



UDDINGSTON GRAMMAR SCHOOL

NATIONAL
QUALIFICATIONS
Tuesday 6th February 2018
10:30 AM – 12:00 NOON

MATHEMATICS
HIGHER PRELIM
Paper 2

Total marks - 70

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Answers obtained by readings from scale drawings will not receive any credit.

Write your answers in the spaces in the answer booklet provided.

Use **blue** or **black** ink.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\begin{aligned}\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2 \sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2 \cos^2 A - 1 \\ &= 1 - 2 \sin^2 A\end{aligned}$$

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b} .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

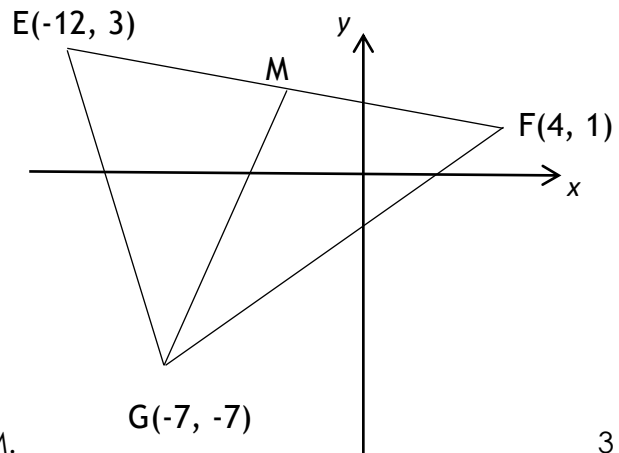
$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

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MARKS

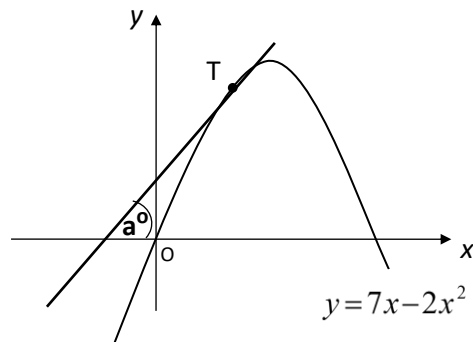
1. Triangle EFG points E(-12, 3), F(4, 1) and G(-7, -7) are the vertices of a triangle, as shown in the diagram opposite.



- (a) Find the equation of median GM. 3
- (b) Find the equation of altitude FP. 3
- (c) Hence, find the coordinate point of intersection between the 2 lines. 3
2. Find $\int \sqrt{6x - 1} \, dx$ 3

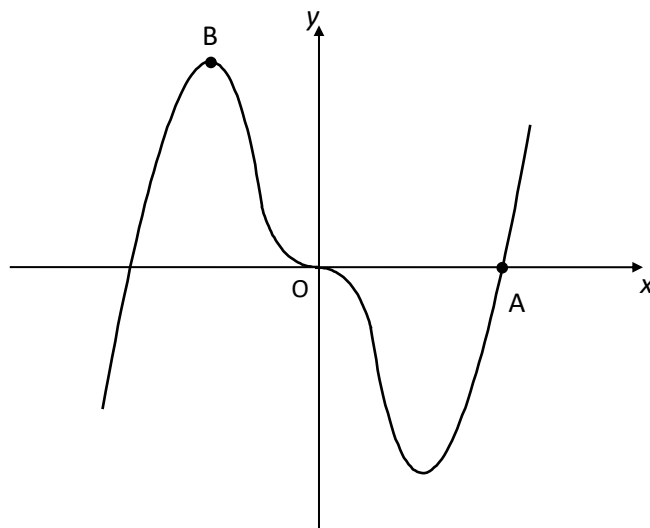
[Turn over

3. The diagram below, which is not drawn to scale, shows part of the graph of the parabola with equation $y = 7x - 2x^2$ and the line which is a tangent to the curve at the point $T(1,5)$.



- (a) What is the gradient of the tangent to the parabola at point T ? 3
- (b) Hence find the size of the angle marked α° to the nearest degree. 1
4. The curve shown below has as its equation $y = 2x^5 - 30x^3$.

The diagram is not drawn to scale.

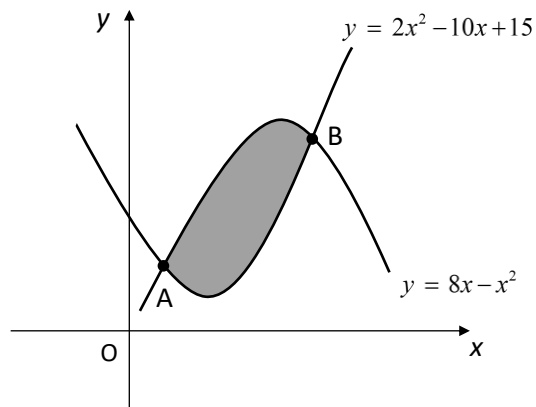


Find algebraically the coordinates of the points A and B .

7

[Turn over

5. The diagram shows parts of the graphs of the parabolas $y = 8x - x^2$ and $y = 2x^2 - 10x + 15$.



- (a) Establish the coordinates of the two intersection points A and B. 3
- (b) Find the area enclosed between the two curves. 5

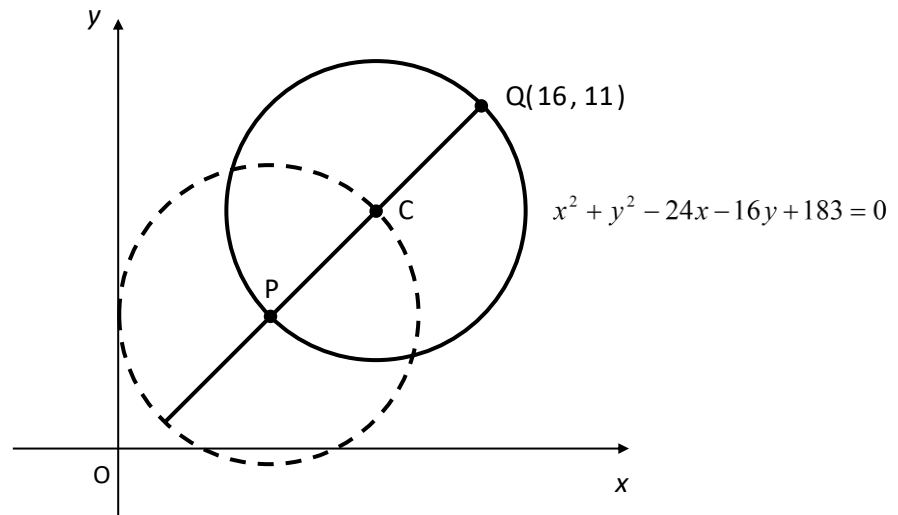
6. Solve **algebraically** the equation

$$2 \cos 2x^\circ - 8 \cos x^\circ + 6 = \cos^2 x^\circ \quad \text{for } 0 \leq x \leq 180^\circ \quad 6$$

[Turn over

7. The diagram below, which is not drawn to scale, shows a circle, centre C , with equation $x^2 + y^2 - 24x - 16y + 183 = 0$.

PQ is a diameter of this circle.



- | | | |
|-----|---|---|
| (a) | Write down the coordinates of C . | 1 |
| (b) | Hence find the coordinates of P . | 2 |
| (c) | Find the equation of the circle, centre P and passing through C . | 3 |
8. A sequence of numbers is defined by the recurrence relation $U_{n+1} = aU_n + 18$, where a is a constant.
- | | | |
|-----|---|---|
| (a) | Given that $U_0 = 10$, show that, in terms of a , $U_2 = 2(5a^2 + 9a + 9)$ | 2 |
| (b) | Hence find a , where $a > 0$, given that $U_2 = 22$. | 3 |

[Turn over

9. A function is defined as $g(x) = 3\sin 2x - 6\cos^2 x$.

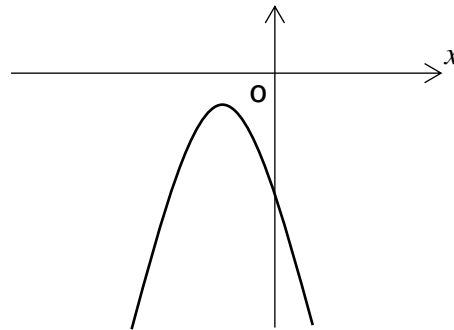
(a) Show that $g'(x)$ can be written in the form

$$g'(x) = 6(\cos 2x + \sin 2x) \quad 4$$

(b) Hence evaluate $g'\left(\frac{\pi}{2}\right)$ 2

10. The diagram opposite shows a curve with equation of the form

$$y = -1 + kx - 9x^2.$$



What are the possible values of k ?

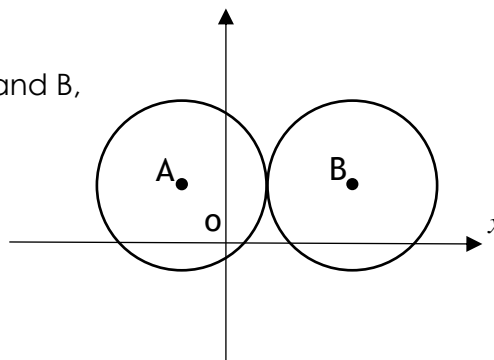
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[Turn over

11. The diagram opposite shows two congruent circles with centres A and B, which touch at a single point..

The circle with centre A has equation $x^2 + y^2 + 4x - 4y - 28 = 0$.

The line through A and B is parallel to the x-axis.



- (a) Determine the coordinates of B.

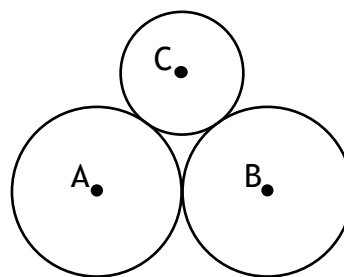
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A third circle, with centre C, touches both of the other circles as shown.

The equation of the line AC is

$$4x - 3y + 14 = 0.$$

- (b) Find the equation of the line through B and C.



3

- (c) Find the equation of the circle with centre C.

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[END OF QUESTION PAPER]