H

UDDINGSTON GRAMMAR SCHOOL

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NATIONAL
QUALIFICATIONS
Tuesday 6th February 2018
9:00 AM – 10:10 AM

MATHEMATICS
HIGHER PRELIM
Paper 1 (Non-Calculator)

Total marks - 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Answers obtained by readings from scale drawings will not receive any credit.

Write your answers in the spaces in the answer booklet provided.

Use **blue** or **black** ink.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2\sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Scalar Product: $a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.

or

$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\dfrac{1}{a}\cos ax + C$
$\cos ax$	$\dfrac{1}{a}\sin ax + C$

MARKS

1. (a) Show that (x – 2) is a factor of $f(x) = 2x^3 + 3x^2 - 29x + 30$

    and factorise $f(x)$ fully.   4

    (b) Hence solve $f(x) = 0$ 1

2. Given $f(x) = (2x + 3)^{-3}$, find $f'(x)$, expressing your answer with
    positive indices.   3

3. Find the value of k given the quadratic equation $2x^2 - 3x + (k + 1) = 0$
    has **equal roots**.   3

4. Find the equation of the line through the point (-2, 3) which is
    perpendicular to the line with equation 5x – y + 1 = 0.   3

5. The diagram below shows part of the graph of $y = f(x)$
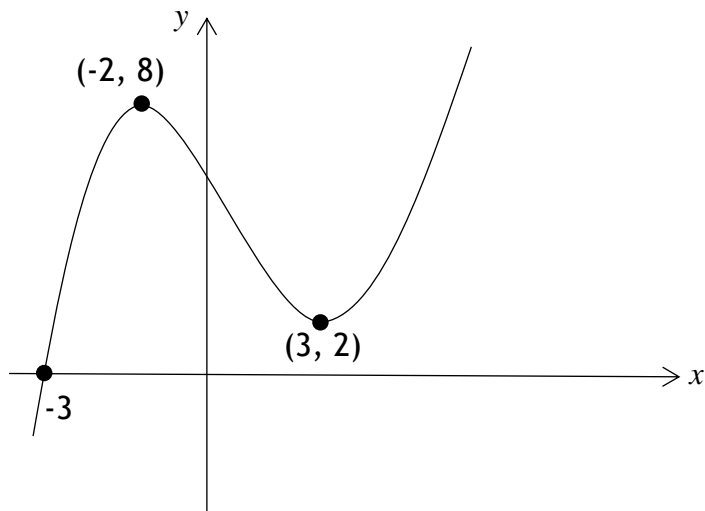
Sketch the graph of $y = f'(x)$ 2

6. Show that the line with equation $y = x - 10$ is a tangent to the circle with equation $x^2 + y^2 - 4x + 8y + 12 = 0$ and state the coordinates of the point of contact. 5

7. A function is defined as $f(x) = 3x^2 - 18x - 2$.

   (a) Express $f(x)$ in the form $f(x) = a(x + b)^2 + c$. 3

   (b) Hence write down the stationary point of $f(x)$ and state its nature 2

8. A function is defined on a suitable domain as $f(x) = 2\sqrt{x} + x^2$.

   Calculate the rate of change of $f(x)$ when $x = 4$. 4

9. The elephant population in a nature reserve is falling by 25% every year due to poaching.
   A successful breeding program has resulted in approximately 250 elephant calves being born every year.

   (a) Construct a recurrence relation in the form $U_{n+1} = aU_n + b$, to model this situation where n is the time in years. 1

   (b) The population is currently estimated to be 2600 elephants.
   Calculate the estimated elephant population in 2 years. 1

   (c) The breeding program will be deemed successful if the population of elephants is greater than 1100 in the long term.

   Is the program likely to succeed?

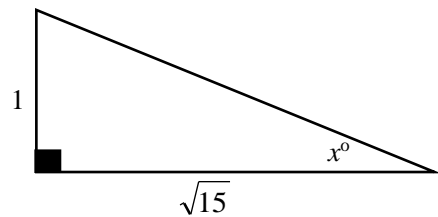   **Explain your answer with words and appropriate working.** 4

10. A curve has as its derivative $\dfrac{dy}{dx} = 2x - \dfrac{9}{x^2}$.

   (a) Given that the point $(-1, 6)$ lies on this curve, express y in terms of x.   4

   (b) Hence find p if the point T $(1, p)$ also lies on this curve.   1

11. In the diagram opposite $\tan x° = \dfrac{1}{\sqrt{15}}$.

   Show that the exact value of $\sin (x + 60)° = \dfrac{1 + 3\sqrt{5}}{8}$ 5

[Turn over

12. Livingston Zoo are looking to build a new aviary against the wall of the giraffe house.

The aviary is to have 3 specially manufactured net sides and a roof as shown below.

The height of the aviary is to be 4 metres, and the volume is to be 800 m³.

Take the length and breadth of the base to be y metres and x metres respectively as shown.

Giraffe house

y

4

x
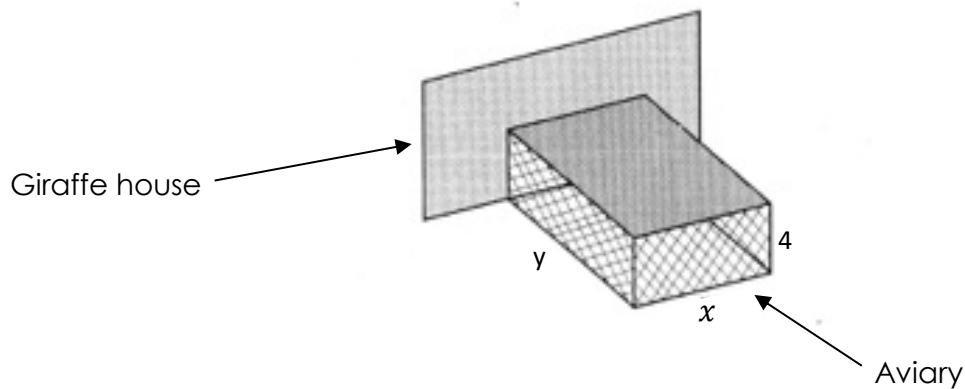
Aviary

(a) Show that the surface area of materials required is given by

$$S(x) = 200 + 4x + \frac{1600}{x}$$

3

(b) Find the dimensions of the aviary which will keep the surface area to a minimum.

6

13. Find a given that $\displaystyle\int_{a}^{2a} (10 - 2x)\ dx\ =\ 8$, where a is a positive whole number.

5

[END OF QUESTION PAPER]