HIGHER PAPER 1

1 a)

$$\begin{array}{c|cccc} 2 & 2 & 3 & -29 & 30 \\ & 0 & 4 & 14 & -30 \\ \hline & 2 & 7 & -15 & \boxed{0} \end{array}$$

• sets up polynomial correctly.

• remainder = 0 so $(x-2)$ is a factor

• $(x-2)(2x^2+7x-15)$

• $(x-2)(2x-3)(x+5)$ /4

b) • $x = 2,\ 3/2$ or -5 /1

2. $f(x) = (2x+3)^{-3}$

$$f'(x) = \underbrace{-3(2x+3)^{-4}}_{\bullet}\underbrace{(2)}_{\bullet}$$

$$= -\frac{6}{(2x+3)^4} \ \bullet$$ /3

3. $a = 2 \quad b = -3 \quad c = (k+1)$

• knows $b^2-4ac = 0$ for equal roots

$(-3)^2 - 8(k+1) = 0$ •

$9 - 8k - 8 = 0$

$8k = 1$

$k = 1/8$ • /3

4. $5x - y + 1 = 0$

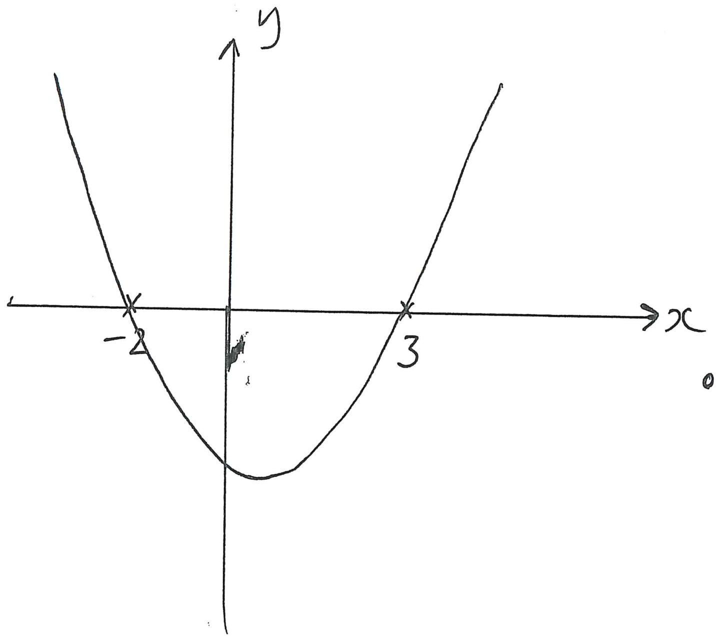
$y = 5x + 1$

- $m = 5$
- $m_{perp} = -\frac{1}{5}$
- $y - 3 = -\frac{1}{5}(x + 2)$

$5y - 15 = -x - 2$

$5y = -x + 13$

$/3$



- -2 and 3 marked on x-axis
- correct shape of graph.

$/2$

6. $x^2 + (x-10)^2 - 4x + 8(x-10) + 12 = 0$ • substitutes

$x^2 + x^2 - 20x + 100 - 4x + 8x - 80 + 12 = 0$

• $2x^2 - 16x + 32 = 0$

$2(x^2 - 8x + 16) = 0$

$/5$

• $2(x-4)^2 = 0$ $(2x-8)(x-4)=0$.

$x = 4$ • only 1 pt of contact so line is tangent

$y = x - 10$, so $y = -6$ • $(4, -6)$

7. a) $f(x) = 3\left[x^2 - 6x - \frac{2}{3}\right]$

$= 3\left[(x-3)^2 - 9 - \frac{2}{3}\right]$

$= 3(x-3)^2 - 27 - 2$

$= 3(x-3)^2 - 29$

/3

b) minimum T.P $\quad f'(x) = 6x - 18.$

(3, −29)

/2

8. $f(x) = 2x^{1/2} + x^2$

$f'(x) = x^{-1/2} + 2x$

$= \frac{1}{\sqrt{x}} + 2x$

$f'(4) = \frac{1}{\sqrt{4}} + 2(4)$

$= \frac{1}{2} + 8 = 8\frac{1}{2}$

/4

9. a) $u_{n+1} = 0.75u_n + 250$ •

1/1

b) $u_1 = 0.75 \times 2600 + 250$

$= 2200$

1/1

$u_2 = 0.75 \times 2200 + 250$

$= 1900$ •

c) • Limit exists since $-1 < 0.75 < 1$

$L = \frac{b}{1-a}$ or $\left\{ \begin{array}{l} L = 0.75L + 250 \\ 0.25L = 250 \end{array} \right.$ •

• $= \frac{250}{1-0.75}$

$L = \frac{250}{0.25}$

4/4

$= \frac{250}{0.25}$

• $= 1000$

• $= 1000$

• Since the elephant population is likely to drop to 1000, the breeding program is not going to be successful.

10. a) $\int 2x - \frac{9}{x^2}\, dx$ • knows to integrate

$y = x^2 + 9x^{-1} + C$ •

$y = x^2 + \frac{9}{x} + C$

4/4

$6 = (-1)^2 + \frac{9}{(-1)} + C$ •

$6 = 1 - 9 + C$ $C = 14$ •

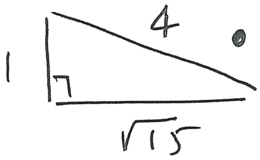
10 b) $y = x^2 + \frac{9}{x} + 14$

$p = 1^2 + 9 + 14 = 24$

$/1$

11. $\sin(x+60)^\circ = \sin x^\circ \cos 60^\circ + \cos x^\circ \sin 60^\circ$

$= \frac{1}{2} \sin x^\circ + \frac{\sqrt{3}}{2} \cos x^\circ$ exact values



$= \frac{1}{2} \times \frac{1}{4} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{15}}{4}$ finds $\sin x$ + $\cos x$

$= \frac{1}{8} + \frac{\sqrt{45}}{8}$

$= \frac{1}{8} + \frac{\sqrt{9}\sqrt{5}}{8}$

$= \frac{1 + 3\sqrt{5}}{8}$

$/5$

12. a) $V = 4xy = 800$

$$xy = 200$$

$$y = \frac{200}{x}$$

S.A

$= xy + 4x + 4y + 4y$

$= xy + 4x + 8y$

$= x\left(\frac{200}{x}\right) + 4x + 8\left(\frac{200}{x}\right)$

$= 200 + 4x + \frac{1600}{x}$

/3

b) $S(x) = 200 + 4x + 1600x^{-1}$

$S'(x) = 4 - 1600x^{-2}$

$= 4 - \frac{1600}{x^2}$

At s.p's
$S'(x) = 0$

$$4 - \frac{1600}{x^2} = 0$$

/6

$$4x^2 - 1600 = 0$$

$$4(x^2 - 400) = 0$$

$$4(x+20)(x-20) = 0$$

$x = \cancel{-20}$ or 20
length.

x	$\rightarrow$	20	$\rightarrow$
$S'(x)$	$-$	0	$+$
shape	\	—	/

∴ surface area minimised when $x = 20$

$20 \times 4 \times 10$ m

13. $\int_a^{2a} (10-2x)\,dx = 8$

$\left[\underbrace{10x - x^2}_{\bullet}\right]_a^{2a} = 8$

u	L
$20a-4a^2$	$10a-a^2$

u-L

$(20a-4a^2)-(10a-a^2)$ • upper-lower

$10a - 4a^2 + a^2 = 8$ • sets = 8

$-3a^2+10a-8=0$

$3a^2-10a+8=0$

$(3a+4)(a-2)=0$ •

$\cancel{a=-4/3}$ $a=2$ •

so $a=2$

/5