MATHEMATICS

H

Expressions and Functions

Unit Assessment Practice

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a,b) and radius r.

Scalar Product: $a.b = |a||b|\cos\theta$, where θ is the angle between a and b

or $a.b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$
$$\sin 2A = 2\sin A\cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2\cos^2 A - 1$$
$$= 1 - 2\sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + c$

1.(a)(i) Simplify $\log_5 7a + \log_5 2b$

(ii) Simplify $\log_6 4b + \log_6 3c$

(iii) Simplify $\log_4 9d + \log_4 5a$

(iv) Simplify $\log_8 7y + \log_8 3s$

1.(b)(i) Express $\log_a x^3 - \log_a x^2$ in the form $k \log_a x$

(ii) Express $\log_a x^5 - \log_a x^2$ in the form $k \log_a x$

(iii) Express $\log_a x^3 - \log_a x$ in the form $k \log_a x$

(iv) Express $\log_a x^6 - \log_a x^5$ in the form $k \log_a x$

2. (a) Solve $\log_2(x - 5) = 5$

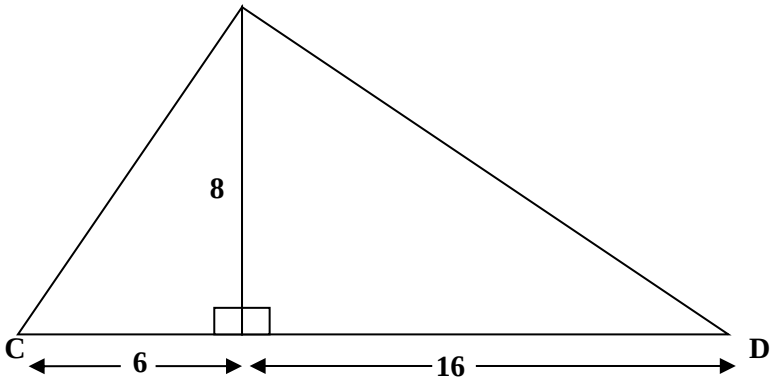
(b) Solve $\log_5(y + 2) = 2$

(c) Solve $\log_3(z - 1) = 3$

(d) Solve $\log_3(d + 2) = 2$

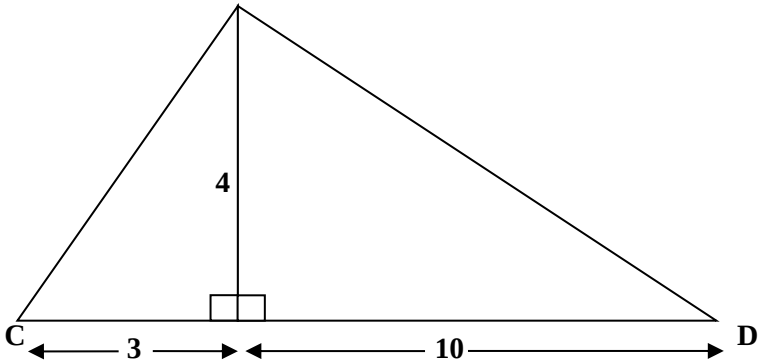
3. (a) The diagram below shows two right-angled triangles.
(i) Write down the values of sin C and cos D.

(ii) Find the exact value of cos (C – D)



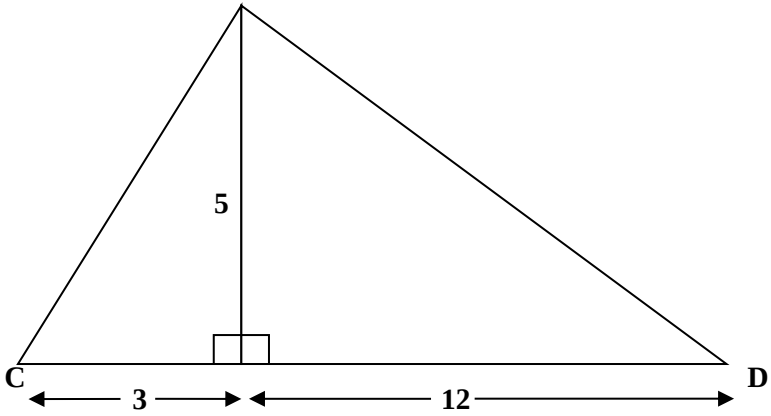
3. (b) The diagram below shows two right-angled triangles.
(i) Write down the values of sin C and cos D.

(ii) Find the exact value of cos (C – D)



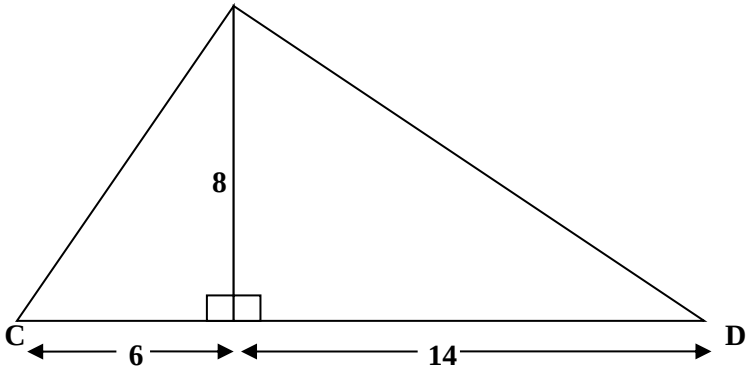
(c) The diagram below shows two right-angled triangles.
(i) Write down the values of cos C and sin D.

(ii) Find the exact value of cos (C + D)



(d) The diagram below shows two right-angled triangles.

(i) Write down the values of cos C and sin D.
(ii) Find the exact value of cos (C + D)



4. **(a)** Show that sin 2x + sin x = sin x (2cos x + 1)

(b) Show that cos x + sin 2x = cos x (1 + 2 sin x)

(c) Show that 2sin x – sin 2x = 2sin x (1- cos x)

(d) Show that 4sin 2x – 2cos x = 2cos x (4sin x – 1)

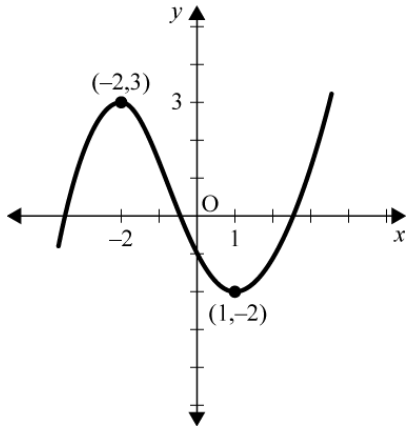
5. **(a)** Express 4 cos x + 8 sin x in the form *k* sin (x – a)
where k > 0 and 0 ≤ a ≤ 360

(b) Express 3 cos x + 8 sin x in the form *k* sin (x – a)
where k > 0 and 0 ≤ a ≤ 360

(c) Express 2 cos x + 6 sin x in the form *k* sin (x – a)
where k > 0 and 0 ≤ a ≤ 360

(d) Express 4 cos x + 7 sin x in the form *k* sin (x – a)
where k > 0 and 0 ≤ a ≤ 360

6. The diagram shows the graph of y = f(x) with a maximum turning point at (–2, 3) and a minimum turning point at (1, –2).



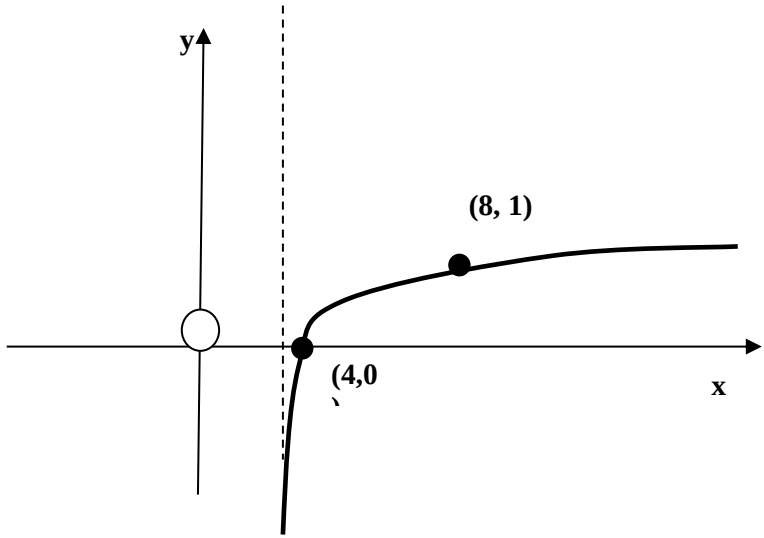
(a) Sketch the graph of y = f(x + 3) – 2

(b) Sketch the graph of y = f(x + 4) – 3

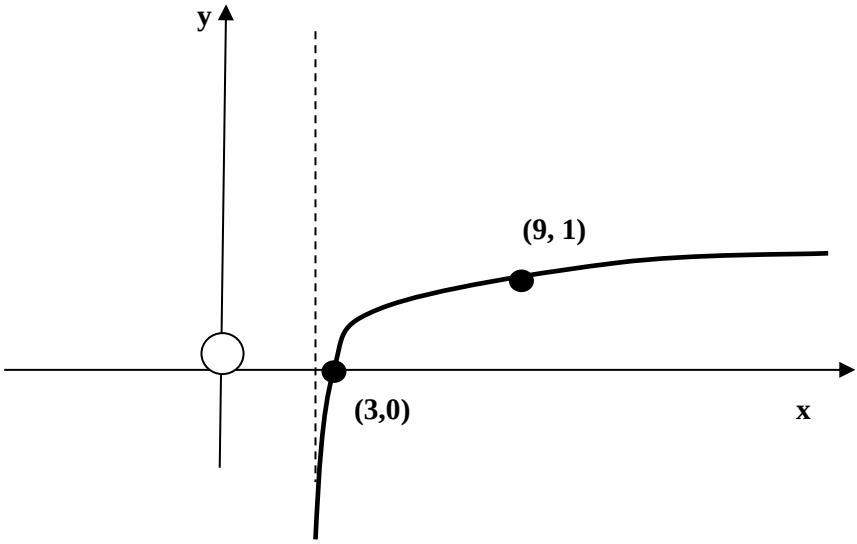
(c) Sketch the graph of y = f(x – 2) + 3

(d) Sketch the graph of y = f(x – 3) – 6

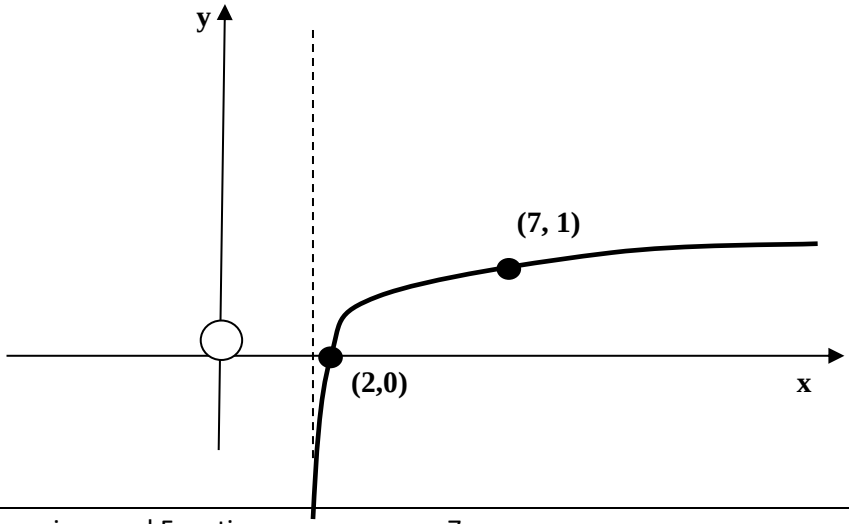
7. **(a)** The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of *a* and *b*



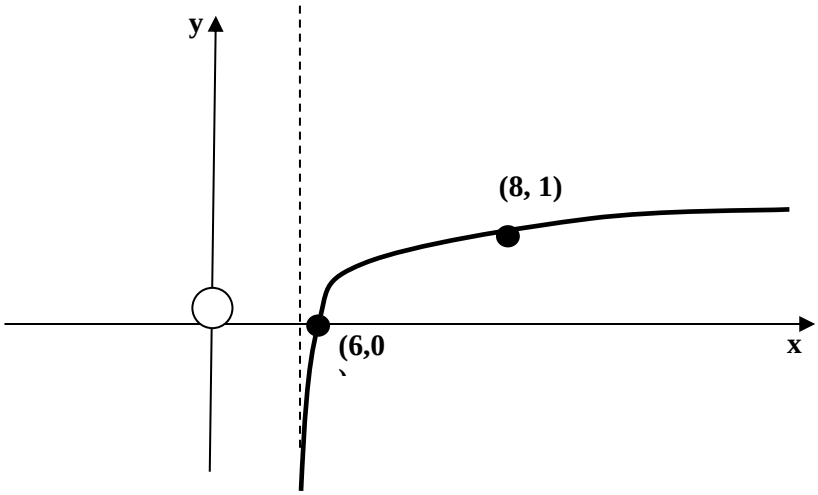
7.(b) The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of *a* and *b*



7.(c) The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of *a* and *b*



7.(d) The diagram shows the graph of $y = \log_b(x - a)$
Determine the values of *a* and *b*



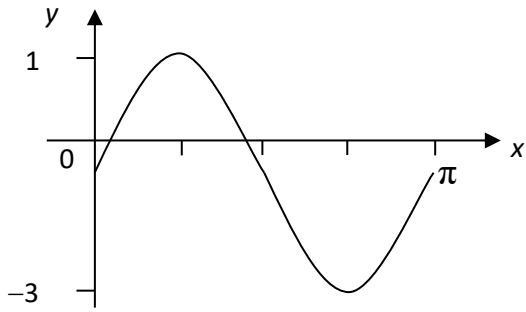
8. **(a)** Sketch the graph of $y = a \cos (x + 60)°$ for $a > 0$ and $0 \leq x \leq 360°$, clearly showing the maximum and minimum values and where it cuts the x-axis.

(b) Sketch the graph of $y = b \sin (x + 45)°$ for $a > 0$ and $0 \leq x \leq 360°$, clearly showing the maximum and minimum values and where it cuts the x-axis.

(c) Sketch the graph of $y = a \cos (x - 60)°$ for $a > 0$ and $0 \leq x \leq 360°$, clearly showing the maximum and minimum values and where it cuts the x-axis.

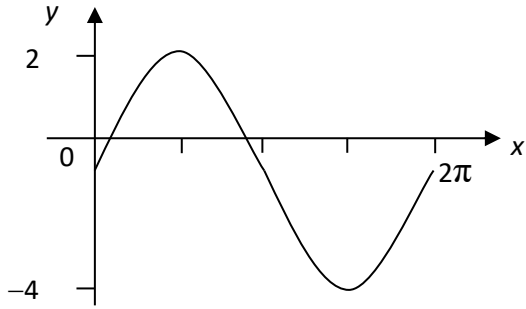
(d) Sketch the graph of $y = b \sin (x - 45)°$ for $a > 0$ and $0 \leq x \leq 360°$, clearly showing the maximum and minimum values and where it cuts the x-axis.

9. (a) The diagram below shows the graph of $y = a \sin(bx) + c$



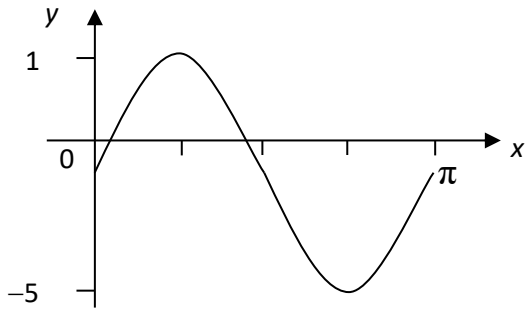
Write down the values of a, b and c.

(b) The diagram below shows the graph of $y = a \sin(bx) + c$



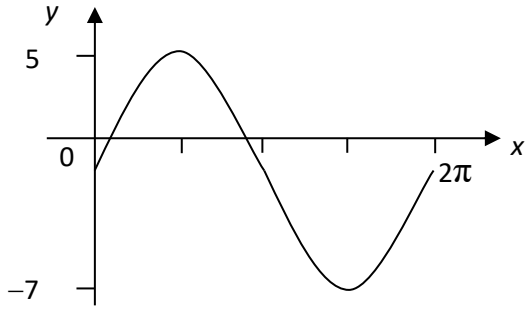
Write down the values of a, b and c.

9. (c) The diagram below shows the graph of $y = a \sin (bx) + c$



Write down the values of a, b and c.

(d) The diagram below shows the graph of $y = a \sin (bx) + c$



Write down the values of a, b and c.

10. (a) The functions f and g defined on suitable domains, are given by

$f(x) = 2x + 5$ and $g(x) = \sqrt{x}$.

Find an expression for $g(f(x))$.

(b) The functions f and g defined on suitable domains, are given by

$f(x) = 3x + 6$ and $g(x) = \sqrt{x}$.

Find an expression for $g(f(x))$.

(c) The functions f and g defined on suitable domains, are given by

$f(x) = 4x + 10$ and $g(x) = \sqrt{x}$.

Find an expression for $g(f(x))$.

(d) The functions f and g defined on suitable domains, are given by

$f(x) = 2x + 7$ and $g(x) = \sqrt{x}$.

Find an expression for $g(f(x))$.

11. (a) Find the range of values of $y = 4 \sin x + 2$

(b) Find the range of values of $y = 3 \sin x - 1$

(c) Find the range of values of $y = 2 \cos x + 3$

(d) Find the range of values of $y = 5 \cos x - 5$

12. (a) Explain why x = 2 is not in the domain of $f(x) = \sqrt{5x - 20}$

(b) Explain why x = 2 is not in the domain of $h(x) = \frac{5}{x-2}$

(c) Explain why x = 5 is not in the domain of $h(x) = \frac{x+3}{x-5}$

(d) Explain why x = 1 is not in the domain of $f(x) = \sqrt{2x - 5}$

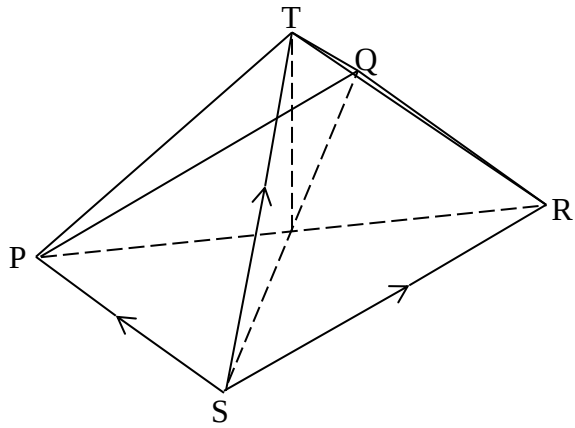
13. **(a)** A function is given by f (x) = 6x + 7. Find the inverse function $f^{-1}(x)$.

(b) A function is given by f (x) = 5x + 8. Find the inverse function $f^{-1}(x)$.

(c) A function is given by f (x) = 8x + 9. Find the inverse function $f^{-1}(x)$.

(d) A function is given by f (x) = 2x + 1. Find the inverse function $f^{-1}(x)$.

14. TPQRS is a pyramid with rectangular base PQRS.



(a) TPQRS is a pyramid with rectangular base PQRS (as above).
If **t**he vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

$$\overrightarrow{SP} = \begin{pmatrix} 3 \\ -8 \\ -6 \end{pmatrix} \qquad \overrightarrow{SR} = \begin{pmatrix} \mathbf{1} \\ \mathbf{12} \\ \mathbf{9} \end{pmatrix} \qquad \overrightarrow{ST} = \begin{pmatrix} \mathbf{-7} \\ \mathbf{0} \\ \mathbf{11} \end{pmatrix}$$

Express $\overrightarrow{PT}$ in component form

(b) TPQRS is a pyramid with rectangular base PQRS (see diagram on left)
If the vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

$$\overrightarrow{SP} = \begin{pmatrix} 4 \\ -6 \\ -5 \end{pmatrix} \qquad \overrightarrow{SR} = \begin{pmatrix} \mathbf{1} \\ \mathbf{12} \\ \mathbf{9} \end{pmatrix} \qquad \overrightarrow{ST} = \begin{pmatrix} \mathbf{-6} \\ \mathbf{2} \\ \mathbf{12} \end{pmatrix}$$

Express $\overrightarrow{PT}$ in component form

(c) TPQRS is a pyramid with rectangular base PQRS (see diagram on left)
If **t**he vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

$$\overrightarrow{SP} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \qquad \overrightarrow{SR} = \begin{pmatrix} \mathbf{1} \\ \mathbf{12} \\ \mathbf{9} \end{pmatrix} \qquad \overrightarrow{ST} = \begin{pmatrix} \mathbf{-4} \\ \mathbf{6} \\ \mathbf{5} \end{pmatrix}$$

Express $\overrightarrow{PT}$ in component form

(d) TPQRS is a pyramid with rectangular base PQRS. (see diagram on left)
If the vectors $\overrightarrow{SP}$, $\overrightarrow{SR}$, $\overrightarrow{ST}$ are given by:

$$\overrightarrow{SP} = \begin{pmatrix} 8 \\ 1 \\ 6 \end{pmatrix} \qquad \overrightarrow{SR} = \begin{pmatrix} \mathbf{1} \\ \mathbf{12} \\ \mathbf{9} \end{pmatrix} \qquad \overrightarrow{ST} = \begin{pmatrix} \mathbf{-2} \\ \mathbf{5} \\ \mathbf{6} \end{pmatrix}$$

Express $\overrightarrow{PT}$ in component form

15. (a) Show that the points P (–1, 4, –8), Q (1, 3, –3), and R (5, 1, 7) are collinear.

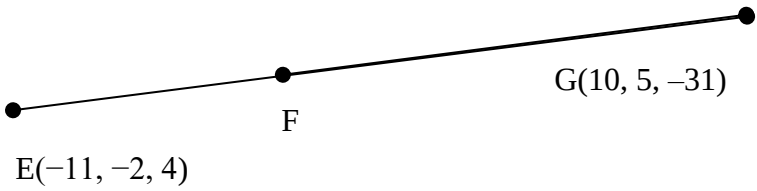
(b) Show that the points P (1, 3, –5), Q (3, 7, –8), and R (7, 15, –14) are collinear.

(c) Show that the points P (–2, 4, 9), Q (1, 2, 3), and R (7, –2, –9) are collinear.

(d) Show that the points P (3, 6, 9), Q (–1, 10, 10), and R (–9, 18, 12) are collinear.

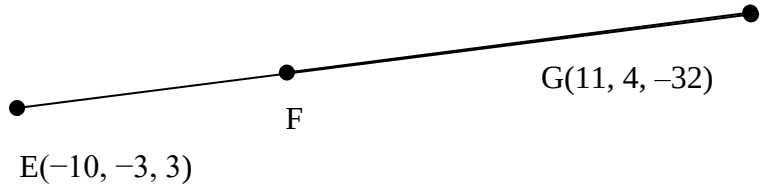
16. (a) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 3:4.

Find the coordinates of F.



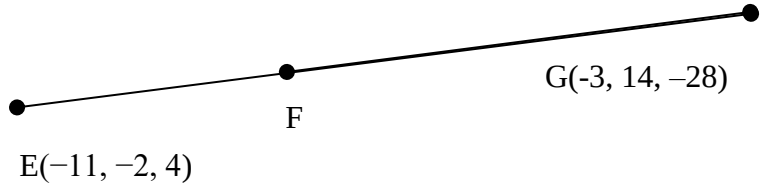
16. (b) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 2:5.

Find the coordinates of F.



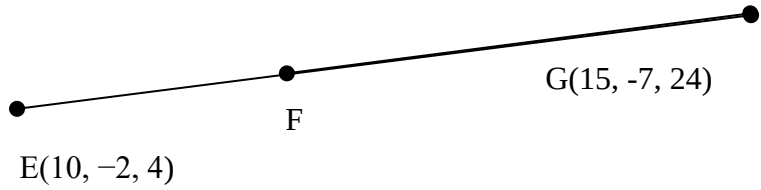
(c) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 3:5.

Find the coordinates of F.

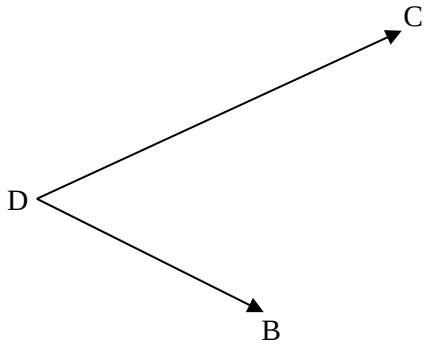


(d) The points E, F and G lie in a straight line, as shown. F divides EG in the ratio 1:4.

Find the coordinates of F.

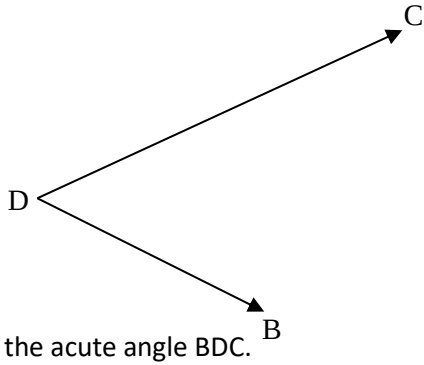


17. **(a)** Points B, C and D have coordinates B(21, –8, 0), C(20, –7, 7) and D(17, –6, 2).



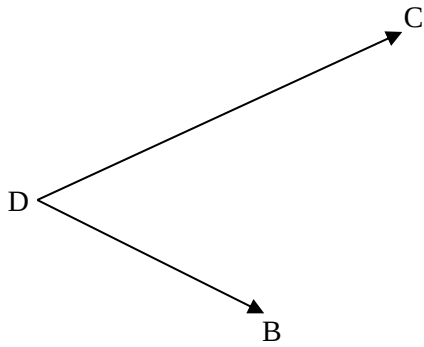
Find the size of the acute angle BDC.

(b) Points B, C and D have coordinates B(15, –7, 4), C(10, –2, 2) and D(18, –1, 3).



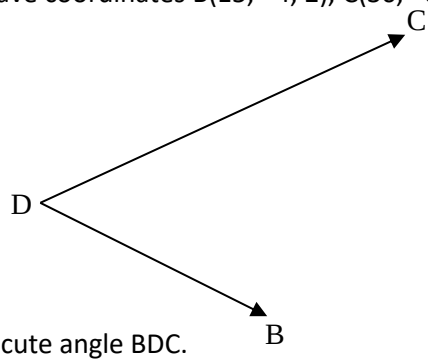
Find the size of the acute angle BDC.

(c) Points B, C and D have coordinates B(11, –4, 1), C(16, –5, 3) and D(12, –8, 1).



Find the size of the acute angle BDC.

(d) Points B, C and D have coordinates B(15, –4, 2), C(30, –8, 8) and D(14, –3, 1).



Find the size of the acute angle BDC.

ANSWERS

1(a) (i) $\log_5 14ab$ (ii) $\log_6 12bc$ (iii) $\log_4 45ad$ (iv) $\log_8 21sy$

1(b) (i) $\log_a x$ (ii) $3\log_a x$ (iii) $2\log_a x$ (iv) $\log ax$

2. (a) x = 37 (b) y = 23

(c) z = 28 (d) d = 7

3. (a)(i) $\frac{4}{5}, \frac{2}{\sqrt{5}}$ (ii) $\frac{2}{\sqrt{5}}$ (b) (i) $\frac{4}{5}, \frac{5}{\sqrt{29}}$ (ii) $\frac{23}{5\sqrt{29}}$ (c) (i) $\frac{3}{\sqrt{34}}, \frac{5}{13}$ (ii) $\frac{11}{13\sqrt{34}}$

(d) (i) $\frac{3}{5}, \frac{4}{\sqrt{65}}$ (ii) $\frac{1}{\sqrt{65}}$

4. Solution is shown

5. (a) $k = 4\sqrt{5}$, a = 333·4° (b) $k = \sqrt{73}$, a = 339·4°

(c) $k = 2\sqrt{10}$, a = 341·6° (d) $k = \sqrt{65}$, a = 330·3°

6. (a) move 3 **left** and **down** 2 (b) move 4 **left** and **down** 3

(c) move 2 **right** and **up** 3 (d) move 3 **right** and **down** 6

7. (a) a = 3, b = 5 (b) a = 2, b = 7 (c) a = 1, b = 6 (d) a = 5, b = 3

8. (a) cos graph moved 60°left. Cuts x at 30° and 210°. Max/min a and –a.

(b) sin graph moved 45°left. Cuts x at 135° and 315°. Max/min b and –b.

(c) cos graph moved 60°right. Cuts x at 150° and 330°. Max/min a and –a.

(d) sin graph moved 30° right. Cuts x at 30 and 210. Max/min b and –b.

9. (a) a = 2, b = 2, c = -1 (b) a = 3, b = 1, c = -1

(c) a = 3, b = 2, c = 2 (d) a = 1, b = 1, c = 1

10. (a) $g(f(x)) = \sqrt{2x+5}$,

(b) $g(f(x)) = \sqrt{3x+6}$ (c) $g(f(x)) = \sqrt{4x+10}$ (d) $g(f(x)) = \sqrt{2x+7}$

11. (a) $-2 \le 4\sin x + 2 \le 6$ (b) $-4 \le 3\sin x - 1 \le 2$

(c) $1 \le 2\cos x + 3 \le 5$ (d) $-10 \le 5\cos x - 5 \le 0$

12. (a) & (d) can't take square root of a negative number

(b) & (c) you can't have zero as the denominator of a fraction.

13. (a) $f^{-1}(x) = \frac{x-7}{6}$ (b) $f^{-1}(x) = \frac{x-8}{5}$

(c) $f^{-1}(x) = \frac{x-9}{8}$ (d) $f^{-1}(x) = \frac{x-1}{2}$

14. (a) $\begin{pmatrix} \mathbf{-10} \\ \mathbf{8} \\ \mathbf{17} \end{pmatrix}$ (b) $\begin{pmatrix} \mathbf{-10} \\ \mathbf{8} \\ \mathbf{17} \end{pmatrix}$

(c) $\begin{pmatrix} \mathbf{-6} \\ \mathbf{5} \\ \mathbf{5} \end{pmatrix}$ (d) $\begin{pmatrix} \mathbf{-10} \\ \mathbf{4} \\ \mathbf{0} \end{pmatrix}$

15. For each question show they are collinear and interpret ratio

16. (a) F = (-2, 1,-11) (b) F = (-4, -1,-7) (c) F = (-8, 4,-8) (d) F = (11, -3, 8)

17. (a) 82.1° (b) 58.2 °

(c) 68.9 ° (d) 27.1 °