Higher

Mathematics

Exponentials and Logarithms

Contents

CfE Edition

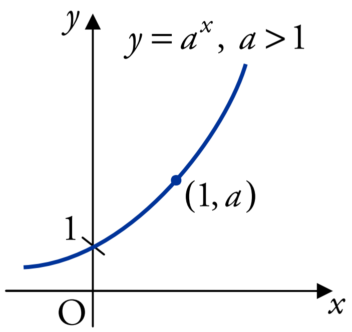
Exponentials and Logarithms

1 Exponentials

We have already met exponential functions in the notes on Functions and Graphs..

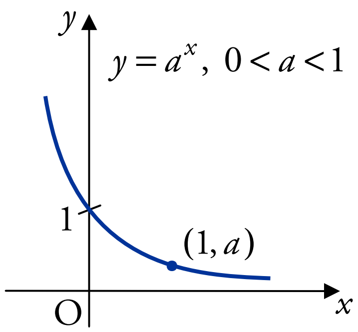
A function of the form $f(x) = a^x$, where $a > 0$ is a constant, is known as an **exponential function** to the base a.

If $a > 1$ then the graph looks like this:



This is sometimes called a **growth** function.

If $0 < a < 1$ then the graph looks like this:



This is sometimes called a **decay** function.

Remember that the graph of an exponential function $f(x) = a^x$ always passes through $(0, 1)$ and $(1, a)$ since:

$$f(0) = a^0 = 1, \qquad f(1) = a^1 = a.$$

EXAMPLES

1. The otter population on an island increases by 16% per year. How many full years will it take the population to double?

Let u_0 be the initial population.

$$u_1 = 1{\cdot}16u_0 \quad \text{(116\% as a decimal)}$$
$$u_2 = 1{\cdot}16u_1 = 1{\cdot}16(1{\cdot}16u_0) = 1{\cdot}16^2 u_0$$
$$u_3 = 1{\cdot}16u_2 = 1{\cdot}16(1{\cdot}16^2 u_0) = 1{\cdot}16^3 u_0$$
$$\vdots$$
$$u_n = 1{\cdot}16^n u_0.$$

For the population to double after n years, we require $u_n \geq 2u_0$.

We want to know the smallest n which gives $1{\cdot}16^n$ a value of 2 or more, since this will make u_n at least twice as big as u_0.

Try values of n until this is satisfied.

If $n = 2,\ 1{\cdot}16^2 = 1{\cdot}35 < 2$

If $n = 3,\ 1{\cdot}16^3 = 1{\cdot}56 < 2$

If $n = 4,\ 1{\cdot}16^4 = 1{\cdot}81 < 2$

If $n = 5,\ 1{\cdot}16^5 = 2{\cdot}10 > 2$

On a calculator: $\boxed{1}\ \boxed{\cdot}\ \boxed{1}\ \boxed{6}\ \boxed{=}$

$\boxed{1}\ \boxed{\cdot}\ \boxed{1}\ \boxed{6}\ \boxed{\text{ANS}}\ \boxed{=}$

$\boxed{=}$

$\vdots$

Therefore after 5 years the population will double.

2. The efficiency of a machine decreases by 5% each year. When the efficiency drops below 75%, the machine needs to be serviced. After how many years will the machine need to be serviced?

Let u_0 be the initial efficiency.

$$u_1 = 0{\cdot}95u_0 \quad \text{(95\% as a decimal)}$$
$$u_2 = 0{\cdot}95u_1 = 0{\cdot}95(0{\cdot}95u_0) = 0{\cdot}95^2 u_0$$
$$u_3 = 0{\cdot}95u_2 = 0{\cdot}95(0{\cdot}95^2 u_0) = 0{\cdot}95^3 u_0$$
$$\vdots$$
$$u_n = 0{\cdot}95^n u_0.$$

When the efficiency drops below $0{\cdot}75u_0$ (75% of the initial value) the machine must be serviced. So the machine needs serviced after n years if $0{\cdot}95^n \leq 0{\cdot}75$.

Try values of n until this is satisfied:

If $n = 2,\ 0{\cdot}95^2 = 0{\cdot}903\ \ > 0{\cdot}75$
If $n = 3,\ 0{\cdot}95^3 = 0{\cdot}857\ \ > 0{\cdot}75$
If $n = 4,\ 0{\cdot}95^4 = 0{\cdot}815\ \ > 0{\cdot}75$
If $n = 5,\ 0{\cdot}95^5 = 0{\cdot}774\ \ > 0{\cdot}75$
If $n = 6,\ 0{\cdot}95^6 = 0{\cdot}735\ \ < 0{\cdot}75$

Therefore after 6 years, the machine will have to be serviced.

2 Logarithms

Having previously defined what a logarithm is (see the notes on Functions and Graphs) we now look in more detail at the properties of these functions.

The relationship between logarithms and exponentials is expressed as:

$$y = \log_a x \iff x = a^y \qquad \text{where } a,\ x > 0.$$

Here, y is the power of a which gives x.

EXAMPLES

1. Write $5^3 = 125$ in logarithmic form.

$$5^3 = 125 \iff 3 = \log_5 125.$$

2. Evaluate $\log_4 16$.

The power of 4 which gives 16 is 2, so $\log_4 16 = 2$.

3 Laws of Logarithms

There are three laws of logarithms which you must know.

Rule 1

$$\log_a x + \log_a y = \log_a (xy) \qquad \text{where } a, x, y > 0.$$

If two logarithmic terms with the same base number (a above) are being added together, then the terms can be combined by multiplying the arguments (x and y above).

EXAMPLE

1. Simplify $\log_5 2 + \log_5 4$.

$$\begin{aligned} &\log_5 2 + \log_5 4 \\ &= \log_5 (2 \times 4) \\ &= \log_5 8. \end{aligned}$$

Rule 2

$$\log_a x - \log_a y = \log_a\left(\tfrac{x}{y}\right) \qquad \text{where } a, x, y > 0.$$

If a logarithmic term is being subtracted from another logarithmic term with the same base number (a above), then the terms can be combined by dividing the arguments (x and y in this case).

Note that the argument which is being taken away (y above) appears on the bottom of the fraction when the two terms are combined.

EXAMPLE

2. Evaluate $\log_4 6 - \log_4 3$.

$$\begin{aligned} &\log_4 6 - \log_4 3 \\ &= \log_4\left(\tfrac{6}{3}\right) \\ &= \log_4 2 \\ &= \tfrac{1}{2} \quad (\text{since } 4^{\frac{1}{2}} = \sqrt{4} = 2). \end{aligned}$$

Rule 3

$$\log_a x^n = n\log_a x \qquad \text{where } a, x > 0.$$

The power of the argument (n above) can come to the front of the term as a multiplier, and vice-versa.

EXAMPLE

3. Express $2\log_7 3$ in the form $\log_7 a$.

$$\begin{aligned} &2\log_7 3 \\ &= \log_7 3^2 \\ &= \log_7 9. \end{aligned}$$

Squash, Split and Fly

You may find the following names are a simpler way to remember the laws of logarithms.

- $\log_a x + \log_a y = \log_a(xy)$ – the arguments are **squashed** together by multiplying.
- $\log_a x - \log_a y = \log_a\left(\tfrac{x}{y}\right)$ – the arguments are **split** into a fraction.
- $\log_a x^n = n\log_a x$ – the power of an argument can **fly** to the front of the log term and vice-versa.

Note

When working with logarithms, you should remember:

$\log_a 1 = 0 \quad \text{since } a^0 = 1, \qquad \log_a a = 1 \quad \text{since } a^1 = a.$

EXAMPLE

4. Evaluate $\log_7 7 + \log_3 3$.

$$\begin{aligned} &\log_7 7 + \log_3 3 \\ &= 1 + 1 \\ &= 2. \end{aligned}$$

Combining several log terms

When adding and subtracting several log terms in the form $\log_a b$, there is a simple way to combine all the terms in one step.

$$\log_a\left(\frac{\text{arguments of \textbf{positive} log terms}}{\text{arguments of \textbf{negative} log terms}}\right)$$

- Multiply the arguments of the positive log terms in the numerator.
- Multiply the arguments of the negative log terms in the denominator.

EXAMPLES

1. Evaluate $\log_{12} 10 + \log_{12} 6 - \log_{12} 5$

$$\begin{aligned} &\log_{12} 10 + \log_{12} 6 - \log_{12} 5 \\ &= \log_{12}\left(\frac{10 \times 6}{5}\right) \\ &= \log_{12} 12 \\ &= 1. \end{aligned}$$

$$\log_{12}\left(\frac{+\log_{12} 10 \qquad +\log_{12} 6}{-\log_{12} 5}\right)$$

2. Evaluate $\log_6 4 + 2\log_6 3$.

$$\begin{aligned} &\log_6 4 + 2\log_6 3 \\ &= \log_6 4 + \log_6 3^2 \\ &= \log_6 4 + \log_6 9 \\ &= \log_6 (4 \times 9) \\ &= \log_6 36 \\ &= 2 \qquad (\text{since } 6^2 = 36). \end{aligned}$$

OR

$$\begin{aligned} &\log_6 4 + 2\log_6 3 \\ &= \log_6 2^2 + 2\log_6 3 \\ &= 2\log_6 2 + 2\log_6 3 \\ &= 2(\log_6 2 + \log_6 3) \\ &= 2(\log_6 (2 \times 3)) \\ &= 2\log_6 6 \\ &= 2 \qquad (\text{since } \log_6 6 = 1). \end{aligned}$$

4 Exponentials and Logarithms to the Base *e*

The constant e is an important number in Mathematics, and occurs frequently in models of real-life situations. Its value is roughly 2·718281828 (to 9 d.p.), and is defined as:

$$e = \left(1 + \frac{1}{n}\right)^n \quad \text{as } n \to \infty .$$

If you try very large values of n on your calculator, you will get close to the value of e. Like π, e is an irrational number.

Throughout this section, we will use e in expressions of the form:

- e^x, which is called an exponential to the base e;
- $\log_e x$, which is called a logarithm to the base e. This is also known as the **natural logarithm** of x, and is often written as $\ln x$ (i.e. $\ln x \equiv \log_e x$).

EXAMPLES

1. Calculate the value of $\log_e 8$.

$\log_e 8 = 2{\cdot}08$ (to 2 d.p.). On a calculator: $\boxed{\ln}\ \boxed{8}\ \boxed{=}$

2. Solve $\log_e x = 9$.

$$\begin{aligned} \log_e x &= 9 \\ \text{so } x &= e^9 \\ x &= 8103{\cdot}08 \text{ (to 2 d.p.).} \end{aligned}$$

On a calculator: $\boxed{e^x}\ \boxed{9}\ \boxed{=}$

3. Simplify $4\log_e(2e) - 3\log_e(3e)$ expressing your answer in the form $a + \log_e b - \log_e c$ where a, b and c are whole numbers.

$$\begin{aligned} & 4\log_e(2e) - 3\log_e(3e) \\ &= 4\log_e 2 + 4\log_e e - 3\log_e 3 - 3\log_e e \\ &= 4\log_e 2 + 4 - 3\log_e 3 - 3 \\ &= 1 + 4\log_e 2 - 3\log_e 3 \\ &= 1 + \log_e 2^4 - \log_e 3^3 \\ &= 1 + \log_e 16 - \log_e 27. \end{aligned}$$

OR

$$\begin{aligned} & 4\log_e(2e) - 3\log_e(3e) \\ &= \log_e(2e)^4 - \log_e(3e)^3 \\ &= \log_e\left(\frac{(2e)^4}{(3e)^3}\right) \\ &= \log_e\left(\frac{16e^4}{27e^3}\right) \\ &= \log_e\left(\frac{16e}{27}\right) \\ &= \log_e e + \log_e 16 - \log_e 27 \\ &= 1 + \log_e 16 - \log_e 27. \end{aligned}$$

Remember
$(ab)^n = a^n b^n$.

5 Exponential and Logarithmic Equations

Many mathematical models of real-life situations use exponentials and logarithms. It is important to become familiar with using the laws of logarithms to help solve equations.

EXAMPLES

1. Solve $\log_a 13 + \log_a x = \log_a 273$ for $x > 0$.

$$\begin{aligned} \log_a 13 + \log_a x &= \log_a 273 \\ \log_a 13x &= \log_a 273 \\ 13x &= 273 \qquad (\text{since } \log_a x = \log_a y \Leftrightarrow x = y) \\ x &= 21. \end{aligned}$$

2. Solve $\log_{11}(4x+3) - \log_{11}(2x-3) = 1$ for $x > \frac{3}{2}$.

$$\begin{aligned} \log_{11}(4x+3) - \log_{11}(2x-3) &= 1 \\ \log_{11}\left(\frac{4x+3}{2x-3}\right) &= 1 \\ \frac{4x+3}{2x-3} &= 11^1 = 11 \qquad (\text{since } \log_a x = y \Leftrightarrow x = a^y) \\ 4x+3 &= 11(2x-3) \\ 4x+3 &= 22x-33 \\ 18x &= 36 \\ x &= 2. \end{aligned}$$

3. Solve $\log_a(2p+1) + \log_a(3p-10) = \log_a(11p)$ for $p > 4$.

$$\begin{aligned} \log_a(2p+1) + \log_a(3p-10) &= \log_a(11p) \\ \log_a\left[(2p+1)(3p-10)\right] &= \log_a(11p) \\ (2p+1)(3p-10) &= 11p \\ 6p^2 - 20p + 3p - 10 - 11p &= 0 \\ 6p^2 - 28p - 10 &= 0 \\ (3p+1)(p-5) &= 0 \end{aligned}$$

$$3p+1 = 0 \quad \text{or} \quad p-5 = 0$$

$$p = -\tfrac{1}{3} \qquad\qquad p = 5.$$

Since we require $p > 4$, $p = 5$ is the solution.

Dealing with Constants

Sometimes it may help to write constants as logs to solve equations.

EXAMPLE

4. Solve $\log_2 7 = \log_2 x + 3$ for $x > 0$.

Write 3 in logarithmic form:

$$\begin{aligned} 3 &= 3 \times 1 \\ &= 3\log_2 2 \quad (\text{since } \log_2 2 = 1) \\ &= \log_2 2^3 \\ &= \log_2 8. \end{aligned}$$

Use this in the equation:

$$\begin{aligned} \log_2 7 &= \log_2 x + \log_2 8 \\ \log_2 7 &= \log_2 8x \\ 7 &= 8x \\ x &= \tfrac{7}{8}. \end{aligned}$$

OR

$$\begin{aligned} \log_2 7 &= \log_2 x + 3 \\ \log_2 7 - \log_2 x &= 3 \\ \log_2\left(\tfrac{7}{x}\right) &= 3. \end{aligned}$$

Converting from log to exponential form:

$$\begin{aligned} \tfrac{7}{x} &= 2^3 \\ x &= \tfrac{7}{2^3} \\ &= \tfrac{7}{8}. \end{aligned}$$

Solving Equations with unknown Exponents

If an unknown value (e.g. x) is the power of a term (e.g. e^x or 10^x), and its value is to be calculated, then we must take logs on both sides of the equation to allow it to be solved.

The same solution will be reached using any base, but calculators can be used for evaluating logs to the base e and 10.

EXAMPLES

5. Solve $e^x = 7$.

Taking $\log_e$ of both sides:

$$\begin{aligned} \log_e e^x &= \log_e 7 \\ x\log_e e &= \log_e 7 \quad (\log_e e = 1) \\ x &= \log_e 7 \\ x &= 1{\cdot}946 \quad (\text{to 3 d.p.}). \end{aligned}$$

OR Taking $\log_{10}$ of both sides:

$$\begin{aligned} \log_{10} e^x &= \log_{10} 7 \\ x\log_{10} e &= \log_{10} 7 \\ x &= \frac{\log_{10} 7}{\log_{10} e} \\ x &= 1{\cdot}946 \quad (\text{to 3 d.p.}). \end{aligned}$$

6. Solve $5^{3x+1} = 40$.

$$\begin{aligned} \log_e 5^{3x+1} &= \log_e 40 \\ (3x+1)\log_e 5 &= \log_e 40 \\ 3x+1 &= \frac{\log_e 40}{\log_e 5} \\ 3x+1 &= 2{\cdot}2920 \\ 3x &= 1{\cdot}2920 \\ x &= 0{\cdot}431 \quad \text{(to 3 d.p.).} \end{aligned}$$

Note
$\log_{10}$ could have been used instead of $\log_e$.

Exponential Growth and Decay

Recall from Section 1 that exponential functions are sometimes known as growth or decay functions. These often occur in models of real-life situations.

For instance, radioactive decay can be modelled using an exponential function. An important measurement is the **half-life** of a radioactive substance, which is the time taken for the mass of the radioactive substance to halve.

EXAMPLE

7. The mass G grams of a radioactive sample after time t years is given by the formula $G = 100e^{-3t}$.

(a) What is the initial mass of radioactive substance in the sample?

(b) Find the half-life of the radioactive substance.

(a) The initial mass was present when $t = 0$:

$$\begin{aligned} G &= 100e^{-3\times 0} \\ &= 100e^{0} \\ &= 100. \end{aligned}$$

So the initial mass was 100 grams.

(b) The half-life is the time t at which $G = 50$, so

$$\begin{aligned} 100e^{-3t} &= 50 \\ e^{-3t} &= \tfrac{1}{2} \\ -3t &= \log_e\left(\tfrac{1}{2}\right) \qquad \text{(converting to log form)} \\ t &= 0{\cdot}231 \text{ (to 3 d.p.).} \end{aligned}$$

So the half-life is $0{\cdot}231$ years, roughly $0{\cdot}231 \times 356 = 84{\cdot}315$ days.

8. The world population, in billions, t years after 1950 is given by $P = 2{\cdot}54e^{0{\cdot}0178t}$.

   (a) What was the world population in 1950?

   (b) Find, to the nearest year, the time taken for the world population to double.

(a) For 1950, $t = 0$:

$$\begin{aligned} P &= 2{\cdot}54e^{0{\cdot}0178\times 0} \\ &= 2{\cdot}54e^{0} \\ &= 2{\cdot}54. \end{aligned}$$

So the world population in 1950 was $2{\cdot}54$ billion.

(b) For the population to double:

$$\begin{aligned} 2{\cdot}54e^{0{\cdot}0178t} &= 2\times 2{\cdot}54 \\ e^{0{\cdot}0178t} &= 2 \\ 0{\cdot}0178t &= \log_e 2 \qquad \text{(converting to log form)} \\ t &= 38{\cdot}94 \text{ (to 2 d.p.).} \end{aligned}$$

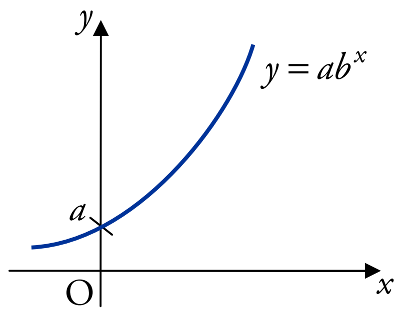
So the population doubled after 39 years (to the nearest year).

6 Graphing with Logarithmic Axes

It is common in applications to find an exponential relationship between variables; for instance, the relationship between the world population and time in the example above. Given some data (e.g. from an experiment) we would like to find an explicit equation for the relationship.

Relationships of the form $y = ab^x$

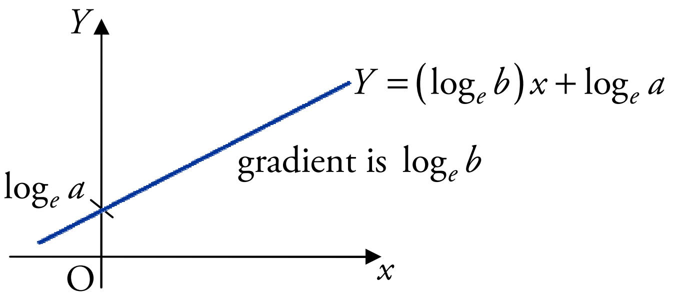
Suppose we have an exponential graph $y = ab^x$, where $a, b > 0$.



Taking logarithms we find that

$$\begin{aligned}\log_e y &= \log_e\left(ab^x\right)\\ &= \log_e a + \log_e b^x\\ &= \log_e a + x\log_e b.\end{aligned}$$

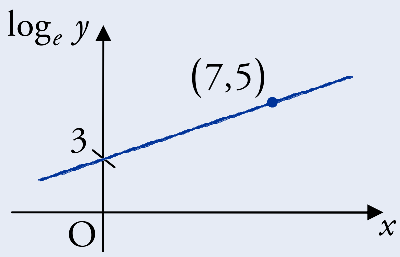
We can scale the y-axis so that $Y = \log_e y$; the Y-axis is called a **logarithmic axis**. Now our relationship is of the form $Y = \left(\log_e b\right)x + \log_e a$, which is a straight line in the (x, Y)-plane.



Since this is just a straight line, we can use known points to find the gradient $\log_e b$ and the Y-axis intercept $\log_e a$. From these we can easily find the values of a and b, and hence specify the equation $y = ab^x$.

EXAMPLES

1. The relationship between two variables, x and y, is of the form $y = ab^x$, where a and b are constants. An experiment to test this relationship produced the data shown in the graph, where $\log_e y$ is plotted against x.



Find the values of a and b.

We ned to obtain a straight line equation:

$$\begin{aligned} y &= ab^x\\ \log_e y &= \log_e ab^x \qquad \text{(taking logs of both sides)}\\ \log_e y &= \log_e a + \log_e b^x\\ \log_e y &= \log_e a + x\log_e b\\ Y &= \left(\log_e b\right)x + \log_e a.\end{aligned}$$

From the graph, the Y-axis intercept is $\log_e a = 3$; so $a = e^3$.

Using the gradient formula:

$$\begin{aligned}\log_e b &= \frac{5-3}{7-0} \\ &= \tfrac{2}{7} \\ b &= e^{\frac{2}{7}}.\end{aligned}$$

2. The results from an experiment were noted as follows:

x	$1{\cdot}30$	$2{\cdot}00$	$2{\cdot}30$	$2{\cdot}80$
$\log_e y$	$2{\cdot}04$	$2{\cdot}56$	$2{\cdot}78$	$3{\cdot}14$

The relationship between these data can be written in the form $y = ab^x$.

Find the values of a and b, and state the formula for y in terms of x.

We need to obtain a straight line equation:

$$\begin{aligned}y &= ab^x \\ \log_e y &= \log_e ab^x \qquad \text{(taking logs of both sides)} \\ \log_e y &= \log_e a + \log_e b^x \\ \log_e y &= \log_e a + x\log_e b \\ \log_e y &= (\log_e b)x + \log_e a.\end{aligned}$$

We can find the gradient $\log_e b$ (and hence b), using two points on the line: using $(1{\cdot}30, 2{\cdot}04)$ and $(2{\cdot}80, 3{\cdot}14)$,

$$\log_e b = \frac{3{\cdot}14 - 2{\cdot}04}{2{\cdot}80 - 1{\cdot}30} = 0{\cdot}73 \text{ (to 2 d.p.).}$$

So $b = e^{0.73} = 2{\cdot}08$ (to 2 d.p.).

Note that $\log_e y = 0{\cdot}73x + \log_e a$. We can work out $\log_e a$ (and hence a) by substituting a point into this equation: using $(1{\cdot}30, 2{\cdot}04)$,

$$\begin{aligned}2{\cdot}04 &= 0{\cdot}73 \times 1{\cdot}30 + \log_e a \\ \log_e a &= 2{\cdot}04 - 0{\cdot}73 \times 1{\cdot}30 \\ &= 1{\cdot}09 \text{ (to 2 d.p.)} \\ a &= e^{1{\cdot}09} \\ &= 2{\cdot}97 \text{ (to 2 d.p.).}\end{aligned}$$

Therefore $y = 2{\cdot}97 \times 2{\cdot}08^x$.

Note
Depending on the points used, slightly different values for a and b may be obtained.

Equations in the form $y = ax^b$

Another common relationship is $y = ax^b$, where $a > 0$. In this case, the relationship can be represented by a straight line if we change *both* axes to logarithmic ones.

EXAMPLE

3. The results from an experiment were noted as follows:

$\log_{10} x$	$1{\cdot}70$	$2{\cdot}29$	$2{\cdot}70$	$2{\cdot}85$
$\log_{10} y$	$1{\cdot}33$	$1{\cdot}67$	$1{\cdot}92$	$2{\cdot}01$

The relationship between these data can be written in the form $y = ax^b$.

Find the values of a and b, and state the formula for y in terms of x.

We need to obtain a straight line equation:

$$\begin{aligned} y &= ax^b \\ \log_{10} y &= \log_{10} ax^b \quad \text{(taking logs of both sides)} \\ \log_{10} y &= \log_{10} a + \log_{10} x^b \\ \log_{10} y &= \log_{10} a + b\log_{10} x \\ Y &= bX + \log_{10} a. \end{aligned}$$

We can find the gradient b, using two points on the line: using $(1{\cdot}70, 1{\cdot}33)$ and $(2{\cdot}85, 2{\cdot}01)$,

$$b = \frac{2{\cdot}01 - 1{\cdot}33}{2{\cdot}85 - 1{\cdot}70} = 0{\cdot}59 \quad \text{(to 2 d.p.)}$$

So $\log_{10} y = 0{\cdot}59 \log_{10} x + \log_{10} a$.

Now we can work out a by substituting a point into this equation: using $(1{\cdot}70, 1{\cdot}33)$,

$$\begin{aligned} 1{\cdot}33 &= 0{\cdot}59 \times 1{\cdot}70 + \log_{10} a \\ \log_{10} a &= 1{\cdot}33 - 0{\cdot}59 \times 1{\cdot}70 \\ &= 0{\cdot}33 \\ a &= 10^{0{\cdot}33} \\ &= 2{\cdot}14 \quad \text{(to 2 d.p.).} \end{aligned}$$

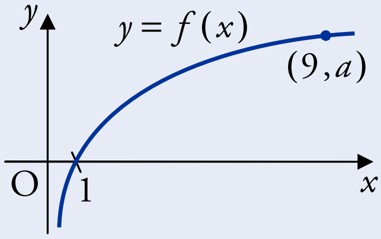
Therefore $y = 2{\cdot}14x^{0{\cdot}59}$.

7 Graph Transformations

Graph transformations were covered in the notes on Functions and Graphs, but we can now look in more detail at applying transformations to graphs of exponential and logarithmic functions.

EXAMPLES

1. Shown below is the graph of $y = f(x)$ where $f(x) = \log_3 x$.



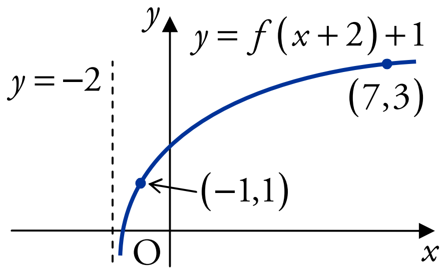
(a) State the value of a.

(b) Sketch the graph of $y = f(x+2)+1$.

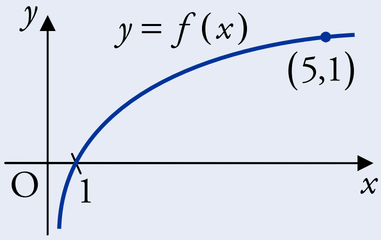
(a) $a = \log_3 9$

$= 2 \quad$ (since $3^2 = 9$).

(b) The graph shifts two units to the left, and one unit upwards:



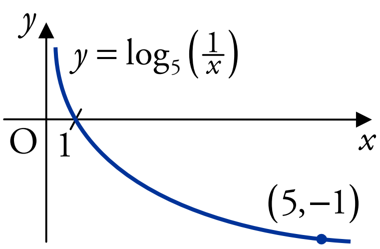
2. Shown below is part of the graph of $y = \log_5 x$.



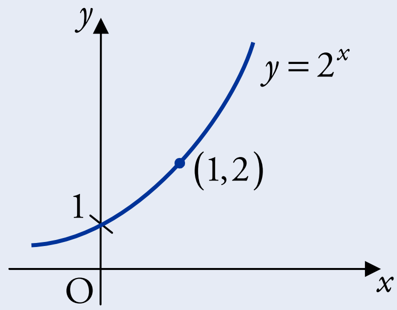
Sketch the graph of $y = \log_5\left(\frac{1}{x}\right)$.

$$y = \log_5\left(\frac{1}{x}\right)$$
$$= \log_5 x^{-1}$$
$$= -\log_5 x$$

So reflect in the x-axis.



3. The diagram shows the graph of $y = 2^x$.

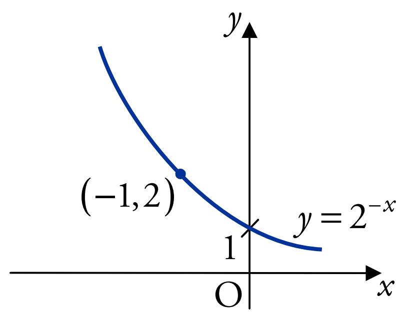


On separate diagrams, sketch the graphs of:

(a) $y = 2^{-x}$;

(b) $y = 2^{2-x}$.

(a) Reflect in the y-axis:



(b) $y = 2^{2-x}$

$= 2^2 2^{-x}$

$= 4 \times 2^{-x}$.

So scale the graph from (a) by 4 in the y-direction: