HIGHER PRELIM - PAPER 2

1 a) MEDIAN

MID (EF) = $(-4, 2)$ •

$M_{GM} = \frac{9}{3} = 3$ •

$y - 2 = 3(x + 4)$ or $y + 7 = 3(x + 7)$ /3

$y = 3x + 14$ • (accept form in previous line)

b) ALTITUDE

$M_{EG} = \frac{-10}{5} = -2$ •

$M_{ALT} = \frac{1}{2}$ •

$y - 1 = \frac{1}{2}(x - 4)$ /3

$2y = x - 2$ •

c) $3x - y = -14 \xrightarrow{\times -2} -6x + 2y = 28$

$x - 2y = 2 \qquad x - 2y = 2$ } • evidence of correct scaling

$-5x = 30$

$x = -6$ •

$-6 - 2y = 2$

$2y = -8$

$y = -4$ • /3

2. $\int \sqrt{6x-1}\ dx$

$\int (6x-1)^{1/2}\ dx$

• $\dfrac{(6x-1)^{3/2}}{3/2 \times 6} + c$ •

•

+ simplifies to

$\dfrac{(6x-1)^{3/2}}{9} + c$

/3

3. a) $y = 7x - 2x^2$

$dy/dx = 7 - 4x$ •

when $x = 1$ $dy/dx = 7 - 4(1)$ • knows to sub in $x = 1$

$= 3$ •

/3

b) $\tan a° = 3$

$a = \tan^{-1}(3) = 72°$ • to nearest degree

/1

4.

A - where curve crosses x axis

$$2x^5 - 30x^3 = 0$$ • sets = 0 and factorises
$$2x^3(x^2 - 15) = 0$$

$x = 0$ $\quad$ $x = \pm\sqrt{15}$ $\quad$ $A(\sqrt{15}, 0)$ • identifies A.

B - S.P.

$$dy/dx = 10x^4 - 90x^2$$ • differentiates

At SP's $dy/dx = 0$

$$10x^4 - 90x^2 = 0$$ • sets = 0 for S.P's

$$10x^2(x^2 - 9) = 0$$

$$10x^2(x-3)(x+3) = 0$$ • factorises + solves.

$$x = 0,\ 3 \text{ or } -3$$

For B $\Rightarrow x = -3$ •

/7

$$y = 2(-3)^5 - 30(-3)^3$$
$$= 324$$ •

$B(-3, 324)$

5. a) $8x - x^2 = 2x^2 - 10x + 15$ •

$8x - x^2 - 2x^2 + 10x - 15 = 0$

$18x - 3x^2 - 15 = 0$

$3x^2 - 18x + 15 = 0$ •

$3(x^2 - 6x + 5) = 0$

$3(x-5)(x-1) = 0$ •

$x = 5$ or 1

so $(5, 15)$ $(1, 7)$

/3

b) TOP − BOTTOM

$8x - x^2 - (2x^2 - 10x + 15)$

$8x - x^2 - 2x^2 + 10x - 15$

$18x - 3x^2 - 15$ •

$$\int_1^5 18x - 3x^2 - 15 \, dx$$ •

/5

$$\left[9x^2 - x^3 - 15x \right]_1^5$$ •

u	L
$9(5)^2 - 5^3 - 15(5)$	$9(1)^2 - 1^3 - 15(1)$
25	−7

← • subs values + equates correctly.

Area = u − L = 32 •

6. $2\cos 2x^\circ - 8\cos x^\circ + 6 = \cos^2 x^\circ$

$2(2\cos^2 x - 1) - 8\cos x^\circ + 6 = \cos^2 x^\circ$ • correct replacement

$4\cos^2 x - 2 - 8\cos x^\circ + 6 - \cos^2 x^\circ = 0$ • breaks bracket + knows to $= 0$

$3\cos^2 x - 8\cos x^\circ + 4 = 0$ •

$(3\cos x - 2)(\cos x - 2) = 0$ •

$\cos x = 2/3$ or $\cancel{\cos x = 2}$ • must disregard $\cos x = 2$

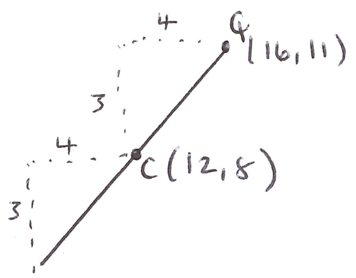
$x = 48.2^\circ, 311.8^\circ$ •

$0 \leq x \leq 180^\circ$, so $x = 48.2^\circ$. accept both.

/6

7. a) $(12, 8)$ • /1

b)



• evidence of 'stepping out' or equiv

• $P(8, 5)$ /2

c) Radius $= \sqrt{4^2 + 3^2}$ • uses pythag / distance

$= 5$ • correct radius /3

$(x-8)^2 + (y-5)^2 = 25$ • do not accept 5^2 for radius.

8. a) $u_0 = 10$

$u_1 = 10a + 18$ • understands method + subs in 10 for u_n.

$u_2 = a(10a + 18) + 18$

$= 10a^2 + 18a + 18$ •

$= 2(5a^2 + 9a + 9)$

/2

b) $2(5a^2 + 9a + 9) = 22$ • sets $= 22$ and gets $= 0$.

$10a^2 + 18a + 18 - 22 = 0$

$10a^2 + 18a - 4 = 0$

/3

$2(5a^2 + 9a - 2) = 0$ • factorises

$2(5a - 1)(a + 2) = 0$

$a = 1/5$ or $\cancel{-2}$ $a > 0$ • solves for a (must disregard -2)

9. a) $g(x) = 3\sin 2x - 6\cos^2 x$

$= 3\sin 2x - 6(\cos x)^2$

$g'(x) = \underbrace{3(2\cos 2x)}_{\bullet} \underbrace{- 12(\cos x)}_{\text{• chain part ①}} \underbrace{(-\sin x)}_{\text{chain part ②}}$ differentiates each part correctly

/4

$= 6\cos 2x + 12\sin x\cos x$

$= 6(\cos 2x + 2\sin x\cos x)$ • tidies up

$= 6(\cos 2x + \sin 2x)$

b) $g'(\pi/2) = 6(\cos \pi + \sin \pi)$ • subs in

$= 6(-1 + 0) = -6$ • equates.

/2

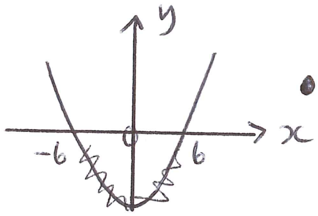
10. No real roots so $b^2 - 4ac < 0$ • correct strategy.

$y = -9x^2 + kx - 1$ $\quad a = -9 \quad b = k \quad c = -1$

$k^2 - 4(-9 \times -1) < 0$ •

$k^2 - 36 < 0$ $\qquad$ /5

$(k+6)(k-6) < 0$ •



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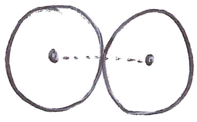
$-6 < k < 6$ •

11. a) Centre A $(-2, 2)$ — •

radius $= \sqrt{4 + 4 + 28}$

$= 6.$ •

/4



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B $(10, 2)$ •

b) • Finds m_{AC}

$-3y = -4x - 14$

$y = 4/3 x + 14/3$ $\quad m = 4/3$

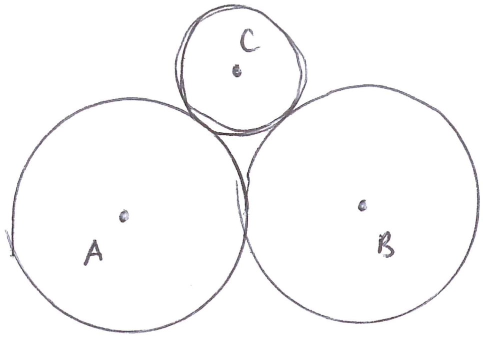
• realises due to symmetry $m_{BC} = -4/3$ $\qquad$ /3

• uses B + gets line equation. $\quad y - 2 = -4/3(x - 10)$

$3y - 6 = -4(x - 10)$

$3y = -4x + 46$

c)



METHOD 1

Lines AC and BC will intersect at C

$$4x - 3y = -14$$
$$4x + 3y = 46$$
$$8x = 32$$

• $x = 4$

* finds x-coord of C

$$16 + 3y = 46$$
$$3y = 30$$

• $y = 10$

finds y-coord.

C(4, 10)

• Distance CB (or CA)

$= \sqrt{(-2-4)^2 + (2-10)^2}$

$= 10$ so $r = 10 - 6 = 4$

/4

• $(x-4)^2 + (y-10)^2 = 16$

METHOD 2

* alternatively to find C.

• pupil may realise the x-coord must be ½ way between A and B $\left(\frac{10-2}{2} = 4\right)$

• sub $x = 4$ into line AC

$$16 - 3y + 14 = 0$$
$$3y = 30$$
$$y = 10$$

C(4, 10)