.

The first number of these two measurements is monitored very carefully and the nurse plots a graph, showing the changes from 8 am.



- a) Find the gradient of the line shown above.
- b) Write down the equation of the line in the form $P = \dots\dots\dots$
- c) It is known that if the blood pressure drops below 70, the patient will be in danger of losing consciousness.

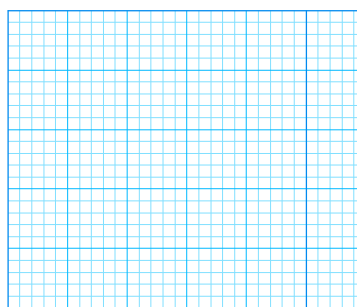
2 KU

2 KU

If David's blood pressure continues to drop in the way indicated, when might he be expected to become unconscious.

3 RE

6. A tank contains 240 litres of water.
When the tap is opened, water flows from the tank at a steady rate of 20 litres per minute.



- a) On the 2mm square-ruled paper provided, draw a graph of the volume V litres, of water in the tank against the time, t minutes.

3 KU

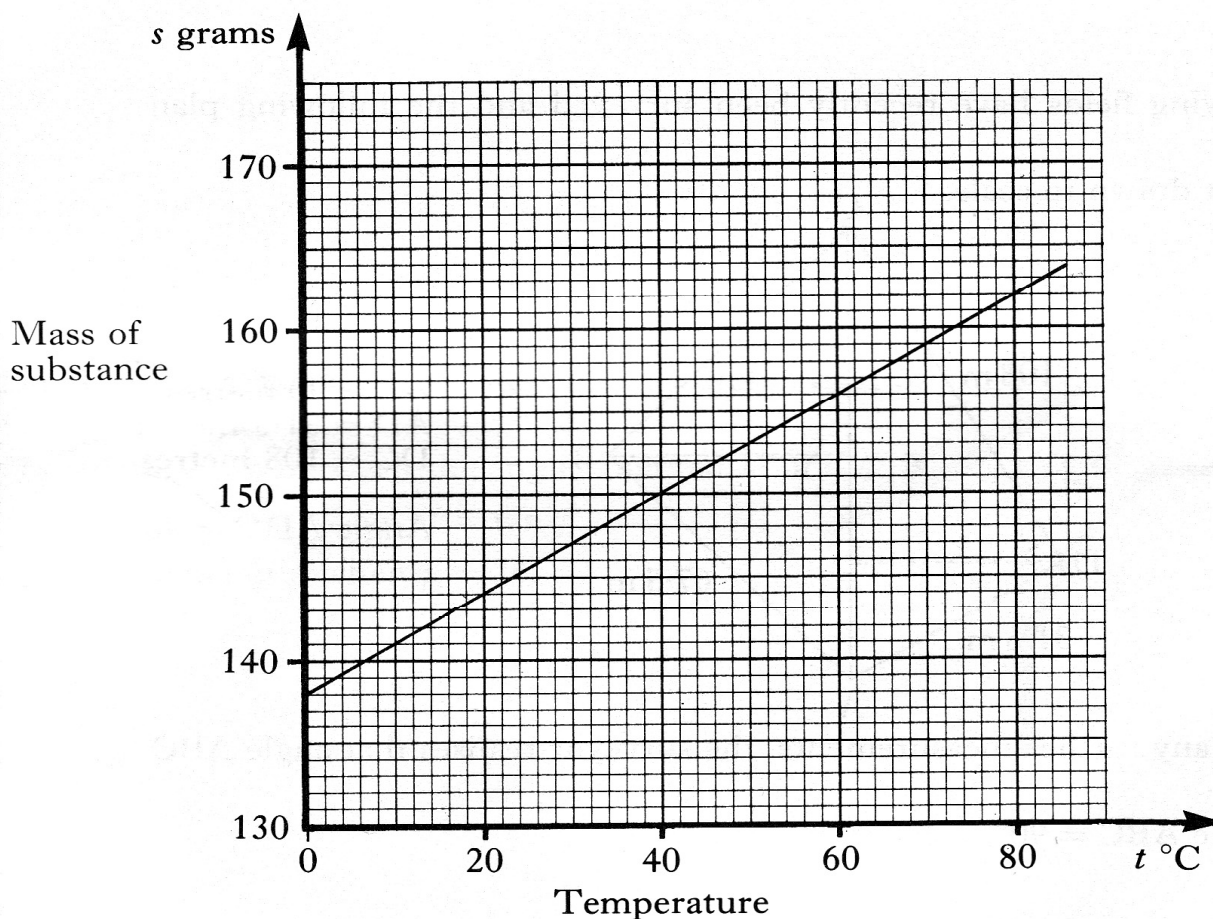
- b) Write down an equation connecting V and t .

2 KU

7. The graph below shows the number of grams, s , of a substance that can be dissolved in a fixed quantity of water when the temperature of the water is $t^\circ\text{C}$.

Find the equation of this straight line in terms of s and t .

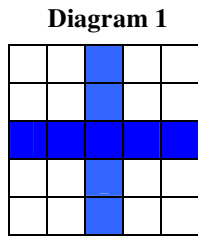
4 KU



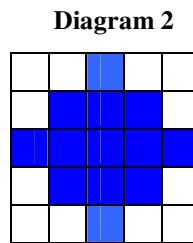
11. Simultaneous Equations

1. Andrew and Doreen each book in at the Sleepwell Lodge.
- a) Andrew stays for 3 nights and has breakfast on 2 mornings.
His bill is £145
Write down an algebraic equation to illustrate this. 1 KU
- b) Doreen stays for 5 nights and has breakfast on 3 mornings.
Her bill is £240.
Write down an equation to illustrate this. 1 KU
- c) Find the cost of one breakfast. 3 RE

2. The reception area in a council office block is to be tiled with a mixture of two types of ceramic tile – white and blue.
- The contractors left two samples, with their cost per square metre, as shown in the diagrams below.



Cost: £25.20

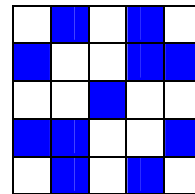


Cost: £26.40

- (a) Using Diagram 1 write down an equation in b and w , where b is the cost of a blue tile and w is the cost of a white tile. 1 KU
- (b) Using Diagram 2 write down a second equation in b and w . 1 KU

Unfortunately the manager did not like any of the samples left and decided to use one of his own.

His choice is shown in the diagram on the right.



- (c) How much per square metre would this design cost ? 4 RE
3. a) 4 peaches and 3 grapefruit cost £1.30
Write down an algebraic equation to illustrate this. 1 KU
- b) 2 peaches and 4 grapefruit cost £1.20.
Write down an algebraic equation to illustrate this. 1 KU
- c) Find the cost of 3 peaches and 2 grapefruit. 4 RE

4. The tickets for a Sports Club Disco cost £2 for members and £3 for non-members.

a) The total ticket money collected was £580.

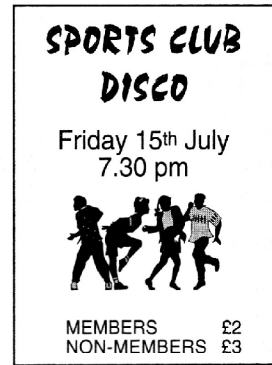
x tickets were sold to members
and y tickets were sold to non-members.

Use this information to write down an equation involving x and y .

b) 250 people bought tickets for the disco.

Write down another equation involving x and y .

c) How many tickets were sold to members ?



2 RE

1 RE

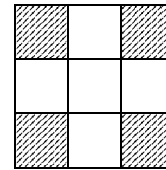
3 RE

5. A small square patio required nine slabs to cover it.

a) The cost of using 4 patterned slabs and 5 plain ones is £15.50

by letting $\pounds x$ be the cost of 1 patterned slab.
 $\pounds y$ be the cost of 1 plain slab.

Write down an algebraic equation to illustrate this.

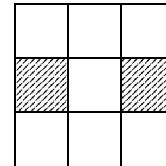


1

1 KU

b) If 2 patterned slabs and 7 plain ones are used instead, the cost becomes £14.50.

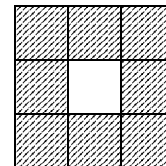
Write down an algebraic equation to illustrate this.



1 KU

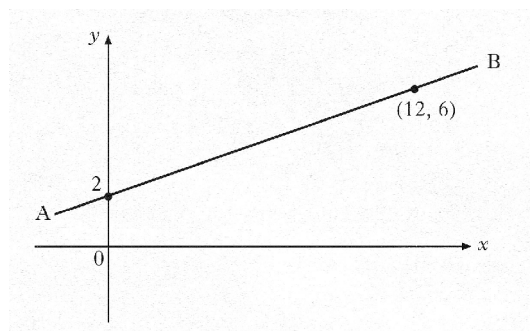
c) Find the cost of this arrangement which is made up by using 8 patterned slabs and 1 plain one.

(show all your working clearly).



4 RE

6. A water pipe runs between two buildings. These are represented by the points A and B in the diagram below.



a) Using the information in the diagram, show that the equation of the line AB is $3y - x = 6$.

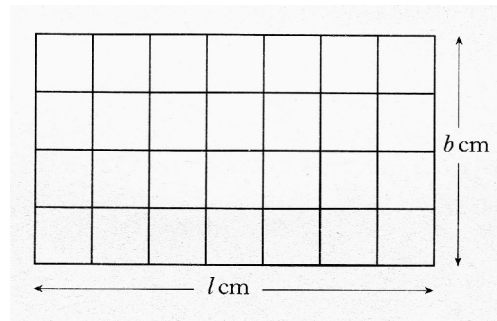
3 KU

b) An emergency outlet pipe has to be built across the main pipe. The line representing this outlet pipe has equation $4y + 5x = 46$

Calculate the coordinates of the point on the diagram at which the outlet pipe will cut across the main water pipe.

4 RE

7. A rectangular window has length, l centimetres and breadth b centimetres
A security grid is made to fit this window.
The grid has 5 horizontal wires and 8 vertical wires.



- a) The perimeter of the window is 260 centimetres.

Use this information to write down an equation involving l and b .

1 RE

- b) In total, 770 centimetres of wire are used.
Write down another equation involving l and b .

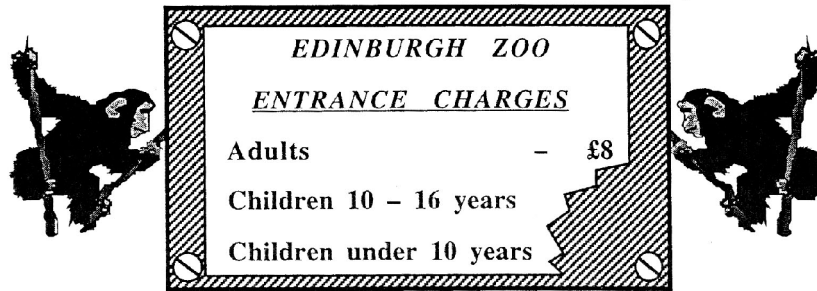
2 RE

- c) Find the length and breadth of the window.

3 RE

8. Gillian and Laura took their children to the zoo.

The entrance cost for the zoo was as shown below, but the charges for children have been torn off.



- a) **Gillian** paid for **herself** and:-
her **2** sons aged 13 and 15, and her **3** daughters all under 10 years of age.

Let the price for each 10 – 16 year old be £ x .

Let the price for each under 10 year old be £ y .

If Gillian paid £19 in total for herself and her own children, explain why the cost can be expressed in the form.

$$2x + 3y = 11$$

1 RE

- b) **Laura** paid for **herself** and:-
her **4** sons aged 10, 12, 13 and 16, and her **1** daughters aged 7.

Laura paid £15 in total.

Write down a second equation in x and y to indicate her total cost.

1 RE

- c) Calculate the cost of:

(i) a single ticket for a 14 year old child.

3 RE

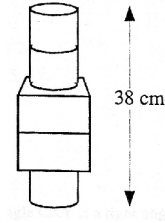
(ii) a single ticket for a 7 year old child.

1 RE

9. A child has built a tower made of two types of brick.

It has three cylinders and two cuboids.
The total height of his tower is 38 centimetres.

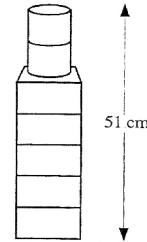
Let x cm be the height of one cylinder and
let y be the height of one cuboid.



a) Construct an equation connecting x and y

He then built this second tower using two cylinders and five cuboids, and its height was 51 centimetres.

b) Form a second equation in x and y and calculate the height of both a cylinder and a cuboid.



1 RE

4 RE

10. A number tower is built from bricks as shown in figure 1.

The number on the brick above is always equal to the sum of the two numbers below.

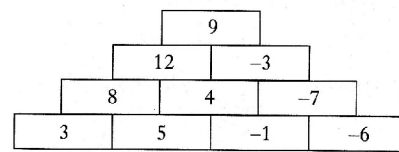


figure 1

a) Find the number on the shaded brick in figure 2.

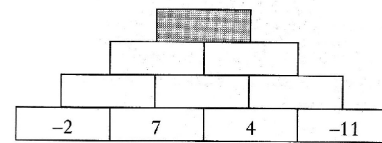


figure 2

1 KU

b) In figure 3, two of the numbers on the base bricks are represented by p and q .

Show that $p + 3q = 10$

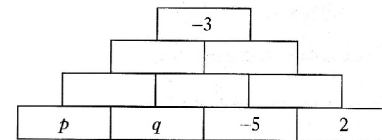


figure 3

2 RE

c) Use figure 4 to write down a second equation in p and q .

d) Find the values of p and q .

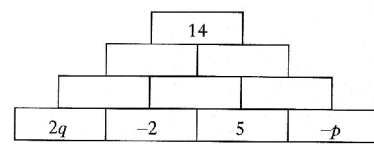


figure 4

2 RE

3 RE

11. Alloys are made by mixing metals. Two different alloys are made using iron and lead.

To make the first alloy, 3 cubic centimetres of iron and 4 cubic centimetres of lead are used.
This alloy weighs 65 grams.

a) Let x grams be the weight of 1 cubic centimetre of iron and y grams be the weight of 1 cubic centimetre of lead.

Write down an equation in x and y which satisfies the above condition.

2 RE

To make the second alloy, 5 cubic centimetres of iron and 7 cubic centimetres of lead are used.
This alloy weighs 112 grams.

b) Write down a second equation in x and y which satisfies this condition.

2 RE


c) Find the weight of 1 cubic centimetre of iron and the weight of 1 cubic centimetre of lead.

3 RE

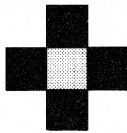
12. A large floor is to be covered with black and grey square tiles to make a chequered pattern.

The person laying the tiles must start at the centre of the floor and work outwards.

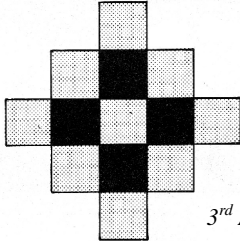
The instructions are as follows.



1st Arrangement



2nd Arrangement



3rd Arrangement

1. Lay a grey tile in the centre of the floor
2. Place black tiles against the edges of the grey tiles
3. Place grey tiles against the edges of all the black tiles
4. Place black tiles against the edges of all the grey tiles.
5. And so on

- a) How many tiles are there in the 4th arrangement? 2 RE
- b) The number of tiles, T , needed to make the N th arrangement is given by the formula

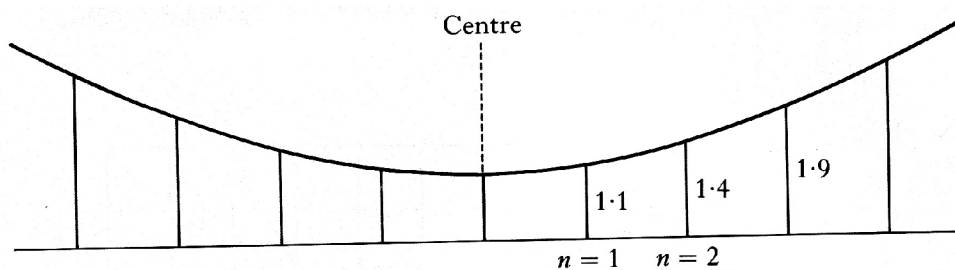
$$T = 2N^2 + aN + b$$

Find the values of a and b .

4 RE

13. The heights in metres of the vertical rods of an early suspension bridge, as you move out from the centre, form the sequence

1.1, 1.4, 1.9, 2.6,



- a) What are the likely heights of the 5th and 6th rods in this sequence 2 RE
- b) The height, h metres, of the n th rod in the sequence is given by the formula

$$h = A + bn^2$$

Find the values of A and b and write down the formula.

4 RE

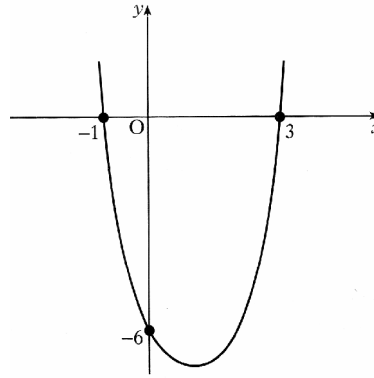
12. Functions

Properties of the parabola

1. The diagram shows part of the graph of a quadratic function, with equation of the form

$$y = k(x-a)(x-b)$$

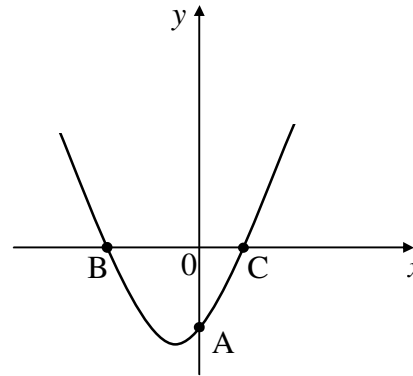
The graph cuts the y -axis at $(0, -6)$ and the x -axis at $(-1, 0)$ and $(3, 0)$



- a) Write down the values of a and b . 2 KU
 b) Calculate the value of k . 2 KU
 c) Find the coordinates of the minimum turning point of the function 2 RE

2. The graph shown has equation $y = x^2 + x - 12$.

- (a) Find the coordinates of A, the point where the curve cuts the y -axis.
 (b) Find the coordinates of B and C, the points where the curve cuts the x -axis.
 (c) Find the coordinates of the minimum turning point.



- 1 RE
 3 RE
 2 RE

3. The graph shows the parabola

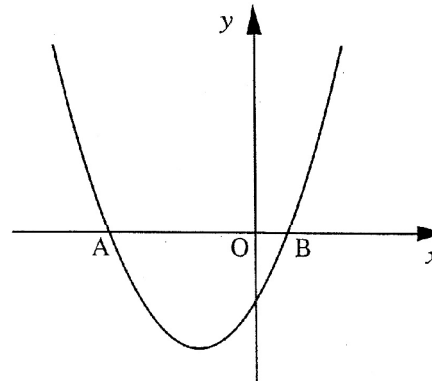
$$y = 3x^2 + 7x - 2$$

By solving the quadratic equation

$$3x^2 + 7x - 2 = 0$$

find the coordinates of point A.

Give your answer **correct to 2 decimal places**.

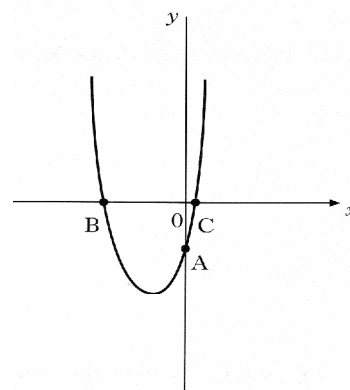


4 KU

4. The diagram below shows part of the graph of $y = 4x^2 + 4x - 3$

The graph cuts the y -axis at A and the x -axis at B and C.

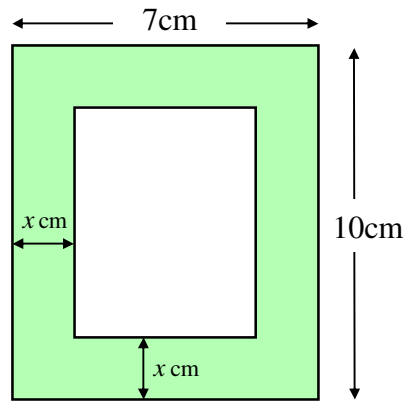
- a) Write down the coordinates of A
 b) Find the co-ordinates of B and C.
 c) Calculate the minimum value of $4x^2 + 4x - 3$



- 1 KU
 3 KU
 2 RE

Applications of the parabola

1. Jane found a small photo-frame and decided to put one of her favourite photographs in it. The diagram below shows the dimensions of the frame.



The width of the wooden surround is x cm.

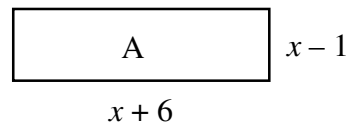
Unfortunately the glass in the centre of the frame was cracked and had to be replaced.

- (a) Show that the area of glass needed for the centre of the frame can be given by the formula

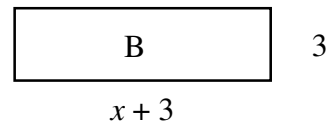
$$A = (4x^2 - 34x + 70) \text{ cm}^2 \quad 4 \text{ RE}$$

- (b) If the area of glass needed was 28cm^2 , find a possible value for x . 3 KU

2. Rectangle A, shown opposite, has length $x + 6$ units and breadth $x - 1$ units.



Rectangle B has length $x + 3$ units and breadth 3 units.



- a) Write down expressions, in terms of x , for the area of Rectangle A and the area of Rectangle B. 2 KU
- b) Given that both rectangles have the same area for a particular value of x , form an equation using your answers to part (a) and solve it to find this value of x . 3 RE

3. A frog is sitting 2 feet to the left of a snake.

The frog then notices a fly sitting on a rock on the other side of the snake. As the frog leaps over the snake to catch the fly, its path is described by the parabola with equation

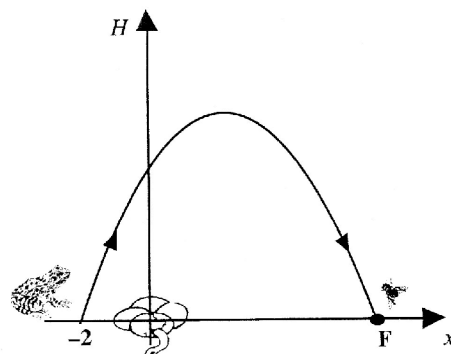
$$H = 8 + 2x - x^2$$

where H is the height of the frog above the ground.

- a) By considering the quadratic equation:

$$8 + 2x - x^2 = 0$$

find the co-ordinates of the point F, where the fly is sitting, and hence write down how far away the fly is from the frog.



- b) How high above the ground does the frog reach on its jump? 4 KU
- b) How high above the ground does the frog reach on its jump? 3 RE

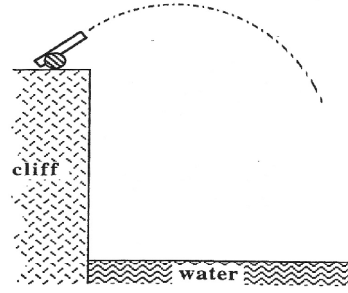
4. When a shell is fired from a cannon on top of a cliff, the height of the shell above the water surface is given by the formula:

$$H(t) = 9 + 6t - 3t^2$$

where t is the time in seconds and $H(t)$ is the height in metres after t seconds.

Calculate the height of the shell after 3 seconds.

Explain what your answer indicates.



2 KU

5. A gardener creates an L-shaped flower bed. He uses the house walls and concrete edging for the boundary as shown in figure 1.

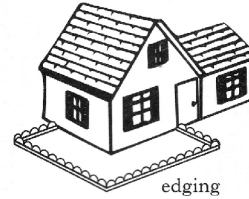


figure 1.

He plans his flower bed as shown in figure 2.

- a) He uses a total of **6 meters of edging**.

$$\begin{aligned} AB &= ED = x \text{ metres.} \\ BC &= DC \end{aligned}$$

Show that the length in metres, of BC, can be expressed as $BC = 3 - x$.

- b) Hence show that the area, A , in square metres, of the flower bed can be expressed as

$$A = 6x - 3x^2$$

- c) Calculate **algebraically** the maximum area of the flower bed.

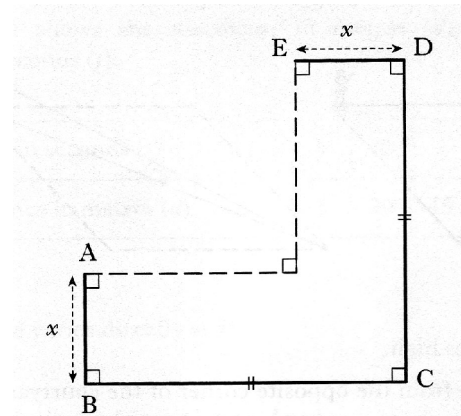


figure 2

2 RE

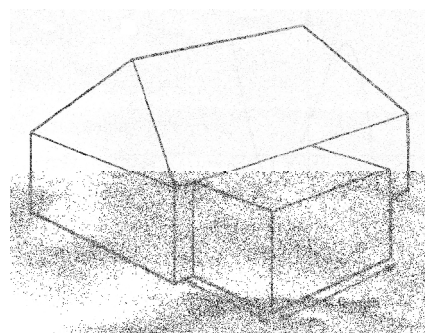
3 RE

3 RE

6. A family want to build an extension at the rear of their house.

An architect advises that the extension should have its length 2 metres more than its width.

- a) If the width of the extension is w metres, write down an expression for its length.



1 KU

Planning regulations state that the area of the ground floor of the extension must not exceed 40% of the area of the ground floor of the original house.

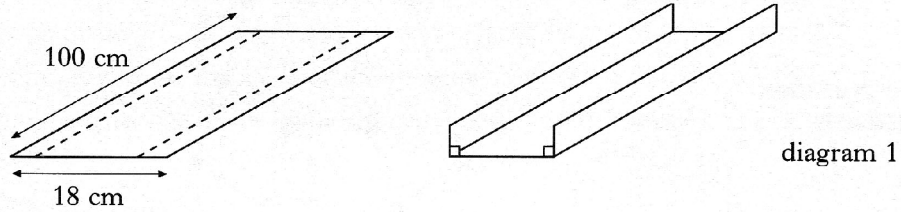
- b) The ground floor of the original house is 12 metres by 10 metres. Show that, if the largest extension is to be built, $w^2 + 2w - 48 = 0$.
- c) Find the dimensions of the largest extension which can be built.

3 RE

2 RE

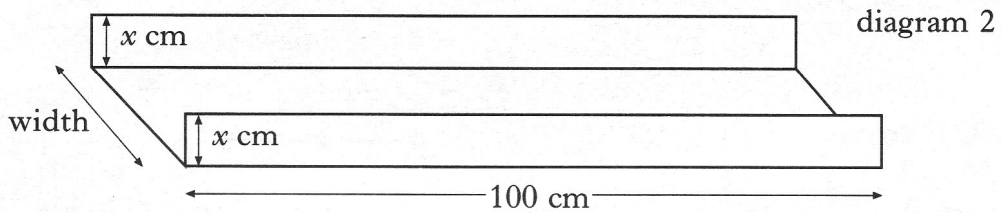
7. A rectangular sheet of plastic 18 cm by 100 cm is used to make a gutter for draining rain water.

The gutter is made by bending the sheet of plastic as shown below in diagram 1.



- a) The depth of the gutter is x centimetres as shown in diagram 2 below.
Write down an expression in x for the width of the gutter.

1 KU



- b) Show that the volume, V cubic centimetres, of this gutter is given by

$$V = 1800x - 200x^2$$

2 RE

- c) Find the dimensions of the gutter which has the largest volume.
Show clearly all your working.

4 RE

13. Making & Using Formulae

1. A rectangular clipboard has a triangular plastic pocket attached as shown in Figure 1.

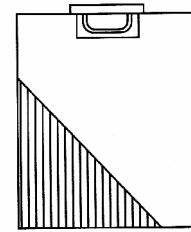


Figure 1

The pocket is attached along edges TD and DB as shown in Figure 2. B is x centimeters from the corner C.

The length of the clipboard is $4x$ centimeters and the breadth is $3x$ centimeters.

The area of the pocket is a quarter of the area of the clipboard.

Find in terms of x , the length of TD.

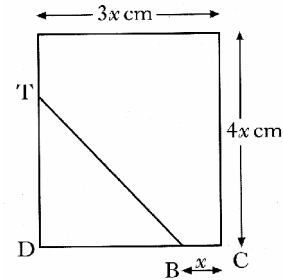


Figure 2

4 RE

2. The number of diagonals, d , in a polygon with n sides is given by the formula:

$$d = \frac{n(n-3)}{2}$$

A polygon has 20 diagonals.

How many sides does it have?

4 RE

3. Esther has a new mobile phone and considers the following daily rates.

<p>Easy Call</p> <p>25 pence per minute for the first 3 minutes</p> <p>5 pence per minute after the first three minutes.</p>
--

<p>Green Call</p> <p>40 pence per minute for the first 2 minutes</p> <p>2 pence per minute after the first two minutes.</p>

- a) For Easy Call, find the cost of ten minutes in a day. 1 KU
- b) For Easy Call, find a formula for the cost of “ m ” minutes in a day, $m > 3$ 1 RE
- c) For Green Call, find a formula for the cost of “ m ” minutes in a day, $m > 2$ 1 RE
- d) Green Call claims that its system is cheaper.
- Find **algebraically**, the least number of minutes (to the nearest minute) which must be used each day for this claim to be true. 3 RE

4. The intensity of light, I , emerging after passing through a liquid with concentration, c , is given by the equation

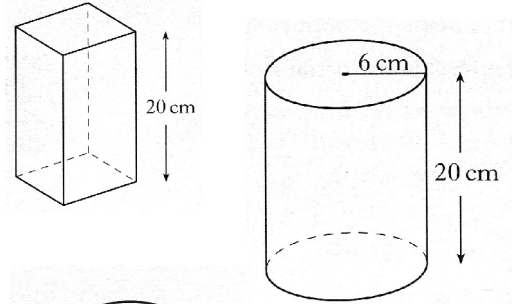
$$I = \frac{20}{2^c} \quad c \geq 0$$

- a) Find the intensity of light when the concentration is 3. 1 KU
- b) Find the concentration of the liquid when the intensity is 10 2 KU
- c) What is the maximum possible intensity? 3 RE

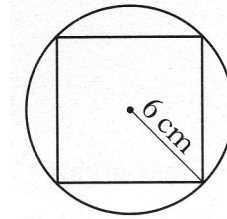
5. A rectangular wall vent is 30 centimetres long and 20 centimetres wide.
It is to be enlarged by increasing **both** the length and the width by x centimetres.
- a) Write down the length of the new vent. 1 RE
- b) Show that the Area, A , square centimeters, of the new vent is given by
- $$A = x^2 + 50x + 600$$
- 2 RE
- c) The area of the new vent **must** be **at least** 40% more than the original area.
Find the minimum dimensions to the nearest centimeters, of the new vent. 5 RE

6. A glass vase, in the shape of a cuboid with a square base is 20 centimetres high.

It is packed in a cardboard cylinder with radius 6 centimetres and height 20 centimetres.



The corners of the vase touch the inside of the cylinder as shown.



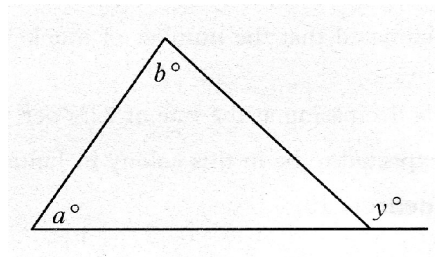
Show that the volume of the space between the vase and the cylinder is $720(\pi - 2)$ cubic centimetres.

7. The cost of renting one of three apartments in Greece depends on the number of people sharing.
- If there are **less** than the standard number of people sharing an apartment, (known as under-occupancy), an extra fee is charged.
- If there are **more** than the standard number, then a reduction is given to **every** person in the room, (*based on each extra adult*).
- The table below shows how the cost is calculated.

Style of Apartment	No. Rooms	Cost per person per week	Based on number sharing	Under-occupancy extra fee per person (£)	Reduction per extra adult (£)
Mailia	1	425	2	40	30 (max. 2 extra)
Mavrikos	2	310	4	45	25 (max. 2 extra)
Tsilivi	3	450	6	55	40 (max. 4 extra)

- a) Find the total cost of 4 adults staying at Malia Apartments for 1 week. 2 RE
- b) Find a formula to calculate the total cost £ C , of P people staying at Malia for 1 week, where P is greater than 2 but less than 5. 3 RE

8. Use the information in the diagram to find a relationship connecting a , b and y



2 RE

9. Anna hired a mobile phone at a fixed charge of £17.50 per month. She is also charged for her total call time each month. 15 minutes of this total call time are **free**. The rest of her call time is charged at 35 pence per minute.

- a) What is the total cost for Anna's phone in a month when her **total call time** is 42 minutes.
- b) Write down a formula for the total cost, £ C , for Anna's phone in a month when her total call time is t minutes, where $t \geq 15$.

2 KU

3 RE

10. A gardener creates an L-shaped flower bed. He uses the house walls and concrete edging for the boundary as shown in figure 1.

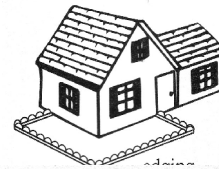


figure 1.

He plans his flower bed as shown in figure 2.

- a) He uses a total of **6 meters of edging**.

$$AB = ED = x \text{ metres.}$$

$$BC = DC$$

Show that the length in metres, of BC, can be expressed as $BC = 3 - x$.

- b) Hence show that the area, A , in square metres, of the flower bed can be expressed as

$$A = 6x - 3x^2$$

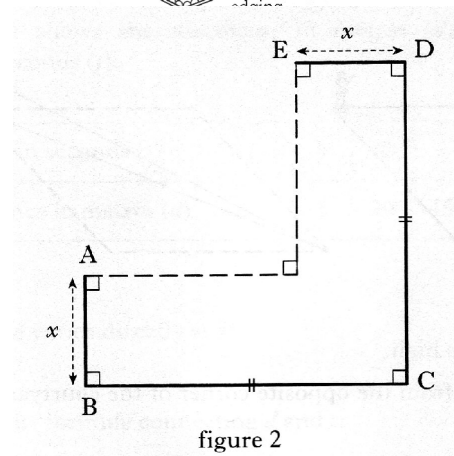


figure 2

2 RE

3 RE

- c) Calculate **algebraically** the maximum area of the flower bed.

3 RE

11. The cost of taking a school group to the theatre can be calculated from the information shown below.

*** 1 adult goes free for every 10 pupils ***

Number of pupils	Cost per pupil	Cost per paying adult
Less than 10	£5.00	£8.00
10 to 19	£4.50	£7.00
20 to 29	£4.00	£6.00
30 to 39	£3.00	£5.00

- a) Find the cost for a group of 12 pupils and 3 adults.
- b) Write down a formula to find the cost, £ C , of taking a group of p pupils and d adults where $20 \leq p \leq 29$.

2 RE

4 RE

12. Traffic authorities are investigating the number of cars travelling along a busy stretch of road.

They assume that all cars are travelling at a speed of v metres per second.

The number of cars, N , which pass a particular point on the road in one minute is given by the formula

$$N = \frac{30v}{2 + v}$$

In one minute, 26 cars pass a point on the road.

Find the speed of the cars in metres per second.

3 RE

13. While on holiday, John's family decide to hire a car.

There are two different schemes for hiring the same type of car, Eurocar and Apex.

<p>EUROCAR HIRE</p> <p>No deposit required £15 per day</p>

<p>APEX HIRE</p> <p>£50 deposit required plus £10 per day</p>

- a) Write down a formula to find the cost, £ C , of hiring the car from Eurocar for d days.

1 KU

- b) Write down a formula to find the cost, £ C , of hiring the car from Apex for d days.

2 KU

- c) John's family have £170 to spend on car hire.

Which scheme should they use to have the car as long as possible?

Show clearly all your working.

4 RE

14. The area, A , of a quadrilateral drawn inside a circle can be found using the formula

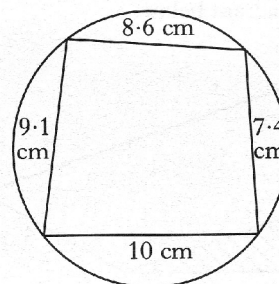
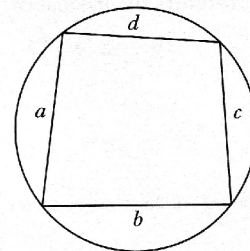
$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

where

$$s = \frac{(a+b+c+d)}{2}$$

Use this formula to find the area of the quadrilateral shown in the diagram.

Give your answer correct to 2 significant figures.



3 KU

15. The travelling expenses claimed by a salesperson depend on the engine capacity of the car and the number of miles travelled per week as shown in the table below.

ENGINE CAPACITY	EXPENSES PER MILE
Less than or equal to 1 litre	£0.25 for each of the first 250 miles travelled
greater than 1 litre but less than or equal to 1.2 litres	£0.27 for each of the first 250 miles travelled
greater than 1.2 litres	£0.29 for each of the first 250 miles travelled
Where the number of miles traveled in a week is greater than 250 , £0.15 can be claimed for each additional mile.	

- a) Find the expenses claimed by a salesperson in a week when 550 miles are travelled and the engine capacity is 1.6 litres. 2 RE
- b) Write down a formula to find the expenses $\pounds E$, claimed for t miles travelled, where t is greater than 250, and the engine capacity is 1.6 litres 3 RE

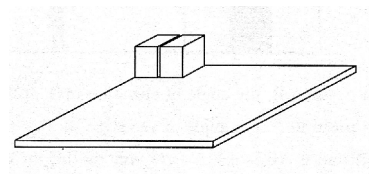
16. The integral part of a positive real number is the part of the number which is an integer.

EXAMPLES **The integral part of 5.6 is 5**
This can be written as $[5.6] = 5$

The integral part of 6.2 is 6
This can be written as $[6.2] = 6$

- a) Find $[16.7]$ 1 RE

- b) Identical boxes are packed on a board for storage. The boxes are all packed the same way round (two boxes are shown in the diagram).

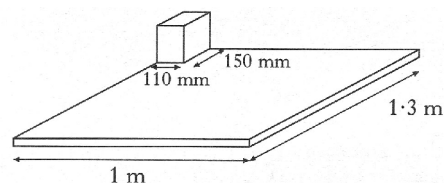


- i) The base of each box measures 150 millimetres by 110 millimetres.

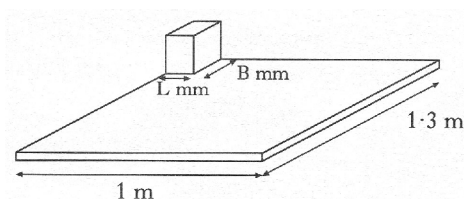
The board measures 1.3 metres by 1 metre.

The number of boxes that can fit along the 1.3 metre length is given by

$$\left[\frac{1300}{150} \right] \quad \text{Find} \quad \left[\frac{1300}{150} \right]$$



- ii) Write down an expression for the number of boxes which can be packed on the board shown on the right.

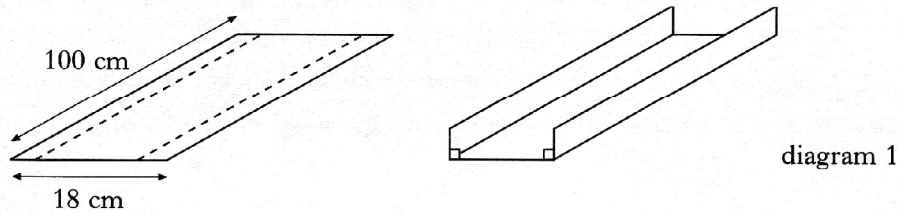


1 RE

2 RE

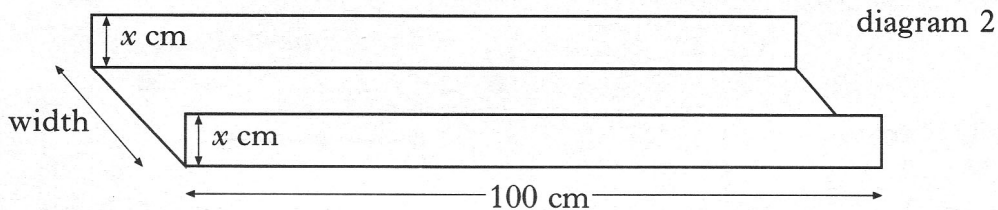
17. A rectangular sheet of plastic 18 cm by 100 cm is used to make a gutter for draining rain water.

The gutter is made by bending the sheet of plastic as shown below in diagram 1.



- a) The depth of the gutter is x centimetres as shown in diagram 2 below.
Write down an expression in x for the width of the gutter.

1 KU



- b) Show that the volume, V cubic centimetres, of this gutter is given by

$$V = 1800x - 200x^2$$

2 RE

- c) Find the dimensions of the gutter which has the largest volume.

Show clearly all your working.

4 RE

18. The cost of sending a parcel depends on the weight of the parcel and the time of delivery. The cost is calculated as shown below.

TIME OF DELIVERY	COST
by 10 am the next working day	£18.20 for 10kg and £0.85 for each extra kg.
by noon the next working day	£13.50 for 10kg and £0.75 for each extra kg.
by 5 pm the next working day	£10.50 for 10kg and £0.50 for each extra kg.

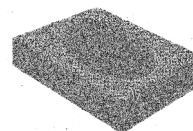
- a) Find the cost of sending a parcel, of weight 14 kg, for delivery **by noon** the next working day.
- b) Write down a formula to find the cost, £ C , of sending a parcel, of weight w kg, where w is greater than 10.

2 RE

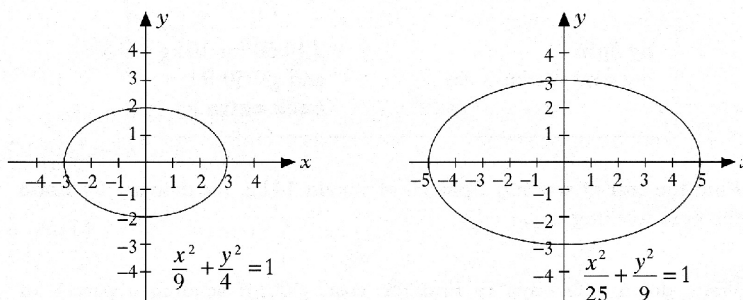
The parcel has to be delivered **by noon** the next working day.

3 RE

19. The opening on this box of tissues is in the shape of an ellipse.



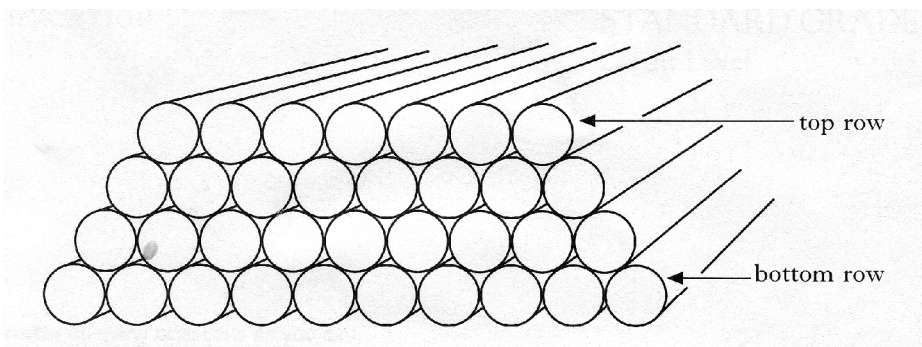
The graphs of two ellipses and their equations are shown below.



Sketch the ellipse with equation $\frac{x^2}{36} + \frac{y^2}{16} = 1$

3 RE

20. Pipes with equal diameters are arranged in a stack.



To find the number of pipes, P , in the stack, the following formula can be used.

$$P = \frac{(b+a)(b-a+1)}{2}$$

where b is the number of pipes on the bottom row and a is the number of pipes on the top row.

- a) Use this formula to find the number of pipes in a stack where $b = 40$ and $a = 15$.
- b) In a particular stack, the number of pipes on the bottom row is twice the number on the top row.

Show that in this stack $P = \frac{3a^2 + 3a}{2}$ where a is the number of pipes on the top row.

- c) Would it be possible to arrange exactly 975 pipes in the kind of stack described in part b)

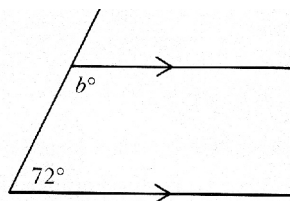
Justify your answer.

1 KU

3 RE

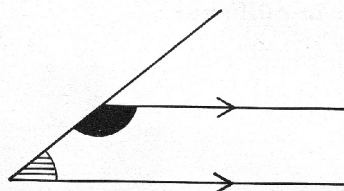
3 RE

21. The diagram opposite shows two parallel lines meeting a third at 72° .



- a) Find the value of b .
- b) The diagram opposite shows the general case of two parallel lines meeting a third line.

Prove that in every case, the sum of the shaded angles is 180° .



1 KU

3 RE

22. An extract from a camping holiday brochure is shown below.

Season	For 14 nights					Over 14 nights
	Two adults	Each extra adult	Each young adult aged 14 to 17	Each child aged 10 to 13	Each child aged 0 to 9	Each additional night per family
Low	£399	£74	£40	Free	Free	£19
Mid	£555	£85	£50	Free	Free	£29
High	£699	£95	£60	£46	Free	£39

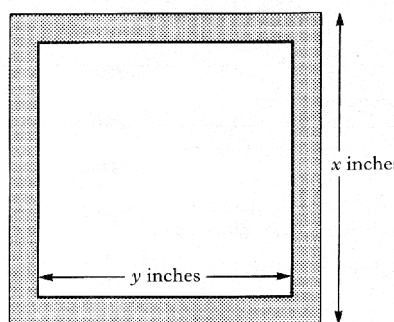
- a) Find the cost of a holiday for 2 adults and a child, aged 8, for 17 nights during mid-season.
- b) Write down a formula to find the cost, $\text{£}C$, of a holiday in mid-season for 2 adults and a child aged 8 lasting t nights, where t is greater than 14.

2 RE

3 RE

23. A square picture frame is shown.

The border of the frame (shaded in the diagram) has uniform width and an area of 48 square inches.



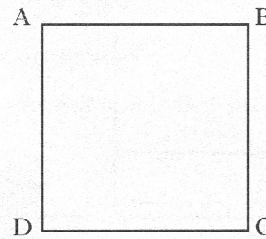
- a) Show that $(x - y)(x + y) = 48$
- b) Given that x and y are whole numbers each greater than 10, find suitable replacements for x and y .

2 RE

3 RE

24. a) ABCD is a square of side 2 cms

Write down the ratio of the length AB to the length of AC.

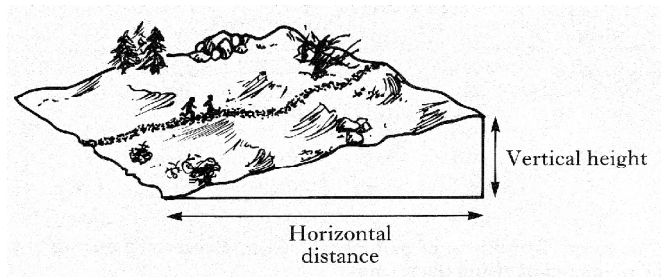


2 KU

- b) Show that in every square, the ratio of the length of a side to the length of a diagonal is $1 : \sqrt{2}$

3 RE

25. The total time a walk takes in hillwalking depends on the horizontal distance covered (h kilometres) and the vertical height climbed v metres.



For **each kilometre** of horizontal distance, 12 minutes should be allowed.

- a) i) Write down the time which should be allowed for h kilometres of horizontal distance.
- ii) for **each 100 metres** of vertical height, 10 minutes should be allowed. Write down the time which should be allowed for v metres of vertical height.
- iii) Show that the **total** time T hours which should be allowed for the walk is given by the formula

$$T = \frac{120h + v}{600}$$

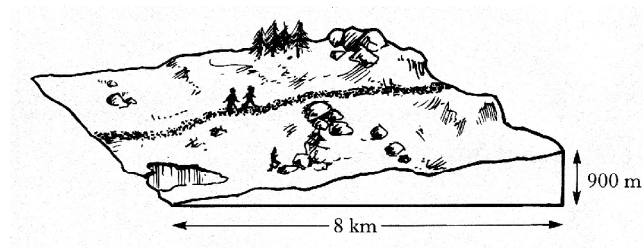
1 KU

2 RE

3 RE

- b) For safety reasons, hillwalkers should be off the hills by 1900 hours.

Would it be safe to start the walk shown at 1300 hours?



3

27. Mr and Mrs Paton want to have their house valued before putting it up for sale.
The fee they have to pay for having this done depends on the value of their house.
The fee is calculated as follows

Value of house	Fee to be paid
First £2000 of value	£5.00
Each additional £500 up to £15000	£1.00 per £500
Each additional £1000 over £15000	£1.00 per £1000

- a) The Paton's house is valued at £33 000
What fee will they have to pay? 4 RE
- b) Write down a formula to find the total fee payable when a house is valued at £ P thousand, where P is a whole number greater than 15. 3 RE

26. The mass, M grams, of a given radio-active isotope decreases with time according to the formula

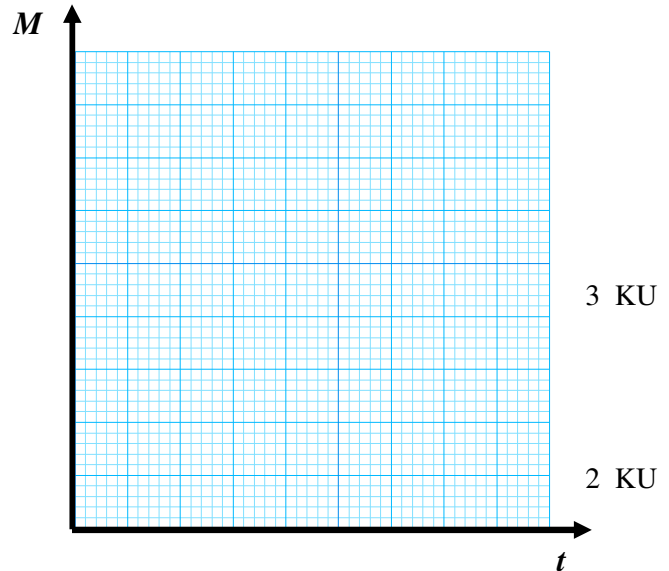
$$M = 80(2)^{-t}$$

where t is the time in years.

- a) The isotope weighs 80 grams at the start.

Show on the grid below,
how the mass of this isotope changes
over the following 4 years.

- b) Calculate how many years it takes for
an isotope weighing 80 grams to
decrease to a weight of $\frac{5}{8}$ of a gram.



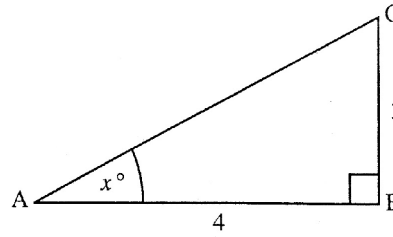
14. Trigonometry 3 - Graphs & Equations

Graphs, triangles, maxima and minima

1. ABC is a right angled triangle with AB = 4 units and BC = 3 units

Prove that for the angle marked x°

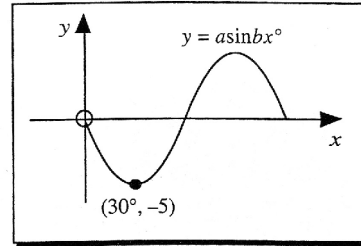
$$\sin^2 x + \cos^2 x = 1$$



2 KU

2. Shown is the graph of $y = a \sin bx^\circ$

Write down the values of a and b .



2 KU

3. On a certain day the depth, D metres, of water at a fishing port, t hours after midnight, is given by the formula

$$D = 12.5 + 9.5 \sin(30t)^\circ$$

- a) Find the depth of water at 1.30 pm
- b) The depth of water in the harbour is recorded each hour. What is the maximum difference in the depths of water in the harbour, over the 24 hour period?

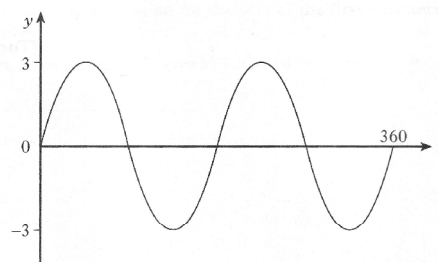
3 RE

Show clearly all your working.

3 RE

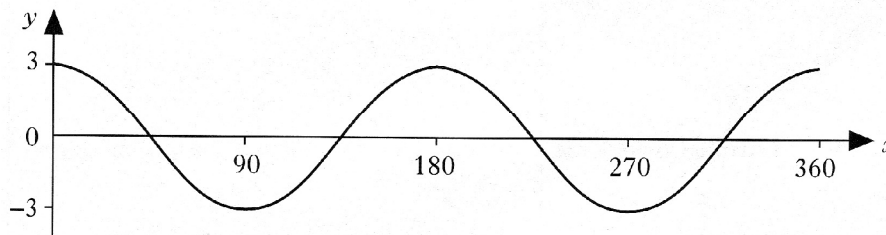
4. The diagram shows the graph of $y = k \sin ax^\circ$, $0 \leq x \leq 360$

Find the values of a and k .



2 RE

- 5.



The diagram shows the graph of $y = a \cos bx^\circ$, $0 \leq x \leq 360$

Find the values of a and b .

2 KU

Solving Equations

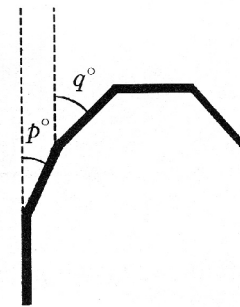
1. Solve the equation $3 \tan x^\circ + 5 = 0$, for $0 \leq x \leq 360$. 4 KU
2. Solve **algebraically** the equation $2 + 3 \sin x^\circ = 0$ for $0 \leq x \leq 360$ 3 KU
3. Solve **algebraically**, the equation $7 \cos x^\circ - 2 = 0$ for $0 \leq x \leq 360$ 3 KU
4. Solve **algebraically**, the equation $5 \tan x - 9 = 0$, for $0 \leq x \leq 360$ 3 KU
5. Solve the equation $5 \sin x^\circ + 2 = 0$, for $0 \leq x \leq 360$ 3 KU
6. Solve algebraically the equation: $\tan 40^\circ = 2 \sin x^\circ + 1$ $0 \leq x \leq 360$ 3 KU

7. The diagram opposite shows part of a natural crystal of topaz.

The relationship between the angles marked p° and q° is

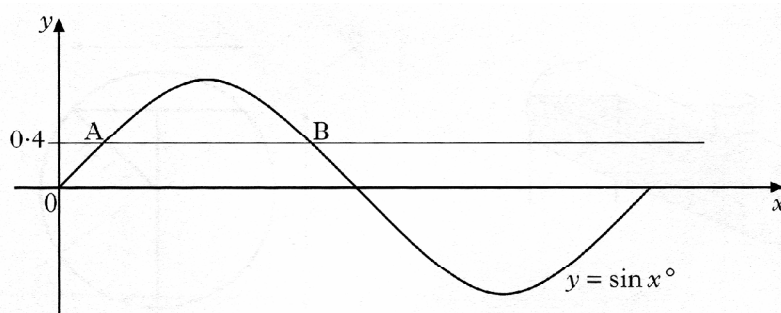
$$2 \tan p^\circ = \tan q^\circ$$

Find the value of q when $p = 24$.



3 KU

8. The diagram shows part of the graph of $y = \sin x$.



The line $y = 0.4$ is drawn and cuts the graph of $y = \sin x$ at A and B.

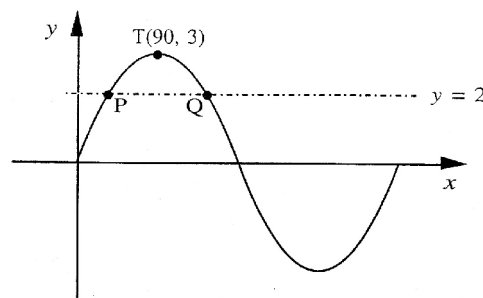
Find the x -coordinates of A and B.

3 RE

9. The graph shown has equation $y = a \sin bx^\circ$.

It has a maximum at the point T(90, 3).

- a) Write down the values of a and b . 1 KU



Also shown in the figure is the line with equation $y = 2$, which meets the curve at the points P and Q.

- b) Find the x -coordinate of the point Q.

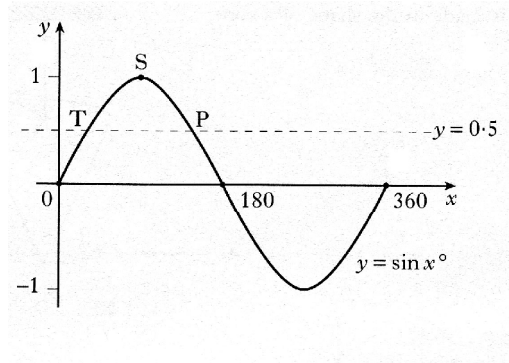
3 RE

10. The diagram shows the graph of $y = \sin x^\circ$, $0 \leq x \leq 360$

- a) Write down the coordinates of point S.

The straight line $y = 0.5$ cuts the graph at T and P.

- b) Find the coordinates of T and P.



1 KU

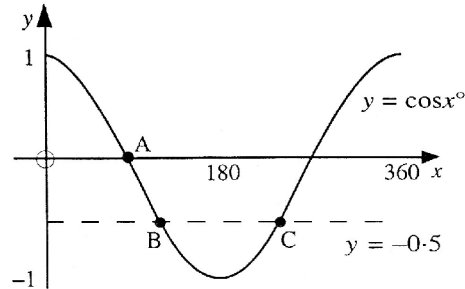
2 KU

11. The diagram shows the graph of $y = \cos x^\circ$, $0 \leq x \leq 360$.

- a) Write down the coordinates of point A.

The straight line $y = -0.5$ cuts the graph at B and C.

- b) Find the coordinates of B and C.



1 KU

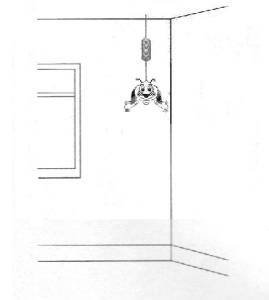
3 KU

12. A toy is hanging by a spring from the ceiling.

Once the toy is set moving, the height, H metres, of the toy above the floor is given by the formula

$$h = 1.9 + 0.3 \cos(30t)^\circ$$

t seconds after starting to move.



- a) State the maximum value of H .
 b) Calculate the height of the toy above the floor after 8 seconds.
 c) When is the height of the toy first 2.05 metres above the floor?

1 KU

3 RE

3 RE

13. The volume of water, V millions of gallons, stored in a reservoir during any month is to be predicted by using the formula

$$V = 1 + 0.5 \cos(30t)^\circ$$

where t is the number of the month. (For January $t = 1$, February $t = 2 \dots$)

- a) Find the volume of water in the reservoir in October.
 b) The local council would need to consider water rationing during any month in which the volume of water stored is likely to be less than 0.55 million gallons.

3 RE

Will the local council need to consider water rationing?

Justify your answer.

4 RE


15. Ratio & Proportion


1. School theatre visits are arranged for parents, teachers and pupils.
The ratio of parents to teachers to pupils **must** be 1 : 3 : 15.
- a) 45 pupils want to go to the theatre.
How many teachers must accompany them? 1 KU
- b) The theatre gives the school 100 tickets for a play.
What is the maximum number of pupils who can go to the play? 3 RE

2. A coffee shop blends its own coffee and sells it in one-kilogram tins.
One blend consists of two kinds of coffee, Brazilian and Columbian,
in the ratio 2 : 3.
The shop has 20 kilograms of Brazilian and 25 kilograms of Columbian in stock.
What is the maximum number of one-kilogram tins of this blend
which can be made. 3 RE

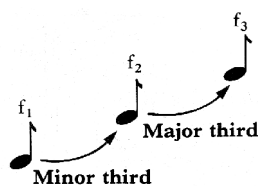
*This is a question from 1990 and is unlikely to be asked today.
However, if you can do it, you are demonstrating a good understanding of ratio and proportion*

3. Each of the examples below gives information about the relation between the frequencies of two musical notes.

 <p>Perfect fifth</p>	<p>When note 2, of frequency f_2, is a perfect fifth above note 1, of frequency f_1,</p> <p>their frequency ratio $f_2 : f_1 = 3 : 2$ i.e. $\frac{f_1}{f_2} = \frac{3}{2}$</p>
---	---

 <p>Perfect fourth</p>	<p>When note 3, is a perfect fourth above note 2, their frequency ratio $f_3 : f_2 = 4 : 3$</p>
--	---

- a) In a given piece of music, note 2 is a **perfect fifth** above note 1,
And note 3 is a **perfect fourth** above note 2.
Show that the frequency ratio of note 3 to note 1 is 2 : 1. 4 RE
- b) It is also known that, when one note is a **minor third** above another note,
their frequency ratio is 6 : 5.



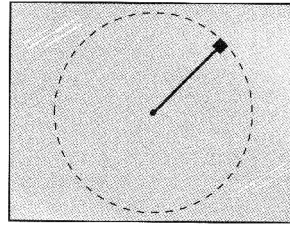
For the notes shown opposite, the second note is a **minor third** above the first and the third note is a **major third** above the second.

If note 3 is a **perfect fifth** above note 1, find the frequency ratio of a **major third**.

Show all your working. 4 RE

16. Variation & Proportion

1. A weight on the end of a string is spun in a circle on a smooth table.
The tension, T , in the string varies directly as the square of the speed, v , and inversely as the radius, r , of the circle.



- a) Write down a formula for T in terms of v and r . 1 KU
- b) The speed of the weight is multiplied by 3 and the radius of the string is halved.
What happens to the tension in the string. 2 RE

2. The electrical resistance, R , of copper wire varies directly as its length, L metres, and inversely as the square of its diameter, d millimetres .
Two lengths of copper wire, A and B, have the same resistance.
Wire A has a diameter of 2 millimetres and a length of 3 metres.
Wire B has a diameter of 3 millimetres
What is the length of wire B. 4 RE

3. A frictional force is necessary for a car to round a bend.
The frictional force, F kilonewtons, varies directly as the square of the car's speed, V metres per second, and inversely as the radius of the bend, R metres.
a) Write down a relationship between F , V and R . 1 KU
- A frictional force of 20 kilonewtons is necessary for a car, travelling at a given speed to round a bend.
- b) Find the frictional force necessary for the same car, travelling at **twice** the given speed, to round the same bend. 2 RE

4. The table below shows the distances, in metres, (d), travelled by a snowboarder in seconds (t).

Time in seconds (t)	1	2	3	4
Distance in metres (d)	5	20	45	80

- a) Explain why d varies directly as t^2 1 RE
- b) Write down the formula connecting d and t . 1 KU
- c) How does the distance change when the time is multiplied by six ? 2 RE
5. The time, T minutes, taken for a stadium to empty varies directly as the number of spectators, S , and inversely as the number of open Exits, E .
- a) Write down a relationship connecting T , S and E . 1 KU
- It takes 12 minutes for a stadium to empty when there are 20 000 spectators and 20 open exits.
- b) How long does it take the stadium to empty when there are 36 000 spectators and 24 open exits ? 3 KU

6. The number of litres of petrol, L , used by a car on a journey varies directly as the distance, D kilometres, travelled, and as the square root of the average speed, S kilometres per hour.
- a) Write down a relationship connecting L , D and S . 1 KU

The car uses 30 litres of petrol for a journey of 550 kilometres when it travels at an average speed of 81 kilometres per hour.

- b) How many litres of petrol does the car use for a journey of 693 kilometres travelling at an average speed of 100 kilometres per hour. 3 KU

7. The surface area of a planet, A square kilometers, varies directly as the square of the diameter, D kilometres of the planet.

The surface area of the Moon is 3.8×10^7 square kilometres.

Calculate the surface area of a planet with diameter double the diameter of the Moon.

Give your answer in scientific notation.

3 KU

8. A table of pairs of values of x and y is shown below.

x	1.5	2	2.5
y	6	4.5	3.6

- a) Explain why y varies inversely as x . 2 KU
 b) Write down the formula connecting x and y 1 KU

9. The number of letters, N , which can be typed on a sheet of paper varies inversely as the square of the size, s , of the letters used.

- a) Write down a relationship connecting N and s . 1 KU
 b) The size of the letters used is doubled.

What effect does this have on the number of letters which can be typed on the sheet of paper.

2 KU

10. The time, T seconds, taken by a child to slide down a chute varies directly as the length, L metres, of the chute and inversely square root of the height, H metres, of the chute above the ground.

It takes 10 seconds to slide down a chute which is 3.75 metres long and 2.25 metres high.



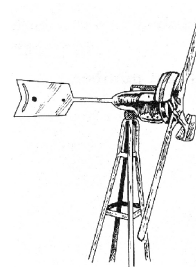
- a) Find a formula connecting T , L and H . 3 KU
 b) How long does it take to slide down a chute which is 5 metres long and 2.56 metres high? 2 KU

11. The power, P watts, produced by a windmill varies directly as the cube of the wind velocity, V metres per second.

At 4 pm on a given day, the wind velocity was 4 metres per second and the windmill was producing 75 watts of electrical power.

By 10 pm the wind velocity had doubled.

How many watts of electrical power were now being produced?



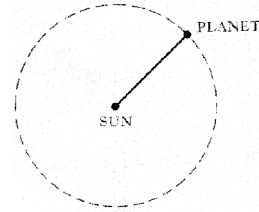
3 KU

17. Distance, Speed & Time and Graphs

Calculations

1. a) A driver travels from A to B, a distance of x miles at a constant speed of 75 kilometres per hour.
Find the time taken for the journey in terms of x . 1 KU
- b) The time for the journey from B to A is $\frac{x}{50}$ hours
Hence calculate the driver's average speed for the whole journey. 4 RE

2. A planet takes 88 days to travel round the Sun.
The approximate path of the planet round the Sun is a circle with diameter 1.2×10^7 kilometres.
Find the speed of the planet as it travels round the Sun.



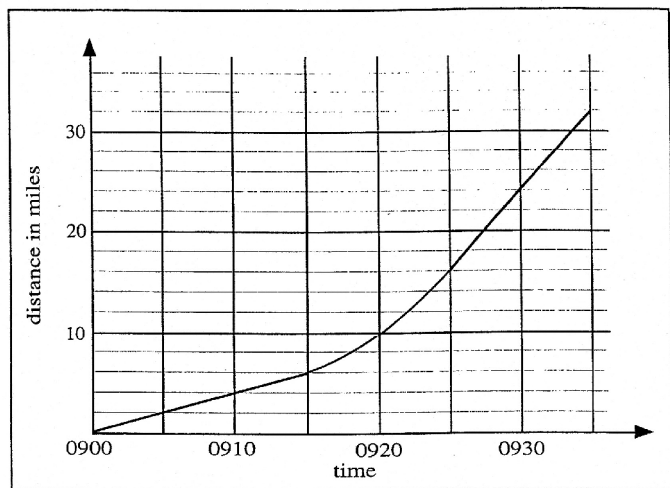
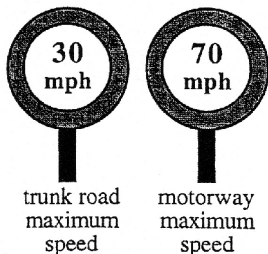
Give your answer in kilometres per hour, correct to 2 significant figures.

4 KU

3. The planet Pluto is at a distance of 5.9×10^9 kilometres from the Sun and the speed of light is 3.0×10^5 kilometres per second.
Calculate, to the nearest half hour, the time taken for light from the Sun to reach Pluto. 4 KU

4. The planet Mars is at a distance of 2.3×10^8 kilometres from the Sun.
The speed of light is 3.0×10^5 km per second.
How long does it take light from the Sun to reach Mars ?
Give your answer to the nearest minute. 3 KU

5. Jennifer is driving to work.
Part of her journey is on a trunk road.
At 0915 she joins the motorway.
The graph shows her journey.

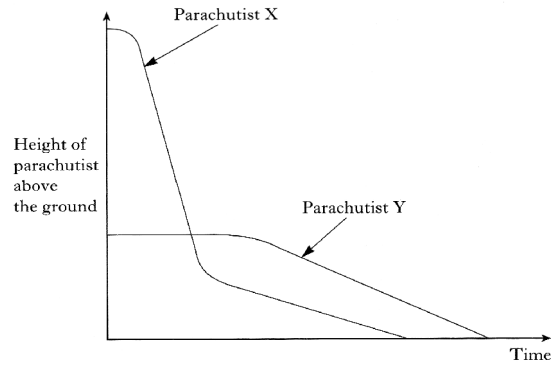


- a) Calculate Jennifer's average speed along the trunk road. 2 KU
- b) Explain what the graph indicates is happening between 0915 and 0925. 1 RE
- c) Where on her way to work, did Jennifer appear to break the speed limit ?
(Give a reason for your answer) 3 RE

Graphs & Interpretation

1. Two parachutists, X and Y, jump from two separate aircrafts at different times.

The graph shows how their height above the ground changes over a period of time.



- a) Which parachutist jumped first ? 1 RE
- b) Which parachutist did not open his parachute immediately after jumping ? 2 RE
Explain your answer clearly.

2. The diagram opposite shows part of the street plan of a town.

Vehicles can travel in both directions along each street.

As a vehicle travels on the straight parts of any street, it can reach the maximum speed.

The speed is always reduced on the bends.

The graph in figure 2 shows how the speed of a vehicle changes as it travels from **A to J**.

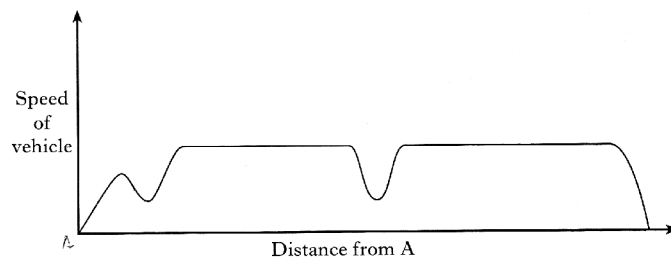
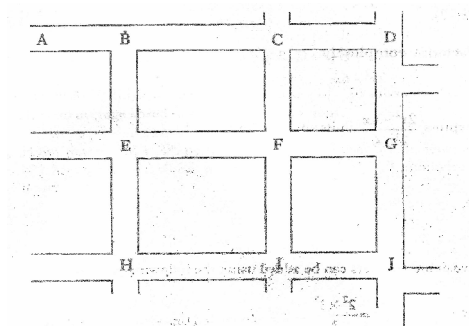


figure 2

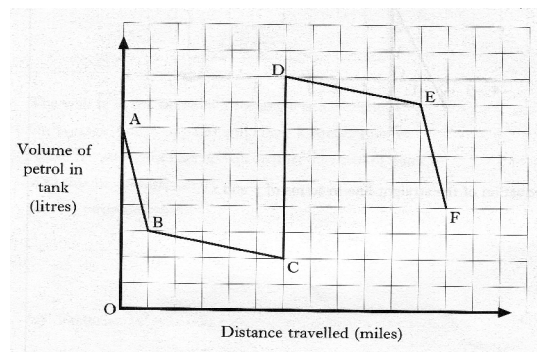
- a) What route did the vehicle travel? Use the letters from figure 1 to indicate this route. 2 RE
- b) Another vehicle took the route A, B, C, F, G and J. Sketch a graph to show how the speed of this vehicle changes during the journey. 3 RE

3. The graph shows the volume of petrol in a car's tank during a journey.

- a) Explain the significance of CD. 1 RE

The journey involves driving through towns and along motorways.

In the towns the car uses more petrol per mile than on the motorways.



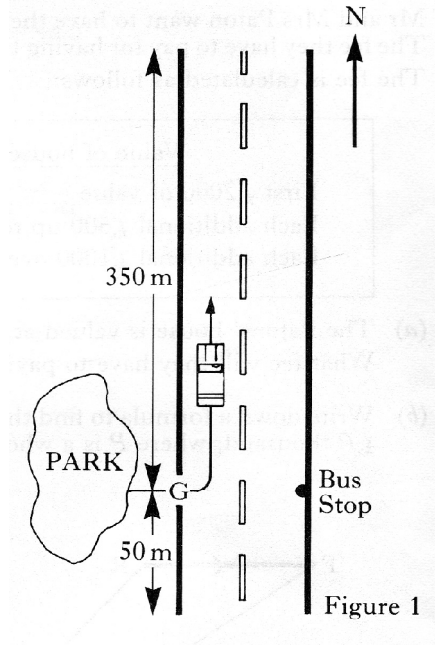
- b) Which two parts of the graph show driving on motorways? 2 RE
Explain your answer clearly.

This is a question from 1990 and would probably be considered to be too long for today's examination. However, it is good practice to attempt it.

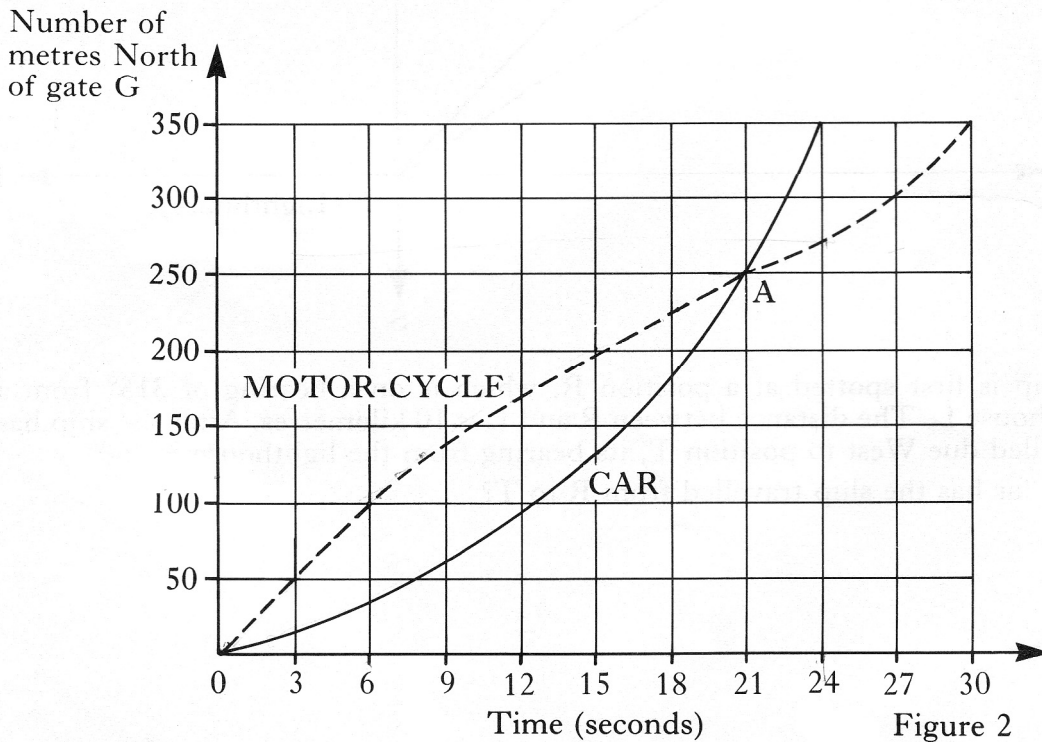
4. The gate G of a country park lies on a 400 metre stretch of road which runs in a north-south direction.

See Figure 1.

A car leaves the park, travels northwards with increasing speed, and reaches the end of the stretch of road 24 seconds later.
 A motor-cycle leaves the park at the same time as the car and also travels northwards.



The progress of the two vehicles is shown on the graph below, Figure 2.

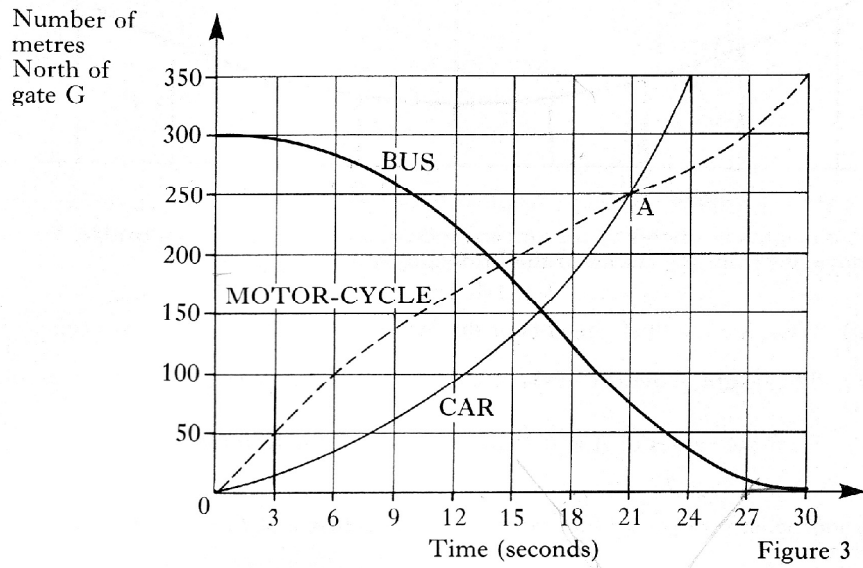


- a) Describe the progress of the motor-cycle as it travels along the road, making particular reference to the significance of the point A.

2 RE

4 (continued)

b) The progress of a bus on the same road is also shown on the graph below, Figure 3.



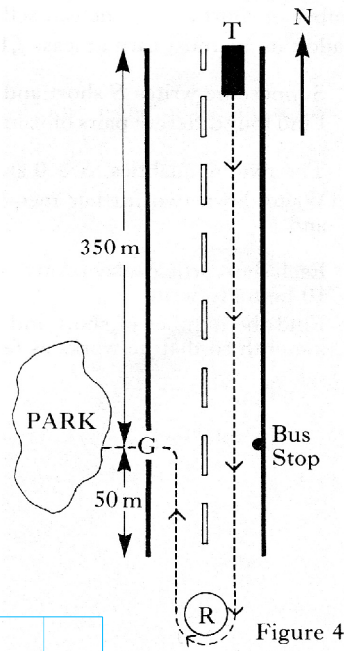
Describe the progress of the bus.

3 RE

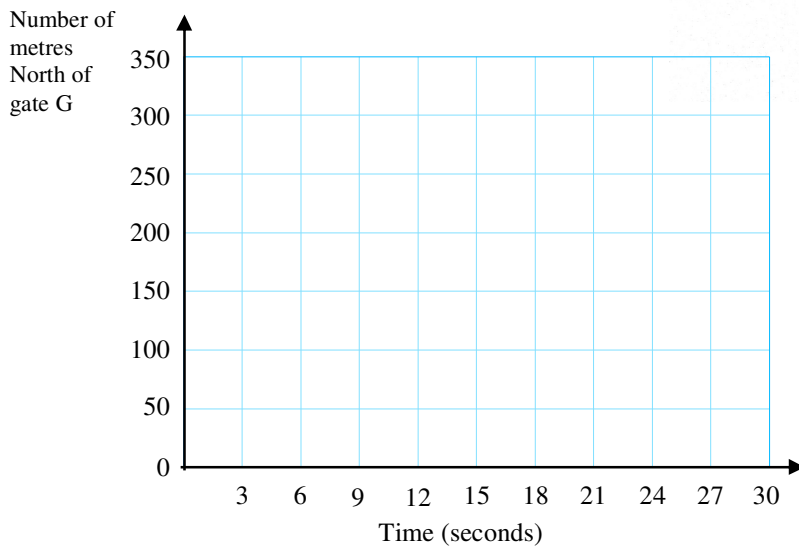
c) Some time later, a taxi enters the same road at point T, in Figure 4, and travels southwards at a steady speed.

It reaches the roundabout R after 18 seconds, drives slowly round the roundabout and enters the gate G, 9 seconds later.

Draw a graph of the progress of the taxi.



3 RE



18. Sequences

1. Using the sequence

$$1, 3, 5, 7, 9, \dots$$

- a) Find S_3 , the sum of the first 3 numbers. 1 RE
 b) Find S_n , the sum of the first n numbers. 2 RE
 c) Hence or otherwise, find the $(n + 1)^{\text{th}}$ term of the sequence 2 RE

2. The median of seven consecutive even integers is $2p + 2$.

- (a) Write down, in terms of p , expressions for the seven integers. 2 RE
 (b) Show that the mean can be expressed as $2(p + 1)$. 2 RE

3. a) Solve the equation

$$2^n = 32$$

1 KU

b) A sequence of numbers can be grouped and added together as shown.

$$\text{The sum of 2 numbers: } (1 + 2) = 4 - 1$$

$$\text{The sum of 3 numbers: } (1 + 2 + 4) = 8 - 1$$

$$\text{The sum of 4 numbers: } (1 + 2 + 4 + 8) = 16 - 1$$

Find a similar expression for the sum of 5 numbers. 1 RE

c) Find a formula for the sum of the first n numbers of this sequence. 2 RE

4. Study the pattern of numbers given below:

1 st pattern	$2 \times (1) - 1$	=	1
2 nd pattern	$2 \times (1 + 2) - 2$	=	4
3 rd pattern	$2 \times (1 + 2 + 3) - 3$	=	9
4 th pattern	$2 \times (1 + 2 + 3 + 4) - 4$	=	16

- a) Write down a similar expression for the 5th pattern. 1 KU
 b) Write down the general formula for the n^{th} pattern. 2 RE
 c) If $2 \times (1 + 2 + 3 + \dots + t) - t = 289$, find the value of t . 3 RE

5. A number pattern is shown below:

$$1^3 + 1 = (1 + 1)(1^2 - 1 + 1)$$

$$2^3 + 1 = (2 + 1)(2^2 - 2 + 1)$$

$$3^3 + 1 = (3 + 1)(3^2 - 3 + 1)$$

- a) Write down a similar expression for $7^3 + 1$ 1 RE
 b) Hence write down an expression for $n^3 + 1$ 1 RE
 c) Hence find an expression for $8p^3 + 1$ 2 RE

6. 1, 3, 5, 7, ...

The **first** odd number can be expressed as $1 = 1^2 - 0^2$

The **second** odd number can be expressed as $3 = 2^2 - 1^2$

The **third** odd number can be expressed as $5 = 3^2 - 2^2$

- a) Express the **fourth** odd number in this form. 1 RE
- b) Express the number 19 in this form 1 RE
- c) Write down a formula for the n^{th} odd number and simplify this expression. 2 RE
- d) **Prove** that the product of two consecutive odd numbers is always odd 3 RE

7. A pattern of numbers is found as follows:

$3 + 2 - 1$	1 st term
$6 + 3 - 3$	2 nd term
$9 + 4 - 5$	3 rd term
.....	
.....	

- a) Write down the next 2 terms in this pattern 2 RE
- b) Write an expression for the n^{th} term in this pattern and express it in its simplest form. 3 RE

8. The difference between squares of any two consecutive whole numbers can be found using the following pattern.

$$2^2 - 1^2 = 3 = 2 + 1$$

$$3^2 - 2^2 = 5 = 3 + 2$$

$$4^2 - 3^2 = 7 = 4 + 3$$

- a) Use this to find the difference between 24^2 and 23^2 2 RE
- b) Write down an expression for the difference between the **squares of any two consecutive numbers**, and simplify it as much as possible. 4 RE

[Hint: let one of the consecutive numbers be n .]

9. A Fibonacci sequence is a sequence of numbers.
After the first two terms, each term is the sum of the previous two terms.

e.g. 2, 3, 5, 8, 13,

$13 = 5 + 8$
↑

$5 = 2 + 3$
↓

- a) Write down the next three terms of this Fibonacci sequence.
5, -1, 4, _____, _____, _____, 1 RE
- b) For the Fibonacci sequence
4, -3, 1, -2, -1, -3, -4,
Show that the sum of the first six terms is equal to four times the fifth term. 2 RE
- c) If p and q are the first two terms of a Fibonacci sequence, **prove** that the sum of the first six terms is equal to four times the fifth term. 3 RE

10. A sequence of terms, starting with 1, is

1, 5, 9, 13, 17,

Consecutive terms in this sequence are formed by adding 4 to the previous term.

The total of consecutive terms of this sequence can be found using the following pattern.

$$\text{Total of the first 2 terms: } 1 + 5 = 2 \times 3$$

$$\text{Total of the first 3 terms: } 1 + 5 + 9 = 3 \times 5$$

$$\text{Total of the first 4 terms: } 1 + 5 + 9 + 13 = 4 \times 7$$

$$\text{Total of the first 5 terms: } 1 + 5 + 9 + 13 + 17 = 5 \times 9$$

- a) Express the total of the first 9 terms of this sequence in the same way. 2 RE
- b) The first n terms of this sequence are added.
Write down an expression, in n , for the total. 3 RE

11. A 3×3 square has been identified on the calendar shown opposite.

The numbers in the diagonally opposite corners of the square are multiplied.

These products are then subtracted in the order shown below.

$$(23 \times 11) - (25 \times 9) = 28$$

- a) Repeat the process for a different 3×3 square.

Show clearly all your working.

M	T	W	T	F	S	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

- b) Prove that in every 3×3 square on the calendar above, the process gives the answer 28. 3 RE

12. Consecutive cubic numbers can be added using the following pattern.

$$1^3 + 2^3 = \frac{2^2 \times 3^2}{4}$$

$$1^3 + 2^3 + 3^3 = \frac{3^2 \times 4^2}{4}$$

$$1^3 + 2^3 + 3^3 + 4^3 = \frac{4^2 \times 5^2}{4}$$

- a) Express $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 + 7^3$ 2 RE
- b) Write down an expression for the sum of the first n consecutive cubic numbers. 3 RE
- c) Write down an expression for $8^3 + 9^3 + 10^3 + \dots + n^3$ 2 RE

13. The sequence of odd numbers starting with 3 is 3, 5, 7, 9, 11,

Consecutive numbers from this sequence can be added using the following pattern.

$$3 + 5 + 7 + 9 = 4 \times 6$$

$$3 + 5 + 7 + 9 + 11 = 5 \times 7$$

$$3 + 5 + 7 + 9 + 11 + 13 = 6 \times 8$$

- a) Express $3 + 5 + \dots + 25$ in the same way. 2 RE
- b) The first n numbers in this sequence are added. Find a formula for the total. 3 RE

14. A sequence of numbers is **1, 5, 12, 22,**

Numbers from this sequence can be illustrated in the following way using dots.

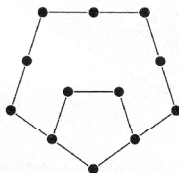
First Number
(N = 1)



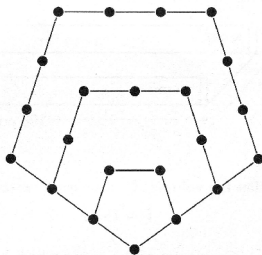
Second Number
(N = 2)



Third Number
(N = 3)



Fourth Number
(N = 4)



a) What is the fifth number in this sequence?

Illustrate this in a sketch.

2 RE

b) The number of dots, D , needed to illustrate the N th number in this sequence is given by the formula

$$D = aN^2 - bN$$

Find the values of a and b .

4 RE

15. Brackets can be multiplied out in the following way.

$$(y+1)(y+2)(y+3) = y^3 + (1+2+3)y^2 + (1 \times 2 + 1 \times 3 + 2 \times 3)y + 1 \times 2 \times 3$$

$$(y+2)(y+3)(y+4) = y^3 + (2+3+4)y^2 + (2 \times 3 + 2 \times 4 + 3 \times 4)y + 2 \times 3 \times 4$$

$$(y+3)(y+4)(y+5) = y^3 + (3+4+5)y^2 + (3 \times 4 + 3 \times 5 + 4 \times 5)y + 3 \times 4 \times 5$$

a) In the same way, multiply out $(y+4)(y+5)(y+6)$

2 RE

b) In the same way, multiply out $(y+a)(y+b)(y+c)$

2 RE

16. The following number pattern can be used to sum consecutive square whole numbers.

$$1^2 + 2^2 = \frac{2 \times 3 \times 5}{6}$$

$$1^2 + 2^2 + 3^2 = \frac{3 \times 4 \times 7}{6}$$

$$1^2 + 2^2 + 3^2 + 4^2 = \frac{4 \times 5 \times 9}{6}$$

a) Express $1^2 + 2^2 + 3^2 + \dots + 10^2$ in the same way.

1 RE

b) Express $1^2 + 2^2 + 3^2 + \dots + 10^2$ in the same way.

2 RE