HIGHER MATHEMATICS

OBJECTIVE QUESTIONS

AUGUST 2007

Items 1-72
These cover the full range of Examinable Content where appropriate.

Items 73-144
These are a repeat of 1-72.

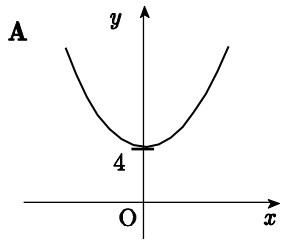
Cat. no	ans	Syll.	Code	item no
1	D	A	1	204
2	C	A	2	244
3	B	A	3	924
4	D	A	4	11
5	C	A	5	926
6	D	A	6	1105
7	C	A	7	273
8	C	A	10	1073
9	A	A	11	1237
10	D	A	12	1364
11	A	A	13	1160
12	B	A	15	1240
13	C	A	16	289
14	B	A	17	1242
15	D	A	18	932
16	A	A	19	1115
17	D	A	21	33
18	C	A	28	66
19	A	A	31	198
20	C	A	32	325
21	B	A	33	249
22	D	C	1	71
23	A	C	2	937
24	D	C	3	378
25	B	C	4	21
26	D	C	6	324
27	B	C	7	48
28	C	C	8	141
29	C	C	11	47
30	C	C	12	53
31	C	C	13	154
32	A	C	14	354
33	C	C	15	1479
34	B	C	16	185
35	C	C	17	166
36	B	C	18	46
37	D	C	20	61
38	B	C	21	45
39	C	C	22	26
40	B	C	23	81
41	D	G	1	946
42	B	G	2	175
43	A	G	3	2
44	B	G	4	372
45	B	G	5	104
46	D	G	6	973
47	C	G	7	949
48	B	G	9	83
49	D	G	10	93
50	B	G	11	357
51	A	G	12	37
52	D	G	16	305
53	C	G	17	90
54	D	G	18	955
55	A	G	19	19
56	D	G	20	351
57	B	G	21	958
58	A	G	22	959
59	D	G	24	315
60	B	G	25	961
61	C	G	26	962
62	C	G	27	162
63	D	G	28	116
64	B	G	29	102
65	C	T	1	67
66	C	T	3	64
67	B	T	4	224
68	D	T	5	131
69	B	T	7	148
70	C	T	8	20
71	D	T	9	393
72	B	T	12	967
73	A	A	1	1325
74	A	A	2	1070
75	A	A	3	1433
76	D	A	4	108
77	B	A	5	1359
78	B	A	6	1462
79	C	A	7	1361
80	B	A	10	1337
81	A	A	11	1322
82	C	A	12	1440
83	C	A	13	1365
84	A	A	15	1366
85	B	A	16	1318
86	A	A	17	1343
87	A	A	18	1344
88	A	A	19	1244
89	D	A	21	257
90	D	A	28	80
91	A	A	31	1117
92	A	A	32	349
93	B	A	33	388
94	A	C	1	1246
95	A	C	2	938
96	A	C	3	1080
97	C	C	4	76
98	C	C	6	940
99	A	C	7	124
100	B	C	8	261
101	A	C	11	201
102	D	C	12	1078
103	B	C	13	943
104	A	C	14	1312
105	D	C	15	1480
106	A	C	16	255
107	A	C	17	1149
108	B	C	18	1015
109	B	C	20	1167
110	B	C	21	126
111	B	C	22	214
112	D	C	23	194
113	D	G	1	977
114	C	T	3	74
115	C	G	3	1263
116	A	G	4	1203
117	D	G	5	132
118	D	G	6	1047
119	A	G	7	1283
120	B	G	9	232
121	D	G	10	1025
122	C	G	11	953
123	C	G	12	1028
124	D	G	16	954
125	B	G	17	231
126	D	G	18	978
127	B	G	19	1051
128	C	G	20	957
129	B	G	21	994
130	A	G	22	991
131	A	G	24	960
132	C	G	25	988
133	A	G	26	1058
134	A	G	27	963
135	B	G	28	1193
136	B	G	29	982
137	C	T	1	112
138	A	T	3	1041
139	C	T	4	964
140	D	T	5	363
141	B	T	7	145
142	A	T	8	984
143	C	T	9	1260
144	B	T	12	981

1. For which real value of x is the function f given by $f(x) = \frac{1}{\sqrt{1-x^2}}$ defined on the set of real numbers ?

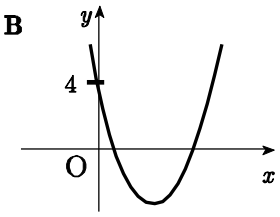
A all x except 1 and -1
B $x < 1$ only
C $x > 1$, $x < -1$ only
D $-1 < x < 1$ only

2. Which of the graphs shown below is most likely to be the graph with equation $y = 3x^2 - 2x + 4$?

A



B



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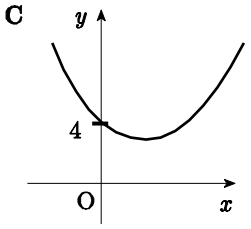

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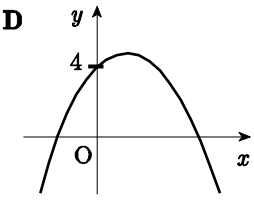
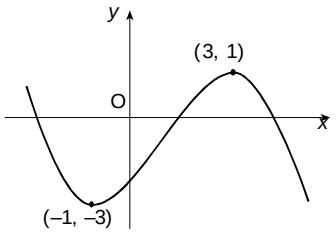
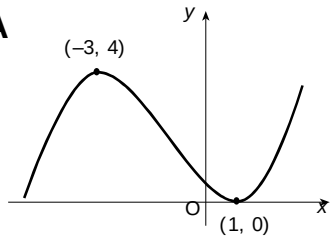


3. The diagram shows part of the graph of a function with equation $y = f(x)$.

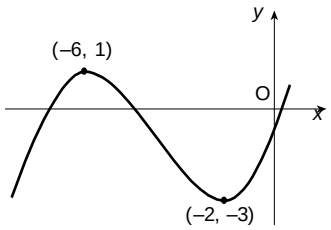


Which of the following diagrams shows the graph with equation $y = f(3 - x)$?

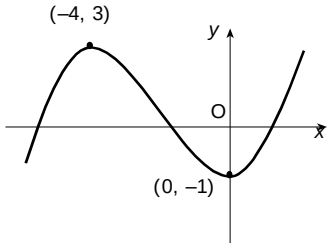
A



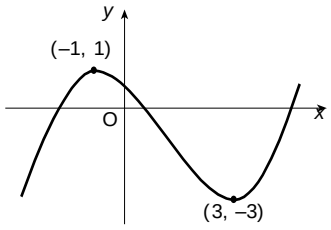
B



C



D



4. $f(x) = 2x^2 - 4$ and $g(x) = 1 - x$ define functions on the set of real numbers.

What is the value of $f(g(2))$?

A 4
B 3
C 0
D −2

5. When $2x^2 - 12x + 13$ is written in the form $2(x + q)^2 + r$, what is the value of r ?

A 13
B 1
C −5
D −13

6. A function f is given by $f(x) = (x - 2)^2 - 3$.

The function g is given by $g(x) = \frac{1}{f(x) + 10}$.

Which of the following statements about the stationary value of g is true ?

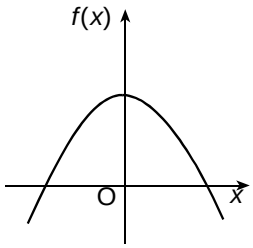
A minimum value of g is 7

B maximum value of g is 7

C minimum value of g is $\frac{1}{7}$

D maximum value of g is $\frac{1}{7}$

7. The diagram shows the graph of the function *f* where $f(x) = p(x-q)^2 + r$.
The line $x = 0$ is an axis of symmetry of the curve. Which of the following is true about *p*, *q* and *r* ?



A $p > 0, q > 0, r > 0$

B $p > 0, q = 0, r < 0$

C $p < 0, q = 0, r > 0$

D $p < 0, q < 0, r = 0$

8. The population of hamsters in a breeding centre increases by 5% during each month.
At the end of each month the breeder sells 30 hamsters.
If u_n represents the hamster population at the beginning of a month, find an expression for u_{n+1}.

A $u_{n+1} = 1 \cdot 5u_n + 30$

B $u_{n+1} = 5u_n - 30$

C $u_{n+1} = 1 \cdot 05u_n - 30$

D $u_{n+1} = 0 \cdot 95u_n + 30$

9. A sequence is defined by the recurrence relation $u_{n+1} = au_n + b$ and $u_0 = 4$.
Express u_2 in terms of *a* and *b*.

A $u_2 = 4a^2 + ab + b$

B $u_2 = 4 + 2b$

C $u_2 = 4a^2 + a^2b$

D $u_2 = 2a + b$

10. A sequence is defined by the recurrence relation $u_{n+1} = 0 \cdot 5u_n + 2$ and $u_0 = 8$.
Here are two statements about this sequence:

(1) A limit exists for this sequence.

(2) No term in the sequence is greater than 8.

Which of the following is true ?

A neither statement is correct

B only statement (1) is correct

C only statement (2) is correct

D both statements are correct

11. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n - 7$ and $u_0 = -2$.
What is the limit of this sequence as $n \to \infty$?

A $-\frac{21}{2}$

B $-\frac{7}{3}$

C $-\frac{1}{18}$

D $-\frac{1}{24}$

12. A parabola has equation $y = x^2 + 6x - 8$.
At what value of *x* does the minimum point of the parabola occur ?

A −8
B −3
C 0
D 3

13. Find the solution of $x^2 + x - 12 < 0$.

A $x < -4$ *or* $x > 3$

B $x < -3$ *or* $x > 4$

C $-4 < x < 3$

D $-3 < x < 4$

14. Here are two statements about the equation $(x-3)^2 = 17$:

(1) the roots of the equation are real

(2) the roots of the equation are equal

Which of the following is true ?

A neither statement is correct

B only statement (1) is correct

C only statement (2) is correct

D both statements are correct

15. The equation $x^2 + 2x + p = 0$ has no real roots.
What is the range of values of p ?

A $p < -1$

B $p < 0$

C $p > 0$

D $p > 1$

16. The roots of a quadratic equation are -1 and p.
Which of the following could be the quadratic equation ?

A $x^2 + (1-p)x - p = 0$

B $x^2 - (1+p)x + p = 0$

C $x^2 + (1+p)x + p = 0$

D $x^2 + (p-1)x - p = 0$

17. If $x-1$ is a factor of $x^3 - 6x^2 + px - 6$, what is the value of p ?

A -6

B -1

C 1

D 11

18. If $\log(x) = 2\log(y) - 3\log(z)$, find an expression for x in terms of y and z.

A $x = 2y - 3z$

B $x = \frac{2y}{3z}$

C $x = \frac{y^2}{z^3}$

D $x = 2y + \frac{z}{3}$

19. Given that $\log_a(64) = \frac{3}{2}$, what is the value of a ?

A 16

B $42\frac{2}{3}$

C 96

D 512

20. Given that $\log_{10}(y) = 2\log_{10}(x) + \log_{10}(3)$, express y in terms of x.

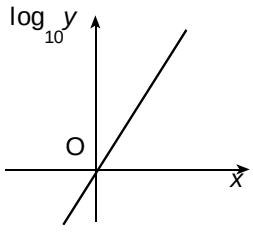
A $y = 2x + 3$

B $y = 6x$

C $y = 3x^2$

D $y = 3 \times 2^x$

21. The diagram shows the graph of $\log_{10}(y)$ plotted against x. The graph is a straight line through the origin with gradient 2.



What is the equation of this line ?

A $y = 2x$

B $y = 10^{2x}$

C $y = 10^{x^2}$

D $y = x^2$

22. If $f(x) = 4x^3 + 5$, what is the value of $f'(2)$?

A 22
B 26
C 37
D 48

23. If $f(x) = 6x^3 - 2x^{-\frac{1}{2}}$ find $f'(x)$.

A $18x^2 + x^{-\frac{3}{2}}$

B $2x^2 + 4x^{\frac{1}{2}}$

C $6x^2 - x^{-\frac{3}{2}}$

D $18x^2 + x^{\frac{1}{2}}$

24. Given that $f(x) = \frac{x^2 + 1}{x}$, $x \neq 0$, find $f'(x)$.

A $2x$

B $2x + 1$

C 1

D $1 - \frac{1}{x^2}$

25. The tangent to the curve with equation $y = 2x^2 - 1$ is drawn at the point where $x = 0$.

What is the gradient of this tangent ?

A -1
B 0
C 1
D 2

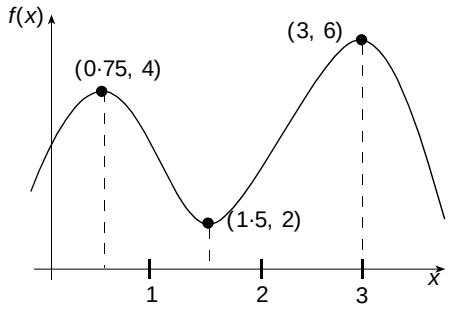
26. The function f is defined by $f(x) = 4x^3 - x^4$, where x is a real number.
What is the rate of change of f with respect to x at $x = -1$?

A -6
B -5
C 5
D 16

27. The graph of $y = f(x)$ is shown with stationary points at $x = 0 \cdot 75$, $x = 1 \cdot 5$ and $x = 3$.

Here are two statements about $f'(x)$:

(1) $f'(1) < 0$
(2) $f'(2) < 0$



Which of the following is true?

A neither statement is correct
B only statement (1) is correct
C only statement (2) is correct
D both statements are correct

28. $f(x) = ax^2 - 2x - 5$ has a stationary value where $x = 3$.
What is the value of a ?

A -1

B 0

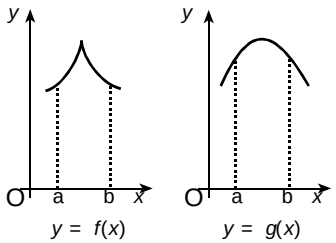
C $\frac{1}{3}$

D $\frac{11}{9}$

29. The diagram shows the graphs of two functions, f and g.
Here are two statements about the functions in the interval $a \leq x \leq b$:

(1) Function f is differentiable for all values of x

(2) Function g is differentiable for all values of x.



Which of the following is true?

A neither statement is correct

B only statement (1) is correct

C only statement (2) is correct

D both statements are correct

30. Find $\int_{-1}^{1} x^4 \, dx$.

A 0

B $\frac{1}{4}$

C $\frac{2}{5}$

D 8

31. Find $\int \left(1 - x^{-\frac{3}{2}}\right)$

A $2x^{-\frac{1}{2}} + c$

B $x + 2x^{-\frac{1}{2}} + c$

C $x - 2x^{\frac{1}{2}} + c$

D $x - 2x^{\frac{3}{2}} + c$

32. Find $\int \left(x^4 + \frac{1}{x^4}\right)\, dx$

A $\frac{x^5}{5} - \frac{1}{3x^3} + c$

B $4x^3 - \frac{4}{x^5} + c$

C $\frac{x^5}{5} + \frac{1}{5x^5} + c$

D $\frac{x^5}{5} + \frac{1}{4x^3} + c$

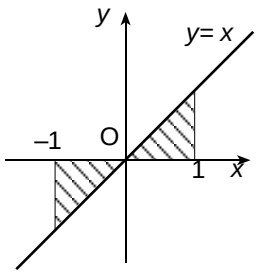
33. What is the value of $\int_{-1}^{3} 3x^2\, dx$?

A 20
B 24
C 28
D 32

34. Here are two statements about the numerical value of the shaded area shown in the diagram:

(1) Shaded area = $2\int_{0}^{1} x\,dx$

(2) Shaded area = $\int_{-1}^{1} x\,dx$.



Which is of the following is true ?

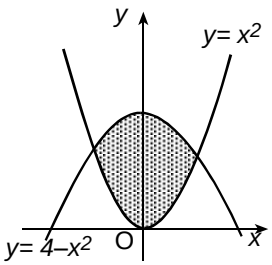
A neither statement is correct

B only statement (1) is correct

C only statement (2) is correct

D both statements are correct

35. The diagram shows the curves with equations $y = x^2$ and $y = 4 - x^2$.



Which of the following integrals gives the shaded area ?

A $\int_0^4 \left(4 - 2x^2\right)\, dx.$

B $\int_{-2}^2 \left(4 - 2x^2\right)\, dx.$

C $\int_{-\sqrt{2}}^{\sqrt{2}} \left(4 - 2x^2\right)\, dx.$

D $\int_0^{\sqrt{2}} \left(2x^2 - 4\right)\, dx.$

36. If $\frac{dy}{dx} = 2x + 1$ and $y = 3$ when $x = 1$, express y in terms of x.

A $y = x^2$

B $y = x^2 + x + 1$

C $y = 2$

D $y = x^2 + 2$

37. Given that $f(x)=\cos\left(3x^2+5\right)$, find $f'(x)$.

A $3\sin\left(3x^2+5\right)$

B $3\cos\left(3x^2+5\right)$

C $-\sin(6x)$

D $-6x\sin\left(3x^2+5\right)$

38. If $f(x)=\left(2x^2-1\right)^3$, find $f'(x)$.

A $\frac{1}{16x}\left(2x^2-1\right)^4$

B $12x\left(2x^2-1\right)^2$

C $48x^5$

D $48x^2$

39. Find $\int (4x-1)^2\ dx$.

A $\frac{1}{3}\left(2x^2-x\right)^3+c$

B $12(4x-1)^3+c$

C $\frac{1}{12}(4x-1)^3+c$

D $\left(2x^2-x\right)^2+c$

40. Find $\int_0^{\frac{\pi}{4}} \cos 2x \ dx$.

A $-2\sqrt{2}$

B $\frac{1}{2}$

C 0

D $\sqrt{2}$

41. What is the distance between the points (−2, 5, 3) and (4, −1, 1) ?

A 6
B 10
C $2\sqrt{14}$
D $2\sqrt{19}$

42. The line joining the points (−2, −3) and (6, *k*) has gradient $\frac{2}{3}$.
What is the value of *k* ?

A $\frac{14}{3}$

B $\frac{7}{3}$

C $-\frac{1}{3}$

D -9

43. A straight line passes through the points P(−5, −2) and Q(−2, −1).
What is the equation of the straight line which passes through P and is perpendicular to PQ ?

A $y+2=-3(x+5)$

B $y-2=-\frac{3}{7}(x-5)$

C $y-1=-\frac{3}{7}(x-2)$

D $y-1=-\frac{1}{3}(x-2)$

44. The equation $ax + y + 4a = 0$ defines a set of straight lines for different values of a, where $a \neq 0$.
Here are two statements about this set of lines:
(1) All cut the *x*-axis at the same point
(2) They are parallel

Which of the following is true ?

A neither statement is correct
B only statement (1) is correct
C only statement (2) is correct
D both statements are correct

45. P and Q are the points (2, 3) and (−1, 4).
What is the gradient of a line perpendicular to PQ ?

A $-\frac{8}{7}$
B 3
C 5
D 7

46. P is the point (a, −2) and Q is (0, b).
M(1, 2) is the midpoint of PQ.
What are the values of a and b ?

	a	b
A	1	−6
B	1	6
C	2	−6
D	2	6

47. Triangle OPQ has vertices at 0(0, 0), P(5, 3) and Q(1, −7).
OS is a median. What are the coordinates of S ?

A (−5, −2)
B (3, −5)
C (3, −2)
D (2, 5)

48. A circle has equation $x^2 + y^2 = 4 - 4x + 2y$.
What is the radius of this circle ?

A 2
B 3
C 4
D 5

49. PQ is a diameter of a circle.
P and Q have coordinates (3, 2) and (7, 2) respectively.
What is the equation of this circle ?

A $(x-3)^2 + (y-2)^2 = 16$

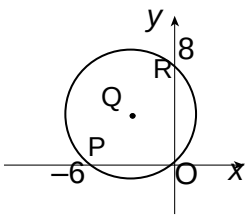
B $(x-4)^2 + y^2 = 2$

C $(x+5)^2 + (y+2)^2 = 2$

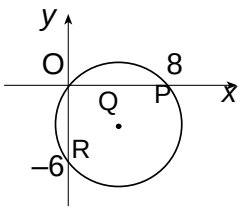
D $(x-5)^2 + (y-2)^2 = 4$

50. The following diagrams each show a circle with centre Q(a, b) and radius 5 units, cutting the x and y axes in P and R respectively. In which diagram would the gradient of the tangent at P equal $-\frac{4}{3}$?

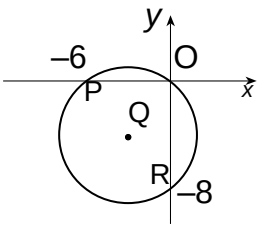
A



B



C



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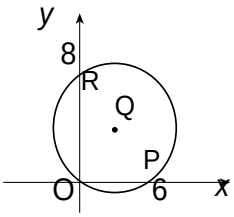


51. The line with equation $y = k$ intersects the circle with equation $x^2 + y^2 = 4$ in at least one point.

What is the range of values of k ?

A $-2 \le k \le 2$

B $-4 \le k \le 4$

C $k \ge 2,\ k \le -2$

D $k \ge 4,\ k \le -4$

52. Given that $\boldsymbol{u} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$, what is the magnitude of $(\boldsymbol{u} - \boldsymbol{v})$?

A 1

B $\sqrt{20}$

C $\sqrt{32}$

D $\sqrt{34}$

53. P, Q and R are points such that $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$, $\overrightarrow{PR} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ and R is (0, 2, 1).

What are the coordinates of Q ?

A (−1, 3, 2)

B (−1, −1, 0)

C (1, 1, 0)

D (2, 0, 1)

54. The vector $\boldsymbol{u}$ is given by $\boldsymbol{u} = \frac{1}{4}\boldsymbol{i} + p\boldsymbol{k}$ where $p > 0$.
If $\boldsymbol{u}$ is a unit vector, what is the value of p ?

A $\frac{3}{4}$

B 1

C $\frac{\sqrt{17}}{16}$

D $\frac{\sqrt{15}}{4}$

55. For what value of z are the vectors $\begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ -9 \\ z \end{pmatrix}$ parallel ?

A -18
B -6
C 14
D 54

56. Given that $\boldsymbol{p} = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}$, $\boldsymbol{q} = \begin{pmatrix} 4 \\ -1 \\ -3 \end{pmatrix}$, and $\boldsymbol{r} = \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}$, what are the components of $\boldsymbol{p} - \boldsymbol{q} + 3\boldsymbol{r}$?

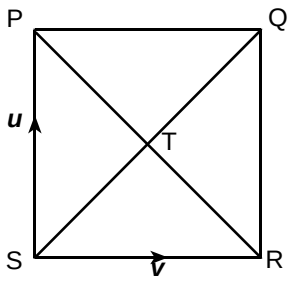
A $\begin{pmatrix} -3 \\ 0 \\ -2 \end{pmatrix}$

B $\begin{pmatrix} 5 \\ 0 \\ -8 \end{pmatrix}$

C $\begin{pmatrix} 0 \\ 0 \\ 54 \end{pmatrix}$

D $\begin{pmatrix} -3 \\ -2 \\ 10 \end{pmatrix}$

57. The diagram shows a square PQRS where $\overrightarrow{SP} = \boldsymbol{u}$ and $\overrightarrow{SR} = \boldsymbol{v}$.



Express $\overrightarrow{ST}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$.

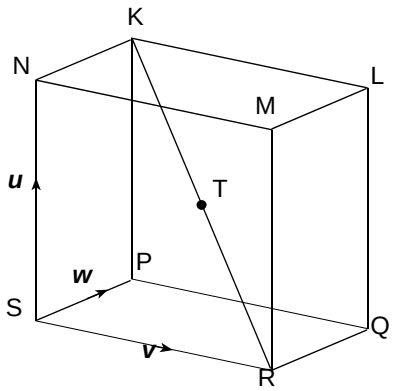
A $\overrightarrow{ST} = \boldsymbol{u} + \frac{1}{2}\boldsymbol{v}$

B $\overrightarrow{ST} = \frac{1}{2}\boldsymbol{u} + \frac{1}{2}\boldsymbol{v}$

C $\overrightarrow{ST} = \boldsymbol{u} - \frac{1}{2}\boldsymbol{v}$

D $\overrightarrow{ST} = \frac{1}{2}\boldsymbol{u} - \frac{1}{2}\boldsymbol{v}$

58. PQRS,KLMN is a cuboid as shown in the diagram.
$\overrightarrow{SN} = \boldsymbol{u}$, $\overrightarrow{SR} = \boldsymbol{v}$ and $\overrightarrow{SP} = \boldsymbol{w}$.
T is the midpoint of KR.



Express $\overrightarrow{KT}$ in terms of $\boldsymbol{u}$, $\boldsymbol{v}$ and $\boldsymbol{w}$.

A $\overrightarrow{KT} = -\frac{1}{2}\boldsymbol{u} + \frac{1}{2}\boldsymbol{v} - \frac{1}{2}\boldsymbol{w}$

B $\overrightarrow{KT} = -\boldsymbol{u} + \boldsymbol{v} - \boldsymbol{w}$

C $\overrightarrow{KT} = \frac{1}{2}\boldsymbol{u} + \frac{1}{2}\boldsymbol{v} + \frac{1}{2}\boldsymbol{w}$

D $\overrightarrow{KT} = \boldsymbol{u} - \boldsymbol{v} + \boldsymbol{w}$

59. The points A(1, 4, 2), B(3, 2, z) and C(7, y, -1) are collinear.
What are the values of y and z ?

	y	z
A	2	−3
B	2	1
C	−2	−3
D	−2	1

60. The point N divides the line LM in the ratio 3 : 1.
L has coordinates (−1, 1, 0) and $\overrightarrow{\text{LM}} = \begin{pmatrix} 4 \\ 4 \\ 4 \end{pmatrix}$.

What are the coordinates of N ?

A $\left(\frac{3}{2}, 2, 1\right)$

B $(2, 4, 3)$

C $\left(\frac{5}{2}, 4, 3\right)$

D $(5, 3, 4)$

61. The components of vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are given by $\boldsymbol{u} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 3 \\ -1 \\ -5 \end{pmatrix}$.

What is the value of $\boldsymbol{u}.\boldsymbol{v}$?

A −10
B − 3
C 3
D 5

62. The vectors $\begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -5 \\ 2 \\ z \end{pmatrix}$ are perpendicular.

What is the value of z ?

A -1

B 0

C $\frac{1}{4}$

D 4

63. What is the angle between the vectors $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$?

A $\frac{\pi}{6}$

B $\frac{\pi}{4}$

C $\frac{\pi}{3}$

D $\frac{\pi}{2}$

64. What is the value of $(\boldsymbol{i} + 2\boldsymbol{j}).(\boldsymbol{j} + 2\boldsymbol{k})$?

A 0
B 2
C 5
D 9

65. Here are two statements about a stationary value for the function $f(x) = 4\sin x - 2$:

(1) f has a stationary value when $x = \frac{\pi}{3}$

(2) f has a stationary value when $x = \frac{\pi}{2}$

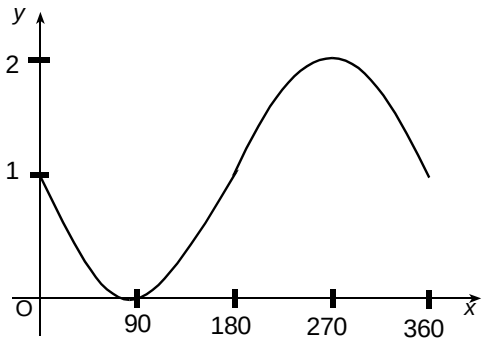
Which of the following is true?

A neither statement is correct
B only statement (1) is correct
C only statement (2) is correct
D both statements are correct

66. What is the exact value of $\sin\frac{2\pi}{3} + \sin\frac{7\pi}{3}$?

A 0
B 1
C $\sqrt{3}$
D 3

67. The diagram shows the graph of a trigonometric function.



Which of the following could be the equation of the graph ?

A $y = 1 + \sin x^\circ$

B $y = 1 - \sin x^\circ$

C $y = 2 - \cos x^\circ$

D $y = 2\cos x^\circ - 1$

68. What is the minimum value of $4\cos\left(x-\frac{\pi}{3}\right)+6$?

A 10
B 9
C 5
D 2

69. Given that $3\cos x^\circ + 4\sin x^\circ = 5\cos(x - 53\cdot 1)^\circ$, which of the following equations has a solution when x is a real number ?

(1) $3\cos x^\circ + 4\sin x^\circ = 2$

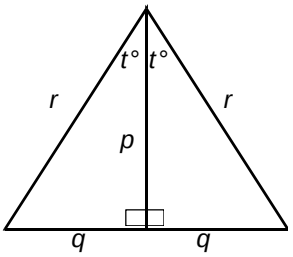
(2) $3\cos x^\circ + 4\sin x^\circ = 8$.

A neither equation has a solution
B only equation (1) has a solution
C only equation (2) has a solution
D both equations have a solution

70. If $\sin x^\circ = \frac{4}{5}$ and $0 < x < 90$, what is the exact value of $\sin 2x^\circ$?

A $\frac{17}{25}$
B $\frac{8}{10}$
C $\frac{24}{25}$
D $\frac{6}{5}$

71. The diagram shows an isosceles triangle with lengths as shown.



Express sin $2t°$ in terms of p, q and r.

A $\sin 2t^\circ = \frac{2q^2}{r^2}$

B $\sin 2t^\circ = \frac{2q}{r}$

C $\sin 2t^\circ = \frac{2p}{r}$

D $\sin 2t^\circ = \frac{2pq}{r^2}$

72. If $\sqrt{3}\cos x + \sin x = k\cos x\cos p + k\sin x\sin p$, where $k > 0$, what is the value of k ?

A 1
B 2
C 3
D 4

73. A function f is defined by $f(x) = 5 + 2\cos 3x$, where x is a real number.
What is the range of f ?

A $3 \le f(x) \le 7$

B $5 \le f(x) \le 7$

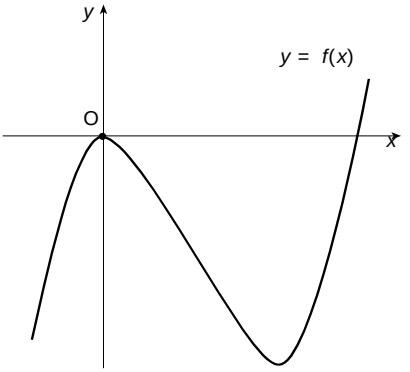
C $5 \le f(x) \le 11$

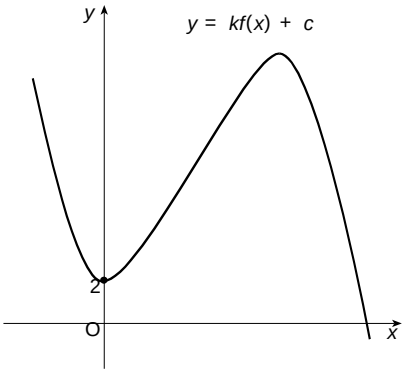
D $-1 \le f(x) \le 11$

74. The graph with equation $y = (x-4)^2 + k$ passes through the point (3, 9).
What are the coordinates of the stationary point of the graph ?

A (4, 8)
B (4, 9)
C (4, 10)
D (4, 11)

75. The diagram shows sketches of $y = f(x)$ and $y = kf(x) + c$.





What are the values of *k* and *c* ?

	k	*c*
A	−1	2
B	−1	−2
C	1	2
D	1	−2

76. $f(x) = 2x - 1$ and $g(x) = 2x + 1$ are functions defined on the set of real numbers. Find an expression for $f(g(x))$.

A $f(g(x)) = 4x^2 - 1$

B $f(g(x)) = 4x^2$

C $f(g(x)) = 4x$

D $f(g(x)) = 4x + 1$

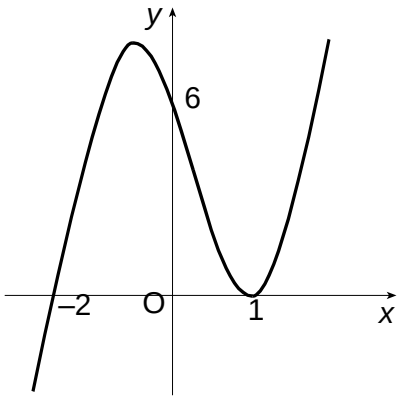
77. When $x^2 + 8x + 5$ is expressed in the form $(x + a)^2 + b$, what is the value of b ?

A −59
B −11
C 0
D 5

78. A function f is given by $f(x) = 4 - 2\cos 3x$ on a suitable domain. What is the minimum value of f ?

A 1
B 2
C 6
D 7

79. The diagram shows part of the graph of a cubic function.



What is the equation of this graph?

A $y = 3(x + 2)^2(x - 1)$

B $y = (x + 2)(x - 1)^2$

C $y = 3(x + 2)(x - 1)^2$

D $y = (x + 2)(x - 1)(x + 1)$

80. A fish farm starts with a stock of 5000 fish. Each Friday 30% of the fish are removed for sale and it is then restocked with 400 new fish.
Let u_n represent the number of fish after restocking n times.

What is the recurrence relation that describes the situation after restocking ?

A $u_{n+1} = 0 \cdot 3u_n + 400$ and $u_0 = 5000$

B $u_{n+1} = 0 \cdot 7u_n + 400$ and $u_0 = 5000$

C $u_{n+1} = 0 \cdot 3(u_n + 400)$ and $u_0 = 5000$

D $u_{n+1} = 0 \cdot 7(u_n + 400)$ and $u_0 = 5000$

81. A sequence is defined by the recurrence relation
$u_{n+1} = 3u_n - 7$ and $u_0 = 1$.

What is the value of u_2 ?

A -19
B -11
C -4
D -1

82. A sequence is generated by the recurrence relation
$2u_{n+1} = ku_n + 7$.
What is the largest range of k for which the sequence has a limit ?

A $-0 \cdot 5 < k < 0 \cdot 5$

B $-1 < k < 1$

C $-2 < k < 2$

D $0 < k < 3$

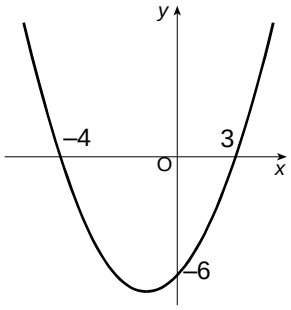
83. A sequence is defined by the recurrence relation
$u_{n+1} = 0 \cdot 6u_n + k$ and $u_0 = 3$.

As $n \to \infty$, the limit of this sequence is 5.

What is the value of k ?

A 0
B 0·88
C 2
D 8

84. The diagram shows the graph of a parabola.



What is the equation of this graph?

A $y = \frac{1}{2}x^2 + \frac{1}{2}x - 6$

B $y = x^2 + x - 12$

C $y = \frac{1}{2}x^2 - \frac{1}{2}x - 6$

D $y = 6x^2 + 6x - 72$

85. What is the solution of $2(x-3)(x+5) > 0$?

A $2 < x < 5$

B $x < -5, x > 3$

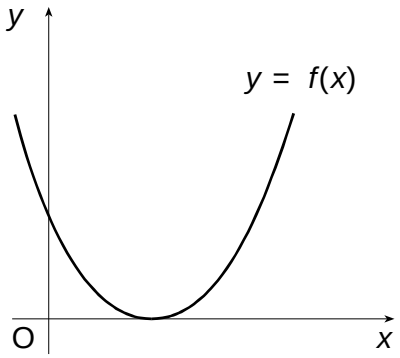
C $-5 < x < 3$

D $x < -3, x > 5$

86. The function g is given by $g(x) = 4x^2 - 12x + 9$.
Which condition describes the nature of the roots of $g(x) = 0$?

A Equal roots
B Exactly three distinct roots
C Exactly two distinct roots
D No real roots

87. The diagram shows part of the graph of a parabola with equation $y = px^2 + qx + r$.
The *x*-axis is a tangent to the parabola.



What is the relationship between *p*, *q* and *r* ?

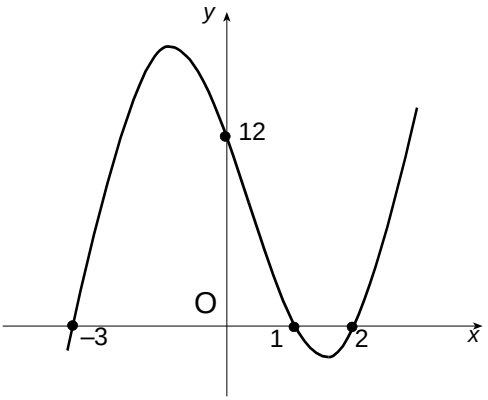
A $q^2 = 4pr$

B $q^2 > 4pr$

C $q^2 < 4pr$

D $q^2 = -4pr$

88. The diagram shows part of the graph of a cubic function.



What is the equation of this graph ?

A $y = 2(x-2)(x-1)(x+3)$

B $y = 12(x-2)(x-1)(x+3)$

C $y = -2(x-3)(x+1)(x+2)$

D $y = 12(x-3)(x+1)(x+2)$

89. What is the remainder on dividing the polynomial $5x^3 - 4x + 8$ by $x - 2$?

A -24
B 0
C 8
D 40

90. What is the value of $\dfrac{\log_3(8)}{\log_3(2)}$?

A $\log_3(4)$
B $\log_3(6)$
C 4
D 3

91. If $\log_9(x) = \dfrac{1}{4}$, what is the value of x?

A $\sqrt{3}$
B $\dfrac{9}{4}$
C $\left(\dfrac{1}{4}\right)^9$
D $\dfrac{3}{2}$

92. Given that $\log_{10}(x) = y\log_{10}(3) + 1$, express x in terms of y.

A $x = 10 \times 3^y$
B $x = 30^{10y}$
C $x = 3y + 10$
D $x = y^3 + 10$

93. Given that $y = kn^x$ where *k* and *n* are constants, what would you plot in order to get a straight line graph ?

A *x* against *y*

B *x* against log (*y)*

C log *(x)* against *y*

D log *(x)* against log (*y)*

94. Given that $f(x) = 2x^3 - 8x$, what is the value of $f'(-1)$?

A -2

B 0

C $4\frac{1}{2}$

D 6

95. If $f(x) = 4x^{-\frac{1}{2}}$, what is the value of $f'(4)$?

A $-\frac{1}{4}$

B $\frac{1}{4}$

C 2

D 4

96. If $f(x) = 3x^2(2x^3 + 4x - 1)$, find $f'(x)$.

A $30x^4 + 36x^2 - 6x$

B $36x^3 + 24x$

C $30x^4 + 12x^3 - 3x^2$

D $x^6 - 3x^4 - x^3$

97. At a point P on the curve $y = 6 - 3x^2$, the gradient is 6.
What is the *x*-coordinate of P ?

A -102

B -3

C -1

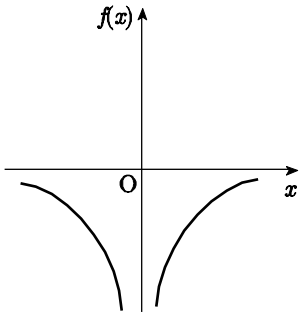
D 6

98. A function f is defined by $f(x) = (x-2)^3$.
What is the rate of change of f with respect to x at $x = 3$?

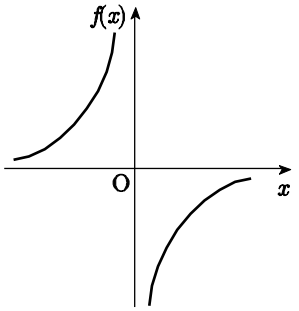
A 0
B 1
C 3
D 19

99. Which of the functions shown satisfies the conditions $f'(x) < 0$ for $x < 0$ *and* $f'(x) > 0$ for $x > 0$, where x is a real number and $x \neq 0$?

A



B



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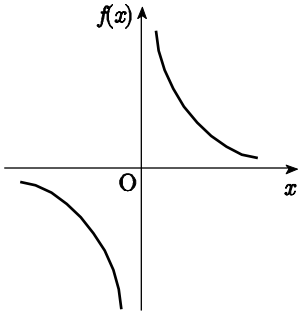

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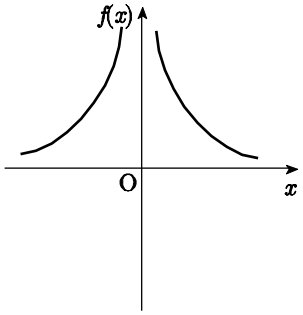


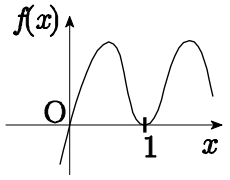
100. A function f is given by $f(x) = (x-1)(x+5)$.
f has a stationary value when $x = a$.

What is the value of a ?

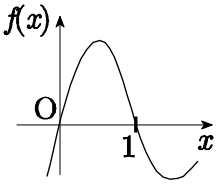
A -5
B -2
C 0
D 1

101. Which of the following could represent a function f such that $f(0) = 0$, $f(1) = 0$, $f'(0) = 1$ and $f'(1) = 0$?

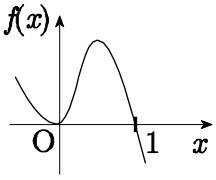
A



B



C



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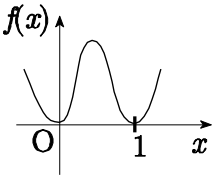


102. The graph of a function f passes through the point (1, 5).

If $f(x) = \int 3x^2\ dx$, find an explanatio n for $f(x)$.

A $f(x) = x^3 - 1$

B $f(x) = 6x + 5$

C $f(x) = x^3 + 5$

D $f(x) = x^3 + 4$

103. If $f'(x) = \frac{1}{\sqrt[4]{x^3}}$, what is $f(x)$?

A $f(x) = \frac{1}{4}x^{\frac{1}{4}} + c$

B $f(x) = 4x^{\frac{1}{4}} + c$

C $f(x) = -\frac{4}{7}x^{-\frac{7}{4}} + c$

D $f(x) = \frac{3}{4}x^{-\frac{7}{4}} + c$

104. Find $\int \frac{1}{5\sqrt{x}}\ dx$.

A $\frac{2}{5}x^{\frac{1}{2}} + c$

B $\frac{5}{2}x^{\frac{1}{2}} + c$

C $-\frac{1}{10}x^{-\frac{3}{2}} + c$

D $\frac{1}{10}x^{-\frac{3}{2}} + c$

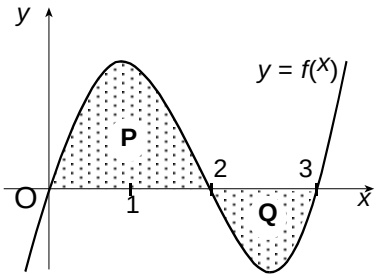
105. What is the value of $\int_0^3 \left(3x^2 + 4x\right)\, dx$?

A 22
B 31
C 39
D 45

106. In the diagram area P = 5 sq. units and area Q = 3 sq. units.
Here are two statements relating to this diagram:

(1) $\int_0^3 f(x)\, dx = 8$

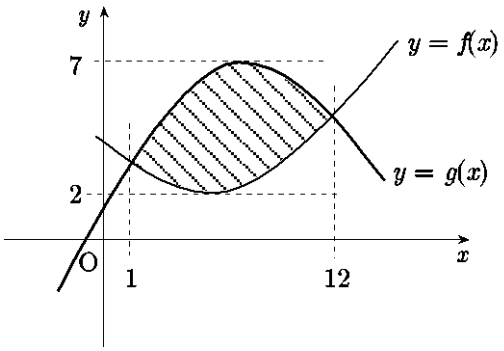
(2) $\int_2^3 f(x)\, dx = 3$



Which of the following is true ?

A neither statement is correct
B only statement (1) is correct
C only statement (2) is correct
D both statements are correct

107. The graphs of functions f and g are shown in the diagram.



Which of the following gives the area of the shaded section ?

A $\int_{1}^{12} (g(x) - f(x))\, dx$

B $\int_{1}^{12} (f(x) - g(x))\, dx$

C $\int_{2}^{7} (g(x) - f(x))\, dx$

D $\int_{2}^{7} (f(x) - g(x))\, dx$

108. A curve passes through the point (2, 3). At every point on the curve $\frac{dy}{dx} = 6x^2$. What is the equation of the curve ?

A $y = 18x^3 - 141$

B $y = 2x^3 - 13$

C $y = 2x^3$

D $y = 12x - 21$

109. If $y = \sin 3x - \cos x$, what is $\frac{dy}{dx}$?

A $-3\cos 3x - \sin x$

B $3\cos 3x + \sin x$

C $\cos 3x - \sin x$

D $3\cos 2x + \sin x$

110. If $f(x) = \left(x^3 + 7\right)^2$, find $f'(x)$.

A $\frac{1}{3}\left(x^3 + 7\right)^3$

B $6x^2\left(x^3 + 7\right)$

C $2\left(3x^2 + 7\right)$

D $6x^2$

111. Find $\int (4x+1)^{-\frac{1}{2}}\, dx$

A $2\left(2x^2 + 1\right)^{\frac{1}{2}} + c$

B $\frac{1}{2}(4x+1)^{\frac{1}{2}} + c$

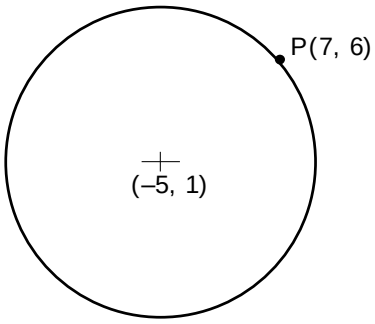
C $\frac{1}{4}(4x+1)^{\frac{1}{2}} + c$

D $-\frac{8}{3}(4x+1)^{\frac{-3}{2}} + c$

112. Find $\int_0^{\pi} (1+\cos x)\, dx$.

A 1
B $\pi - 2$
C 2
D π

113. The point P(7, 6) lies on a circle with centre (−5, 1) as shown in the diagram.



What is the length of the diameter ?

A $2\sqrt{53}$ units
B $2\sqrt{111}$ units
C $2\sqrt{157}$ units
D 26 units

114. What is the exact value of $\tan \frac{7\pi}{6}$?

A $-\sqrt{3}$
B $-\frac{\sqrt{3}}{2}$
C $\frac{1}{\sqrt{3}}$
D $\sqrt{3}$

115. A line L is parallel to the line with equation $4x + 2y = 6$ and passes through the point $(-3, 1)$.
What is the equation of L ?

A $y - 1 = -2(x - 3)$

B $y - 1 = 4(x - 3)$

C $y - 1 = -2(x + 3)$

D $y + 3 = -2(x - 1)$

116. The lines with the equations $ax - 2y + 5 = 0$ and $3x + y - 4 = 0$ are parallel.
What is the value of a ?

A -6

B -2

C $-\frac{1}{3}$

D 3

117. A line L has equation $x + 3y + 7 = 0$.
What is the gradient of a line perpendicular to L ?

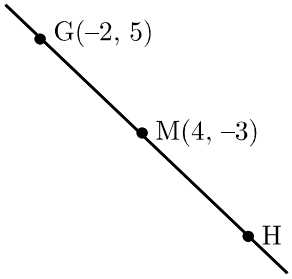
A $-\frac{4}{3}$

B -1

C 1

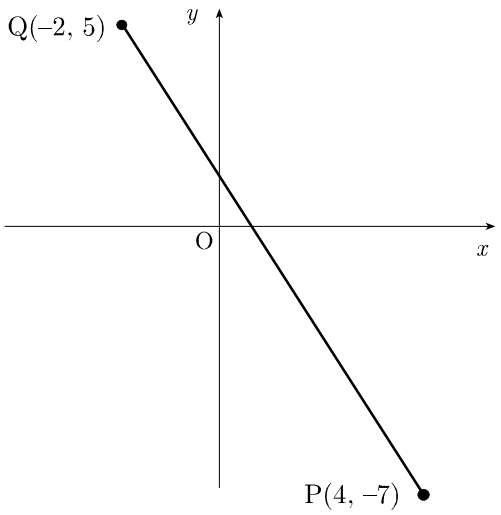
D 3

118. A straight line passes through the points G, M and H
where G = (−2, 5) and M = (4, −3).
M is the midpoint of GH.
What are the coordinates of H ?



A (6, −8)
B (6, 1)
C (−6, 1)
D (10, −11)

119. P and Q have coordinates (4, −7) and (−2, 5) respectively. The perpendicular bisector of PQ has a gradient of $\frac{1}{2}$.



What is the equation of the perpendicular bisector of PQ ?

A $2y = x - 3$
B $y = -2x + 1$
C $y = 2x + 3$
D $2y = -x - 1$

120. Q is the centre of the circle with equation $x^2 + y^2 + 2x - 4y - 15 = 0$ and R(3, 4) lies on the circumference.
What is the gradient of QR ?

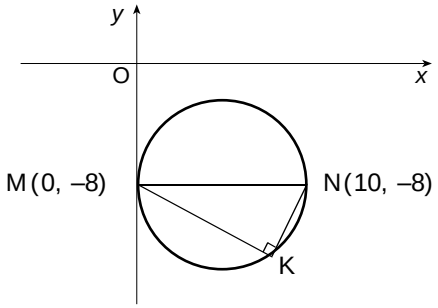
A $\frac{1}{8}$

B $\frac{1}{2}$

C 1

D $\frac{8}{5}$

121. The diagram shows a circle with the *y*-axis as a tangent.
M and N have coordinates (0, −8) and (10, −8) and angle MKN equals 90°.



What is the equation of the circle passing through M, K and N ?

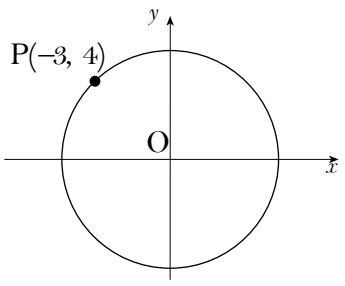
A $(x+5)^2 + (y-8)^2 = 100$

B $(x-10)^2 + (y+8)^2 = 100$

C $(x+5)^2 + (y-8)^2 = 5$

D $(x-5)^2 + (y+8)^2 = 25$

122. The point P(−3, 4) lies on the circle $x^2 + y^2 = 25$ as shown in the diagram.



What is the gradient of the tangent at P ?

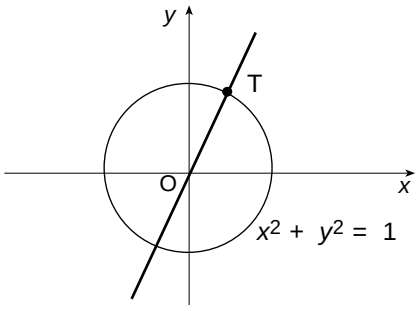
A $-\frac{4}{3}$

B $-\frac{1}{5}$

C $\frac{3}{4}$

D $\frac{5}{3}$

123. The line with the equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 1$ at the point T.
What is the *x*-coordinate of T ?



A $\frac{1}{3}$

B $\frac{1}{\sqrt{6}}$

C $\frac{1}{\sqrt{5}}$

D $\frac{1}{2}$

124. What is the magnitude of the vector $\boldsymbol{v} = -2\boldsymbol{i} + 5\boldsymbol{j} + \boldsymbol{k}$?

A 3
B 4
C $\sqrt{21}$
D $\sqrt{30}$

125. P is the point (1,2,3), $\overrightarrow{\text{PR}}$ represents the vector $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ and $\overrightarrow{\text{RQ}}$ represents the vector $\begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$.

What are the coordinates of Q ?

A (4, 3, 5)
B (5, 4, 6)
C (−2, 0, −1)
D (3, 2, 4)

126. Vector $\boldsymbol{p}$ has components $\begin{pmatrix} \frac{2}{5} \\ \frac{\sqrt{5}}{5} \\ a \end{pmatrix}$, where $a > 0$.

If $\boldsymbol{p}$ is a unit vector, what is possible value of a ?

A $\dfrac{3-\sqrt{5}}{5}$

B $\dfrac{9}{25}$

C $\dfrac{3}{5}$

D $\dfrac{4}{5}$

127. A vector $\boldsymbol{u}$ has components $\begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$.

What are the components of a unit vector parallel to $\boldsymbol{u}$?

A $\begin{pmatrix} \frac{5}{2} \\ -\frac{5}{3} \\ \frac{5}{6} \end{pmatrix}$

B $\begin{pmatrix} \frac{2}{7} \\ -\frac{3}{7} \\ \frac{6}{7} \end{pmatrix}$

C $\begin{pmatrix} -\frac{2}{11} \\ -\frac{3}{11} \\ \frac{6}{11} \end{pmatrix}$

D $\begin{pmatrix} 4 \\ -6 \\ 12 \end{pmatrix}$

128. Vector $\boldsymbol{u}$ and $\boldsymbol{v}$ are given by $\boldsymbol{u} = 2\boldsymbol{i} + \boldsymbol{k}$ and $\boldsymbol{v} = \boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$.

What are the components of vector $2\boldsymbol{u} - \boldsymbol{v}$?

A $\begin{pmatrix} 6 \\ 8 \\ -8 \end{pmatrix}$

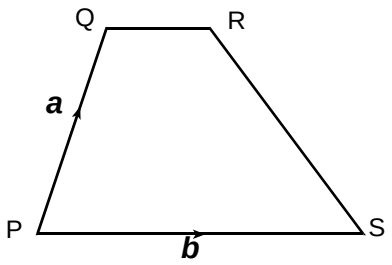
B $\begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix}$

C $\begin{pmatrix} 3 \\ 3 \\ -2 \end{pmatrix}$

D $\begin{pmatrix} 4 \\ 6 \\ -6 \end{pmatrix}$

129. The diagram shows a trapezium PQRS.
PS is parallel to QR and $|\text{PS}| = 3|\text{QR}|$.

$\overrightarrow{\text{PQ}}$ and $\overrightarrow{\text{PS}}$ represent vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ respectively.



Express $\overrightarrow{\text{SR}}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

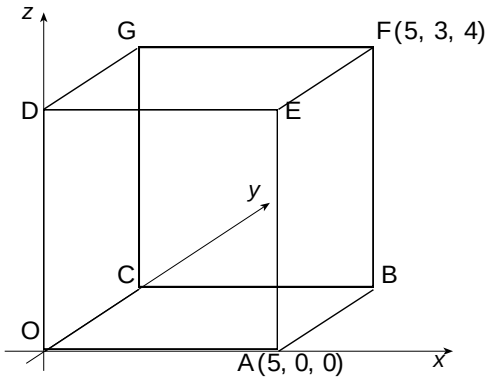
A $\overrightarrow{\text{SR}} = \boldsymbol{a}$

B $\overrightarrow{\text{SR}} = \boldsymbol{a} - \frac{2}{3}\boldsymbol{b}$

C $\overrightarrow{\text{SR}} = -\boldsymbol{a} + \frac{4}{3}\boldsymbol{b}$

D $\overrightarrow{\text{SR}} = \boldsymbol{a} - 4\boldsymbol{b}$

130. OABC,DEFG is a cuboid where A is the point (5, 0, 0) and F is (5, 3, 4), as shown in the diagram.



What are the components of $\overrightarrow{AG}$?

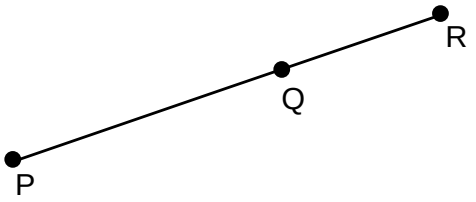
A $\begin{pmatrix} -5 \\ 3 \\ 4 \end{pmatrix}$

B $\begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$

C $\begin{pmatrix} 4 \\ -5 \\ -3 \end{pmatrix}$

D $\begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$

131. The diagram shows three collinear points P, Q and R where $3\overrightarrow{PQ} = 2\overrightarrow{PR}$.



What is the ratio in which Q divides PR ?

A 2 : 1
B 3 : 1
C 3 : 2
D 5 : 3

132. A is the point (1,4,−2) and $\overrightarrow{AB} = \begin{pmatrix} -1 \\ -5 \\ 7 \end{pmatrix}$.

If $\overrightarrow{AC} = 3\overrightarrow{AB}$, what are the coordinates of C ?

A (1, 1, 13)
B (−3, −15, 21)
C (−2, −11, 19)
D (3, 15, −21)

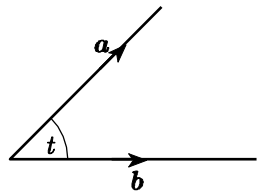
133. Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = \boldsymbol{i} + 2\boldsymbol{j} - 4\boldsymbol{k}$ and $\boldsymbol{v} = 3\boldsymbol{i} + 2\boldsymbol{k}$.
What is the value of $\boldsymbol{u}.\boldsymbol{v}$?

A −5
B −1
C 0
D 3

134. Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are given $\boldsymbol{u} = 2\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$ and $\boldsymbol{v} = 3\boldsymbol{i} + p\boldsymbol{j} - \boldsymbol{k}$.
If $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular, what is the value of p ?

A 1
B 4
C 7
D 8

135. Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are inclined at an angle of t radians to each other, as shown in the diagram.



If $\boldsymbol{a}.\boldsymbol{b} = 2$ and $|\boldsymbol{a}| = |\boldsymbol{b}| = \sqrt{3}$ units, what is the value of cos t ?

A -1

B $\frac{2}{3}$

C $\frac{2}{\sqrt{3}}$

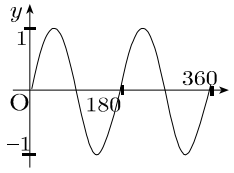
D $\frac{3}{2}$

136. Two vectors, $\boldsymbol{a}$ and $\boldsymbol{b}$, are perpendicular and $|\boldsymbol{a}| = 2$ units, $|\boldsymbol{b}| = 3$ units.
What is the value of $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b})$?

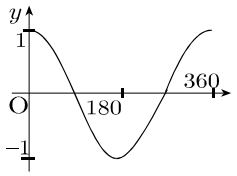
A 0
B 4
C 7
D 10

137. Which of the four graphs is most likely to show the graph of $y = \cos 2x^\circ$ for $0 \leq x \leq 360$?

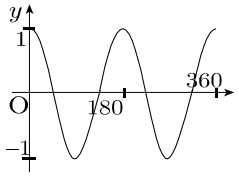
A



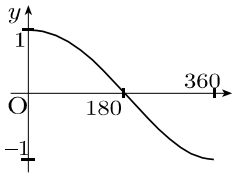
B



C



D



138. If $f(x) = 1 + \cos x$, what is the value of $f'\left(\frac{2\pi}{3}\right)$?

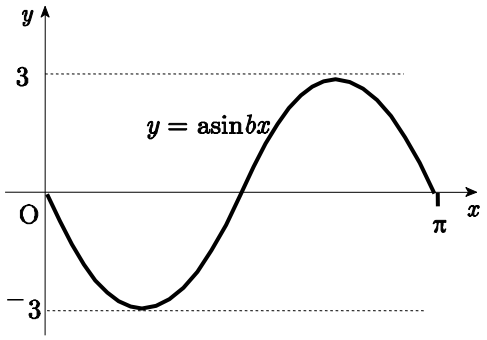
A $-\frac{\sqrt{3}}{2}$

B $-\frac{1}{2}$

C $\frac{1}{2}$

D $\frac{1}{\sqrt{3}}$

139. The diagram shows part of the graph whose equation is of the form $y = a\sin bx$.



What is the equation of this graph?

A $y = -3\sin\frac{1}{2}x$

B $y = 3\sin\frac{1}{2}x$

C $y = -3\sin 2x$

D $y = 3\sin 2x$

140. The maximum value of $1-\cos\left(x-\frac{\pi}{6}\right)$, $0 \leq x < 2\pi$ occurs when $x = t$.

What is the value of t ?

A 0

B $\frac{\pi}{6}$

C $\frac{2\pi}{3}$

D $\frac{7\pi}{6}$

141. What is the solution of the equation $\sqrt{3}\sin x = -\cos x$ where $0 \leq x \leq \frac{3\pi}{2}$?

A $\frac{2\pi}{3}$

B $\frac{5\pi}{6}$

C $\frac{7\pi}{6}$

D $\frac{4\pi}{3}$

142. Expand $\cos\left(x + \frac{\pi}{4}\right)$.

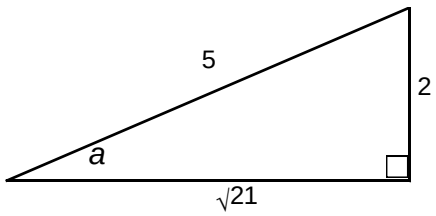
A $\cos\left(x + \frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}\cos x - \frac{1}{\sqrt{2}}\sin x$

B $\cos\left(x + \frac{\pi}{4}\right) = \cos x + \frac{1}{\sqrt{2}}$

C $\cos\left(x + \frac{\pi}{4}\right) = \cos x - \frac{1}{\sqrt{2}}$

D $\cos\left(x + \frac{\pi}{4}\right) = \frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x$

143. The diagram shows a right-angled triangle with side lengths of 2, $\sqrt{21}$ and 5.



What is the exact value of $\sin 2a$?

A $\frac{4}{5}$

B $\frac{17}{25}$

C $\frac{4\sqrt{21}}{25}$

D $\frac{2\sqrt{21}}{5}$

144. k and a are given by

$$k \sin a = 1$$

and $$k \cos a = 1$$

where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$.

What are the values of k and a ?

	k	a
A	$\sqrt{2}$	0
B	$\sqrt{2}$	$\frac{\pi}{4}$
C	2	0
D	2	$\frac{\pi}{4}$