## 12 Optimisation and Areas Between Curves

### Revision Section

This section will help you revise previous learning which is required in this topic.

**R1 I can differentiate algebraic functions which can be simplified to an expression in powers of $x$, including terms expressed as surds.**

**1. Find the derivative of the following**

(a) $y = x^3$ (b) $y = x^5$ (c) $f(x) = x^4$

(d) $y = x^{-6}$ (e) $f(x) = x^{-19}$ (f) $y = x^{-5}$

(g) $y = 6x^6$ (h) $f(x) = \frac{1}{2}x^6$ (i) $y = \frac{2}{3}x^9$

(j) $y = \frac{2}{5}x^{18}$ (k) $f(x) = \frac{2}{x^2}$ (l) $y = \frac{3}{x^3}$

(m) $f(x) = \frac{15}{x^{-3}}$ (n) $f(x) = 3x^{-5}$ (o) $y = \frac{6}{x^{-4}}$

(p) $y = \frac{14}{x^{-4}}$ (q) $y = 9x^4$ (r) $f(x) = \frac{2}{7x^{-1}}$

(s) $y = x^{\frac{1}{2}}$ (t) $f(x) = x^{\frac{2}{3}}$ (u) $y = x^{\frac{3}{4}}$

(v) $y = \sqrt{x}$ (w) $f(x) = \sqrt{x^5}$ (x) $f(x) = \sqrt[3]{x}$

(y) $y = \sqrt[5]{x^4}$ (z) $f(x) = \frac{1}{\sqrt{x}}$ (aa) $f(x) = \frac{1}{\sqrt[3]{x^2}}$

(bb) $y = \frac{2}{\sqrt[3]{x^8}}$ (cc) $y = \frac{1}{2\sqrt[3]{x^2}}$ (dd) $y = \frac{2}{3\sqrt[4]{x^3}}$

**2.** Find the derivative of the following

(a) $y = 3x^5 + 2x^4 - x$ (b) $f(x) = x^{\frac{2}{3}} + 4x^2$ (c) $y = 5x^{-2} - 3x^{\frac{1}{2}}$

(d) $y = 3x^7 - \frac{1}{5\sqrt[4]{x^3}}$ (e) $y = \frac{3}{5\sqrt[2]{x^5}} + 5$ (f) $y = \frac{2}{3\sqrt[4]{x^3}} + x^2 + x$

(g) $y = 4x^{-1} - 4x^{\frac{2}{3}}$ (h) $f(x) = 5x^3 - 6x^{-\frac{1}{2}}$ (i) $y = x^2 - 5 - \frac{1}{x^2}$

# Optimisation and Areas Between Curves

**3.** Find the derivative of the following

**(a)** $y = (x+2)(x-3)$ **(b)** $y = (x+2)^2$ **(c)** $y = x^2(x-2)$

**(d)** $y = (x-5)(2x-2)$ **(e)** $y = (\frac{1}{x}+1)^2$ **(f)** $y = \frac{1}{\sqrt{x}}(\frac{1}{\sqrt{x}}-1)$

**4.** Find the derivative of the following

**(a)** $y = \frac{x+5}{x}$ **(b)** $y = \frac{x^4-6x+x^3}{x^2}$ **(c)** $y = \frac{x+2}{\sqrt{x}}$

**(d)** $y = \frac{(x+1)(x+2)}{x}$ **(e)** $y = \frac{(x-1)(x+3)}{x^2}$ **(f)** $y = \frac{3x^2+5x+1}{2x^2}$

**R2.** **I am able to differentiate expressions which contain terms involving $\sin x$ and $\cos x$, expressed in radians.**

Find the derivative of the following

**(1)** $y = \sin x$ **(2)** $y = \cos x$ **(3)** $y = 5\sin x$

**(4)** $y = -2\cos x$ **(5)** $y = -6\sin x$ **(6)** $y = 9\sin x + 7\cos x$

**(7)** $y = 2\cos x + 4\sin x$ **(8)** $y = 3\cos x - 5\sin x$ **(9)** $y = 3\sin x - 5\cos x$

**R3.** **I can differentiate a composite function using the chain rule.**

Find the derivative of the following

**(1)** $y = (x-8)^5$ **(2)** $y = (x+2)^3$ **(3)** $y = (3x-1)^5$

**(4)** $y = (x+2)^{-8}$ **(5)** $y = (3x-5)^{-3}$ **(6)** $y = \frac{1}{(3x+3)^5}$

**(7)** $y = \sqrt{(x-2)}$ **(8)** $y = \sqrt{(x+2)^3}$ **(9)** $y = \frac{2}{\sqrt{(x+2)^5}}$

# Optimisation and Areas Between Curves

**R4 I can evaluate the definite integral of a polynomial functions with integer limits.**

**1.** Find

**(a)** $\int_0^1 (x^2 - 3x + 4)\ \mathrm{dx}$

**(b)** $\int_0^1 (4x^2 + 3x)\ \mathrm{dx}$

**(c)** $\int_0^1 (x^3 + 2x^2 - 1)\ \mathrm{dx}$

**(d)** $\int_0^2 (2x - 1)(x + 2)\, dx$

**(e)** $\int_{-1}^1 2x^2\,(2x + 1)\, dx$

**(f)** $\int_{-2}^1 (2x^3 - x^2 + 3x)\ dx$

**2.** Find

(a) $\int_{-1}^1 (5x^3 - 2x)\ dx$

(b) $\int_{-1}^1 (3x^2 - 4x + 2)\ dx$

(c) $\int_{-1}^1 (3x + 2)(x - 2)\, dx$

(d) $\int_0^2 (3x^2 + 8x - 5)\ dx$

(e) $\int_{-2}^0 (x - 3)^2\ \ dx$

(f) $\int_{-1}^0 (x^2 - 2x + 7)\ dx$

(g) $\int_0^3 x(x - 2)(x - 3)\, dx$

(h) $\int_{-2}^2 (x + 2)(x - 2)\, dx$

(i) $\int_1^4 (x - 1)(x - 2)\, dx$

**R5 I can evaluate the definite integral of a function with limits in radians, surds or fractions.**

**1.** Evaluate

**(a)** $\int_0^{\pi} \cos 2x\ \ dx$

**(b)** $\int_0^{\pi/2} \cos 2x\ \ dx$

**(c)** $\int_0^{\pi} \sin 2x\ \ dx$

**(d)** $\int_0^{\pi/4} \sin 2x\ \ dx$

**(e)** $\int_0^{\pi/3} \cos 3x\ \ dx$

**(f)** $\int_0^{2\pi} \cos \frac{1}{2} x\ \ dx$

**(g)** $\int_0^{\pi} (\sin t + \cos t)\ \ dt$

**(h)** $\int_0^{\pi/4} \sin 4t + \cos 4t\ dt$

**(i)** $\int_0^{\pi/4} \cos\left(2t + \frac{\pi}{2}\right)\ dt$

**(j)** $\int_{\pi/6}^{\pi/4} \sin\left(2t - \frac{\pi}{3}\right)\ dt$

# Optimisation and Areas Between Curves

**2.** Evaluate

(a) $\int_0^{\frac{1}{2}} (x^3 + 12x^2 + 7)\ dx$

(b) $\int_{-1}^{\frac{1}{2}} (3x^2 - 4x)\ dx$

(c) $\int_0^{\frac{2}{3}} (9x^2 + 8)\ dx$

(d) $\int_{-\frac{1}{2}}^{1} (9x^2 + 2x - 1)\ dx$

(e) $\int_0^{\sqrt{3}} (2x + 4)\ dx$

(f) $\int_1^{\frac{\sqrt{3}}{2}} (10 - 2x)\ dx$

**R6 I can apply a standard integral of the form $f(x) = (px + q)^n$ with $n \neq -1$.**

**1.** Find

**(a)** $\int (x + 2)^8\ dx$

**(b)** $\int (2x + 4)^3\ dx$

**(c)** $\int (5x + 7)^4\ dx$

**(d)** $\int (2x - 1)^5\ dx$

**(e)** $\int 6(5 - 4x)^6\ dx$

**(f)** $\int (10 - x)^{-10}\ dx$

**(g)** $\int 3(4x + 1)^{-3}\ dx$

**(h)** $\int 2(5x - 9)^{-5}\ dx$

**(i)** $\int (3 - 7x)^{-4}\ dx$

**(j)** $\int (x - 1)^{\frac{1}{2}}\ dx$

**(k)** $\int (2x - 1)^{\frac{1}{3}}\ dx$

**(l)** $\int (2x - 1)^{\frac{1}{4}}\ dx$

**(m)** $\int (2x - 2)^{\frac{1}{2}}\ dx$

**(n)** $\int (3x + 4)^{\frac{2}{3}}\ dx$

**2.** Find

**(a)** $\int \frac{1}{(5x+3)^5}\ dx$

**(b)** $\int \frac{dx}{(3x-2)^4}$

**(c)** $\int \frac{3}{(4-2x)^6}\ dx$

**(d)** $\int \frac{2\,dx}{(x-2)^3}$

**(e)** $\int \frac{3\,dx}{(4x+2)^4}$

**(f)** $\int \frac{1}{(5x-2)^{\frac{1}{2}}}\ dx$

**(g)** $\int \sqrt{4x + 2}\ dx$

**(h)** $\int \sqrt[4]{2x + 4}\ dx$

**(i)** $\int \frac{1}{\sqrt{(3x-4)}}\ dx$

**(j)** $\int \frac{2dx}{\sqrt{(2x-5)}}$

# Optimisation and Areas Between Curves

**R7 I can integrate $\sin^2 x$ and $\cos^2 x$ by first making a substitution.**

**Find**

**(1)** $\int \sin^2 x \; dx$

**(2)** $\int \cos^2 x \; dx$

**(3)** $\int 2\sin^2 x \; dx$

**(4)** $\int 2\cos^2 x \; dx$

# Optimisation and Areas Between Curves

**Section B - Assessment Standard Section**

This section will help you practise for your Assessment Standard Test for Optimisation and Areas Between Curves (Applications 1.4)

**1.** The Area of an aviary netting can be represented by the formula

$$A = x^2 + \frac{2000}{x}$$

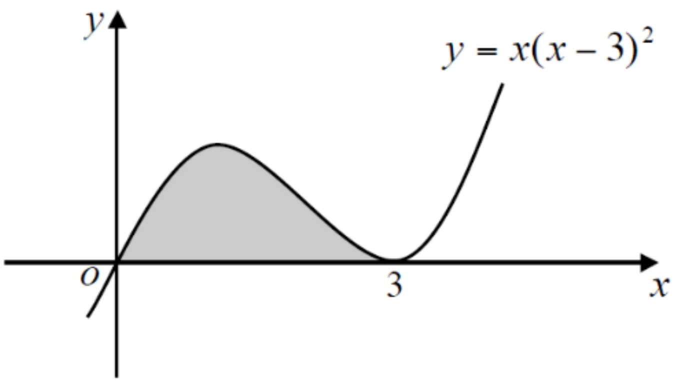
Find the value of $x$ which minimises the cost of netting for the aviary.

**2.** The volume of a trough can be represented by the formula

$$\mathrm{V} = \mathrm{x} - \frac{1}{4}\mathrm{x}^3$$

Find the value of $x$ which maximises the volume.

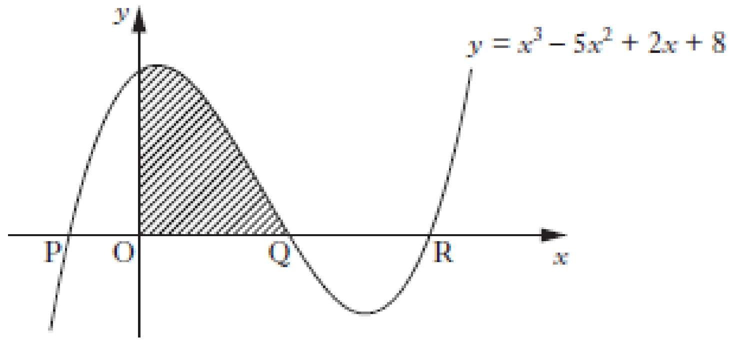
**3.** The curve with equation $y = x(x-3)^2$ is shown in the diagram.




Calculate the shaded area.

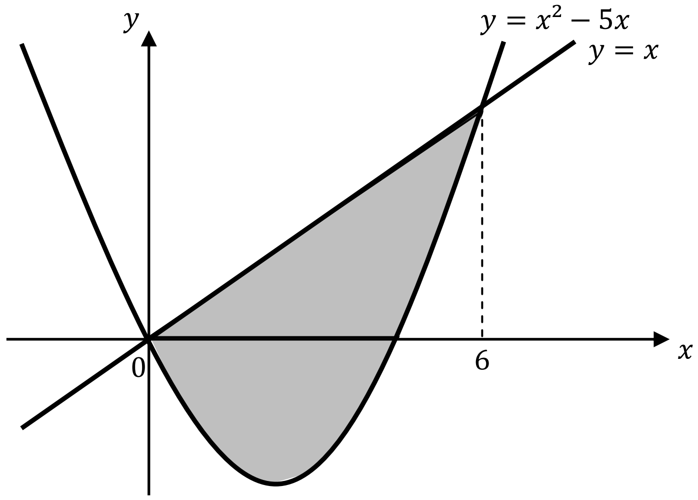
# Optimisation and Areas Between Curves

**4.** The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.




Calculate the shaded area.

**5.** The diagram shows the line $y = x$ and the curve with equation $y = x^2 - 5x$.
The line and the curve meet at the points where $x = 0$ and $x = 6$.

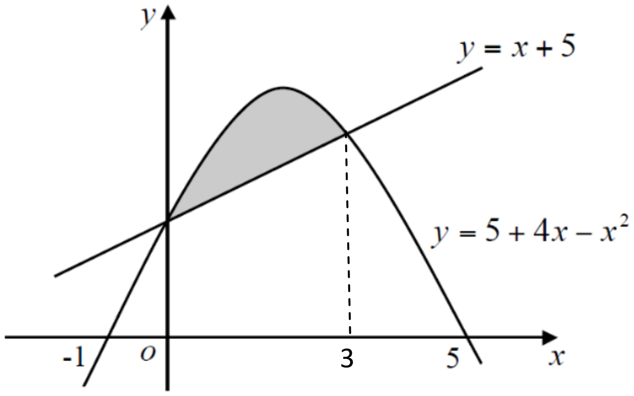



Calculate the shaded area.

# Optimisation and Areas Between Curves

**6.** The diagram shows the line $y = x + 5$ and the curve with equation $y = 5 + 4x - x^2$.

The line and the curve meet at the points where $x = 0$ and $x = 3$.




Calculate the shaded area.

# Optimisation and Areas Between Curves

**Section C – Operational Skills Section**

This section provides problems with the operational skills associated with Optimisation and Areas between Curves

| | |
|---|---|
| **O1** | **I can determine the optimal solution for a given problem.** |

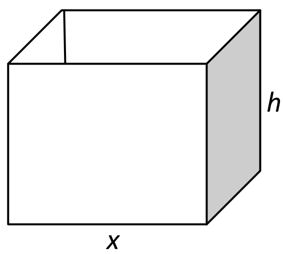
**1.** The height *h* m of a ball thrown upwards is given by the formula $h(x) = 20t - 5t^2$ where *t* is the time in seconds from when the ball is thrown.

**(a)** When does the ball reach its maximum height?

**(b)** Calculate the maximum height of the ball.

**2.** A plastic box with a square base and an open top is being designed. It must have a volume of 108 $cm^3$ .

The length of the base is *x* cm and the height is *h* cm.



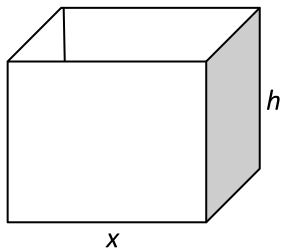

**(a)** Show that the total surface area *A* is given by

$$A(x) = x^2 + \frac{432}{x}$$

**(b)** Find the dimensions of the tray using the least amount of plastic

**3.** An open tank is to be designed in the shape of a cuboid with a square base. It must have a surface area of 100 $cm^2$ .

The length of the base is *x* cm.



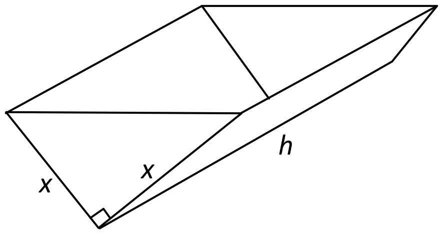

**(a)** Show that the volume *V* is given by

$$V(x) = 2x - \frac{x^3}{4}$$

**(b)** Find the length of the base which gives the tank a maximum volume.

# Optimisation and Areas Between Curves

**4.** An open trough is in the shape of a triangular prism, the trough has a capacity of 256 000 cm$^3$.

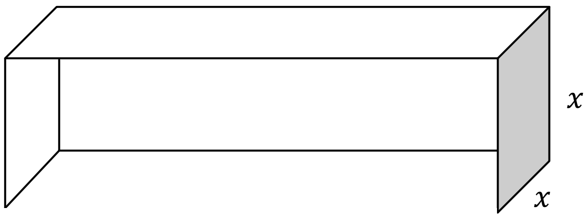



**(a)** Show that the surface area of this trough is given by

$$A(x) = x^2 + \frac{1024\,000}{x}$$

**(b)** Find the value of $x$ such that the surface area is minimised.

**5.** A shelter consists of two square sides ($x$ meters), a rectangular top and back. The total amount of material used to make the shelter is 96 m$^2$



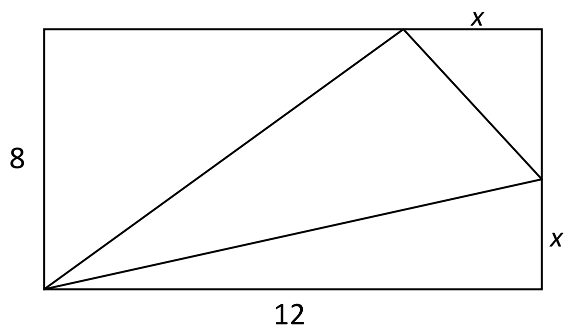

**(a)** Show that the volume of the shelter is given by

$$V(x) = x(48 - x^2)$$

**(b)** Find the dimensions of the shelter with the maximum volume.

# Optimisation and Areas Between Curves

**6.** A triangular piece of material is cut out of a rectangular sheet, the dimensions are shown on the diagram.




**(a)** Show that the area of the triangle is given by the formula

$$A(x) = 48 - 6x + \frac{1}{2}x^2$$

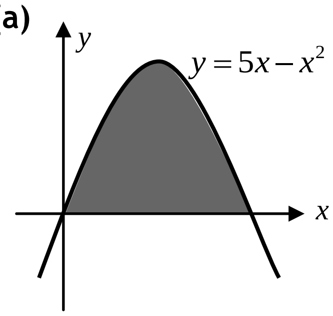
**(b)** Find the biggest area of triangle possible.

# Optimisation and Areas Between Curves

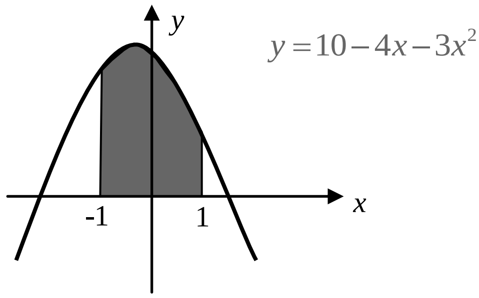
**O2 I can evaluate the area enclosed between a function and the x axis.**

**1.** Find the shaded area in the following diagrams

**(a)**



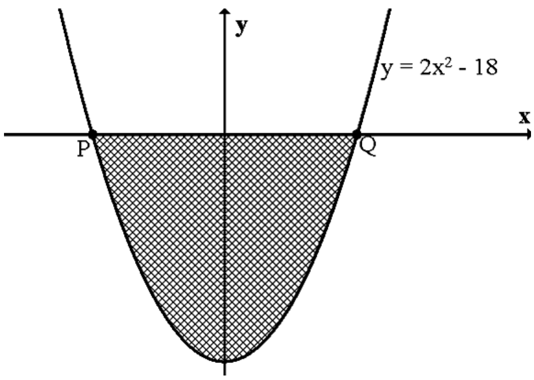

**(b)**




**2.** The diagram shows part of the graph of $y = 2x^2 - 18$.

**(a)** Find the coordinates of P and Q.

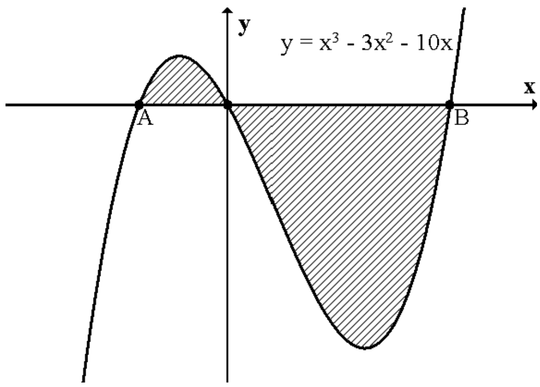
**(b)** Find the shaded area.




**3.** The diagram shows the graph of $y = x^3 - 3x^2 - 10x$.

**(a)** Find the coordinates of A and B.

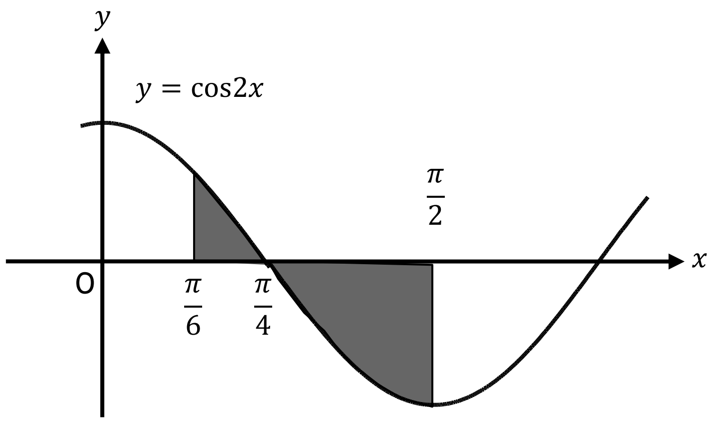
**(b)** Calculate the shaded area.



# Optimisation and Areas Between Curves

**4.** An artist has designed a "bow" shape which he finds can be modelled by the shaded area shown.

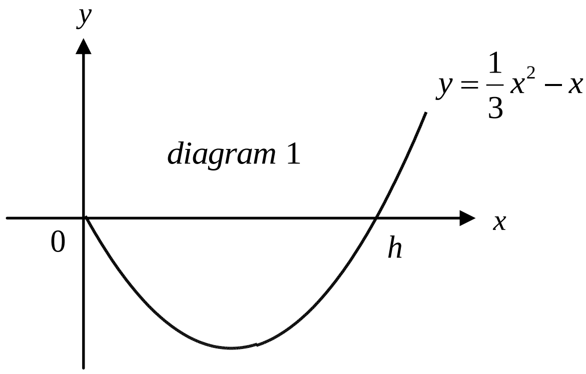
Calculate the area of this shape.




**5.** Diagram 1 shows the profit/loss function for the manufacture of $x$ thousand kitchen units.

The profit/loss is measured in millions of £s and is represented by the area between the function and the $x$-axis .

Any area below the $x$-axis represents a loss; any area above the $x$-axis represents a profit.

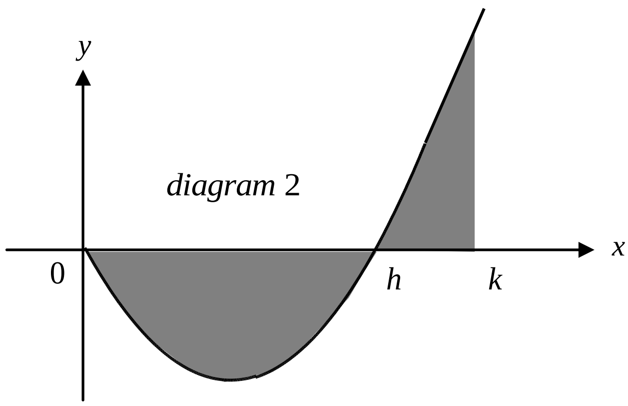



The profit/loss function is given by $f(x)=\frac{1}{3}x^2-x$ where $x\geq 0$.

**(a)** Find the value of ***h***.

**(b)** Diagram 2 (not drawn to scale) represents the breakeven situation where the initial loss made on selling the first ***h*** thousand units is exactly balanced by the later profit.

Calculate the value of $k$.

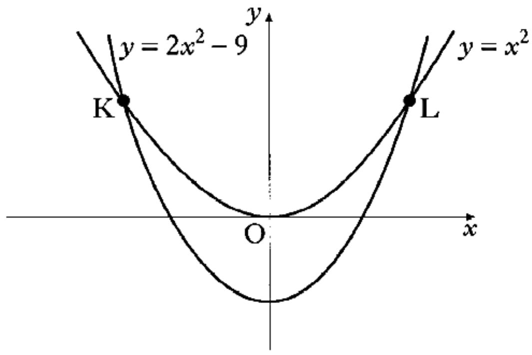


# Optimisation and Areas Between Curves

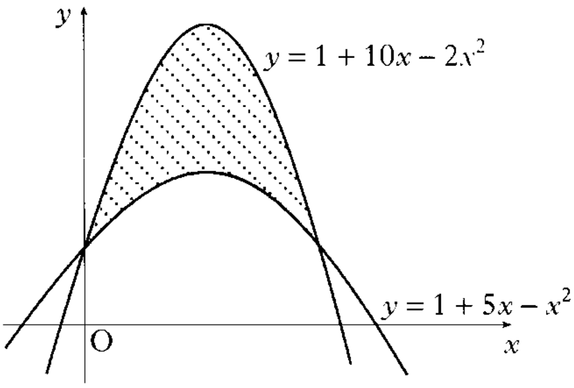
| O3 | I can evaluate the area enclosed between two functions. |
|---|---|

**1.** The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves.

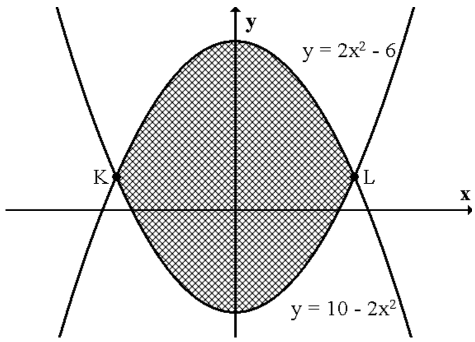



**2.** Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$.




**3.** The curves with equations $y = 2x^2 - 6$ and $y = 10 – 2x^2$ intersect at K and L.

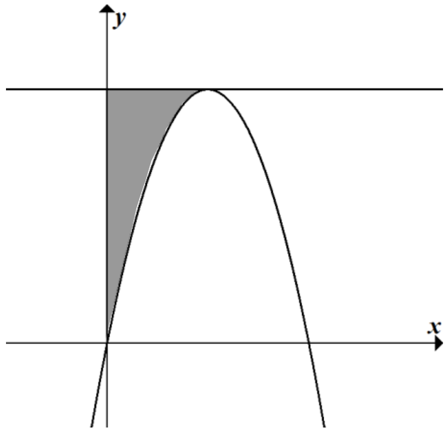
Calculate the area enclosed by these two curves.




**4.** Calculate the area enclosed by the line $\boldsymbol{y = 3(x - 1)}$ and the parabola $\boldsymbol{y = 3 + 2x - x^2}$.

# Optimisation and Areas Between Curves

**5.** Part of the graph of $y = 6x - x^2$ is shown. The tangent at the maximum stationary point has been drawn.

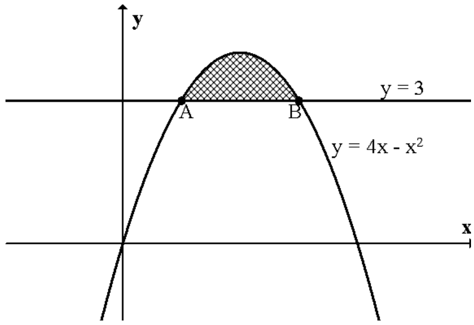



**(a)** Explain clearly why the tangent has equation $y = 9$.

**(b)** Calculate the shaded area enclosed by the curve, the tangent and the $y$ –axis.

**6.** The diagram opposite shows the curve

$y = 4x - x^2$ and the line $y = 3$.

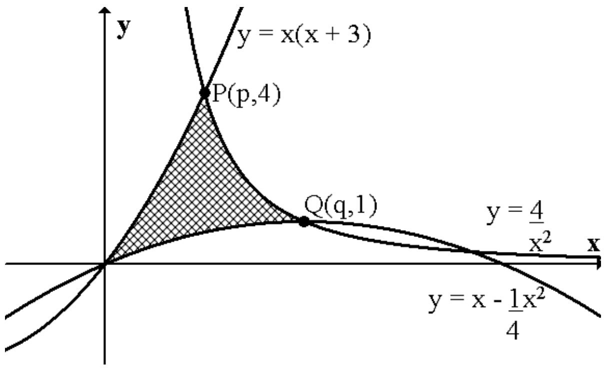



**(a)** Find the coordinates of A and B.

**(b)** Calculate the shaded area.

**7.** The diagram shows an area enclosed by 3 curves:

$\boldsymbol{y = x(x+3)}$, $\boldsymbol{y = \frac{4}{x^2}}$ and $\boldsymbol{y = x - \frac{1}{4}x^2}$




**(a)** P and Q have coordinates $(p,\ 4)$ and $(q,\ 1)$.

Find the values of p and q.

**(b)** Calculate the shaded area.

# Optimisation and Areas Between Curves

**8.** This diagram shows 2 curves $y = f_1(x)$ and $y = f_2(x)$ which intersect at $x = a$.

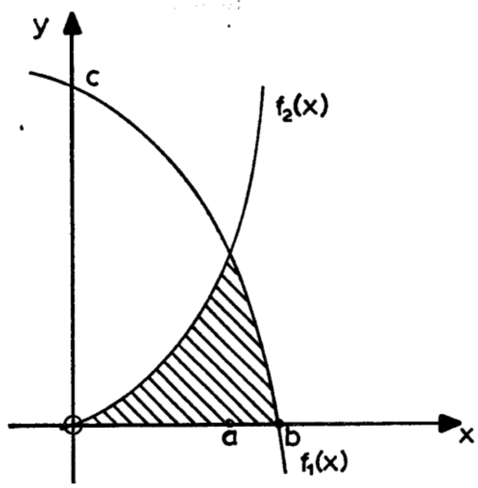
The area of the shaded region is

**A** $\int_0^b f_1(x)dx - \int_0^a f_2(x)dx$

**B** $\int_0^a f_2(x)dx + \int_a^b f_1(x)dx$

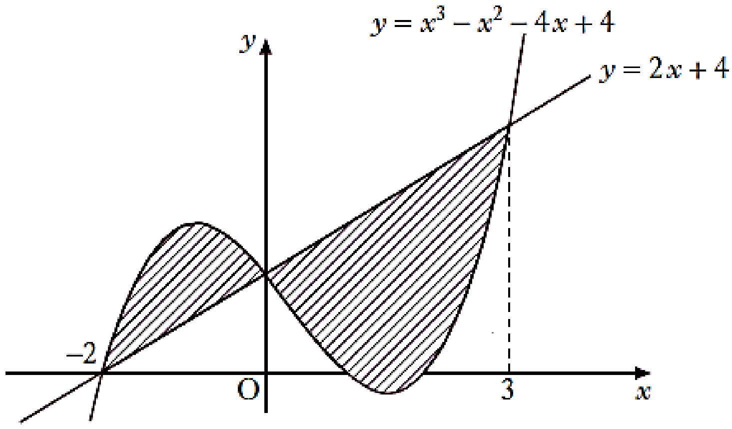
**C** $\int_0^a f_2(x)dx - \int_a^b f_1(x)dx$

**D** $\int_0^a f_1(x)dx + \int_a^b f_2(x)dx$




**9.** The diagram show the curve with equation $\boldsymbol{y = x^3 - x^2 - 4x + 4}$ and the line with equation $\boldsymbol{y = 2x + 4}$.

The curve and the line intersect at the points $(\mathbf{-2, 0})$, $(\mathbf{0, 4})$ and $(\mathbf{3, 10})$.




Calculate the total shaded area.

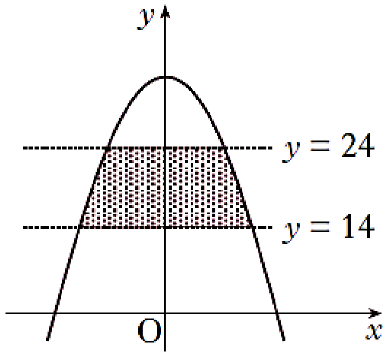
# Optimisation and Areas Between Curves

**10.** The parabola shown in the diagram has equation

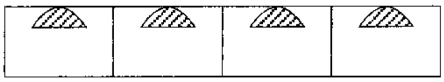
$$y = 32 - 2x^2$$

The shaded area lies between the lines $\boldsymbol{y = 14}$ and $\boldsymbol{y = 24}$.

Calculate the shaded area.



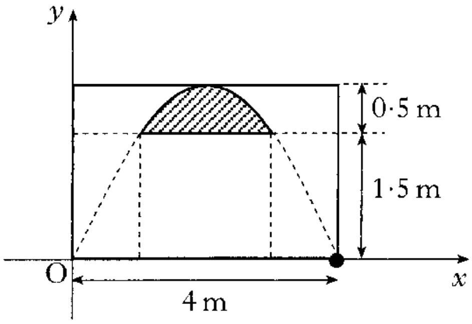

**11.** An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.



The second diagram shows one such windo. The shaded part represents the glass.

The top edge of the window is part of the parabola with equation $\boldsymbol{y = 2x - \frac{1}{2}x^2}$.

Find the area in square metres of the glass in one window.

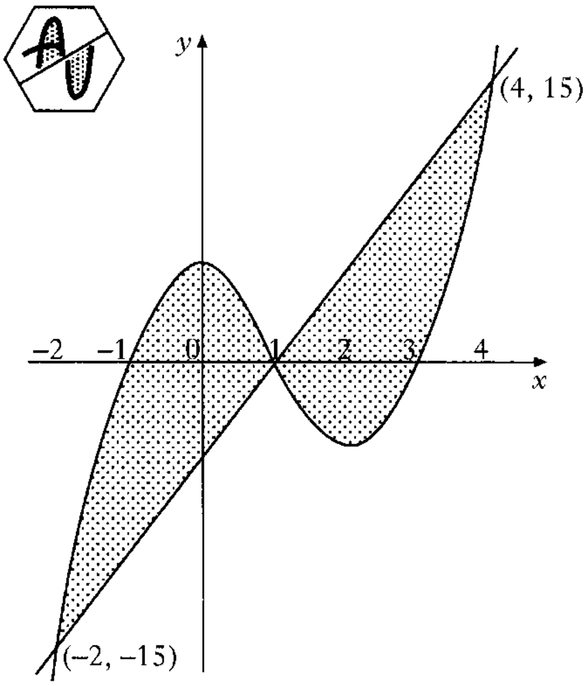



**12.** A firm asked for a logo to be designed involving the letters A and U. Their initial sketch is shown in the hexagon.

A mathematical representation of the final logo is shown in the coordinate diagram.

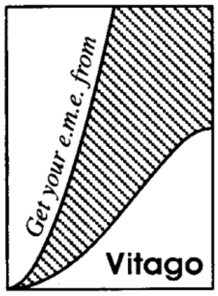
The curve has equation $\boldsymbol{y = (x+1)(x-1)(x-3)}$ and the straight line has equation $\boldsymbol{y = 5x - 5}$. The point $(\mathbf{1},\ \mathbf{0})$ is the centre of half-turn symmetry.

Calculate the total shaded area.

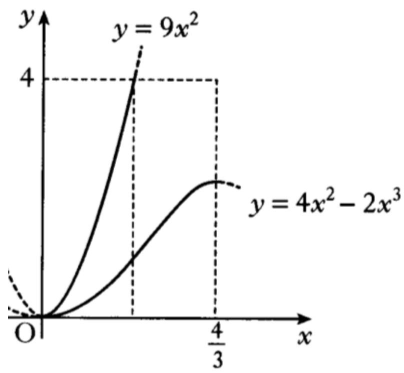


# Optimisation and Areas Between Curves

**13.** The diagram shows the front of a packet of Vitago, a new vitamin preparation to provide early morning energy. The shaded region is red and the rest yellow.




The design was created by drawing the curves $\boldsymbol{y = 9x^2}$ and $\boldsymbol{y = 4x^2 - 2x^3}$ as shown in the diagram below.

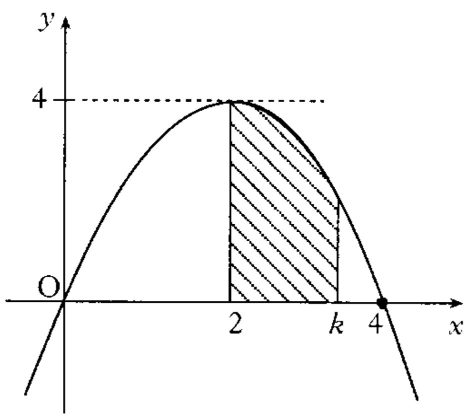



The edges of the packet are represented by the coordinate axes and the lines $\boldsymbol{x = \frac{4}{3}}$ and $\boldsymbol{y = 4}$.

Show that the area of red is $\boldsymbol{\frac{160}{81}}$ square units.

**14.** The parabola shown crosses the $x$-axis at $(0,0)$ and $(4,0)$, and has a maximum at $(2,4)$.

The shaded area is bound by the parabola, the $x$-axis and the lines $x = 2$ and $x = k$.




**(a)** Find the equation of the parabola.

**(b)** Hence show that the shaded area, $A$, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$

# Optimisation and Areas Between Curves

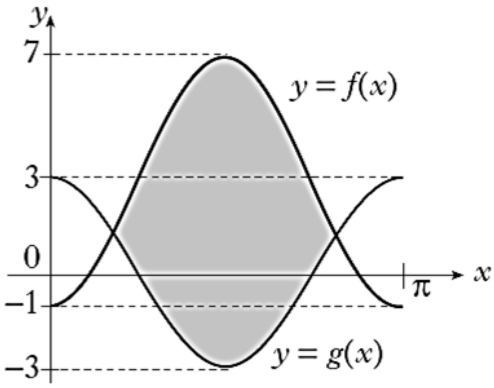
## Section D - Cross Topic Exam Style Questions

### Trigonometry and Areas between curves

**1.** The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = -4\cos 2x + 3$ and $g(x)$ is the form $g(x) = m\cos nx$.

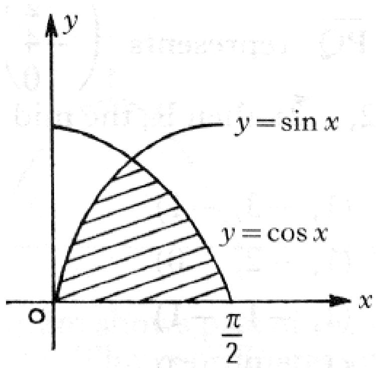
**(a)** Write down the values of $m$ and $n$.




**(b)** Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \leq x \leq \pi$.

**(c)** Calculate the shaded area.

**2.** The diagram shows the curves $y = \sin x$ and $y = \cos x$ for $0 \leq x \leq \pi/2$.




The shaded area is given by

**A** $\int_0^{\pi/4} \sin x\, dx + \int_{\pi/4}^{\pi/2} \cos x\, dx$

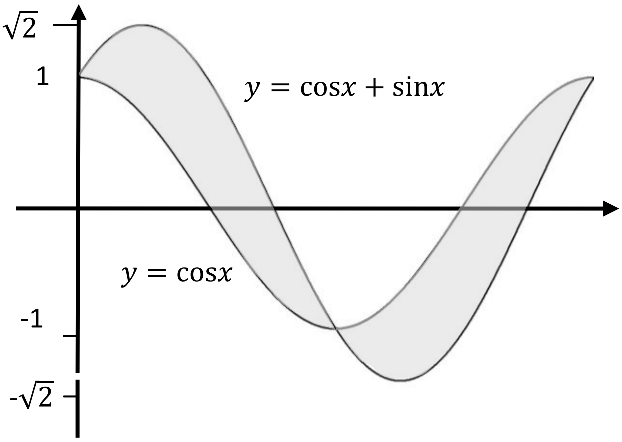
**B** $\int_0^{\pi/4} \cos x\, dx + \int_{\pi/4}^{\pi/2} \sin x\, dx$

**C** $\int_0^{\pi/2} (\sin x - \cos x)\, dx$

**D** $\int_0^{\pi/2} (\sin x + \cos x)\, dx$

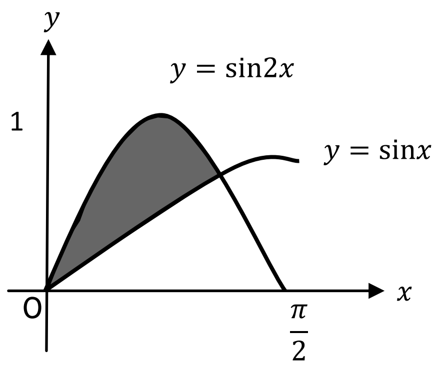
# Optimisation and Areas Between Curves

**3.** **(a)** Find the x-values of the three points of intersection for these curves, for $\mathbf{0 \leq x \leq 2\pi}$.




**(b)** Calculate the area enclosed by the curves.

**4.** **(a)** Write down a formula for $\mathbf{\sin 2x}$, and use it to solve the equation $\mathbf{\sin 2x = \sin x}$ for $\mathbf{0 \leq x \leq \frac{\pi}{2}}$.




**(b)** Find the shaded area enclosed by the curves $\boldsymbol{y = \sin 2x}$ and $\boldsymbol{y = \sin x}$.

**5.** **(a)** The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x + a)$ where $R > 0$ and $0 \leq a \leq 2\pi$.

Calculate the values of $R$ and $a$.

**(b)** Hence find the value of $t$, where $0 \leq t \leq 2$, for which

$$\int_0^t (3\sin x + 5\cos x)\, dx = 3$$

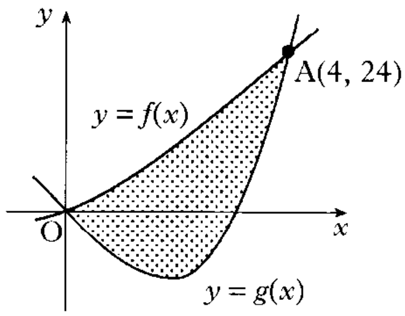
# Optimisation and Areas Between Curves

**Polynomials and Areas between curves**

**6.** The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram.

The graphs intersect at $A(4,\ 24)$ and the origin.

Find the shaded area enclosed between the curves.

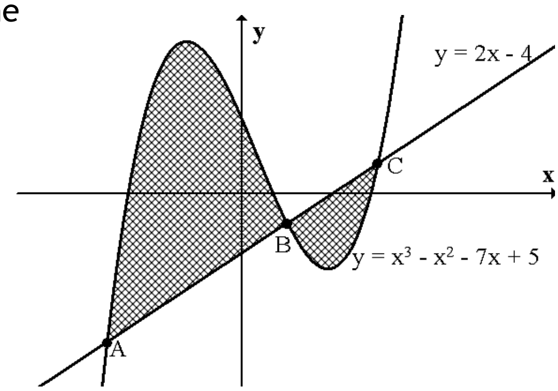



**7.** The curve $\boldsymbol{y = x^3 - x^2 - 7x + 5}$ and the line $\boldsymbol{y = 2x - 4}$ are shown opposite.

**(a)** B has coordinates $(1,\ -2)$.

Find the coordinates of A and C.

**(b)** Hence calculate the shaded area.




**8.** **(a)** **(i)** Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

**(ii)** Factorise $x^3 - 5x^2 + 2x + 8$ fully.

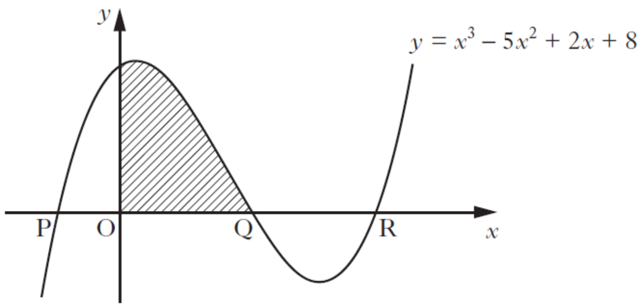
**(iii)** Solve $x^3 - 5x^2 + 2x + 8 = 0$.

**(b)** The diagram shows the curve with equation

$$y = x^3 - 5x^2 + 2x + 8$$

The curve crosses the $x$-axis at $P$, $Q$ and $R$.

Determine the shade area.

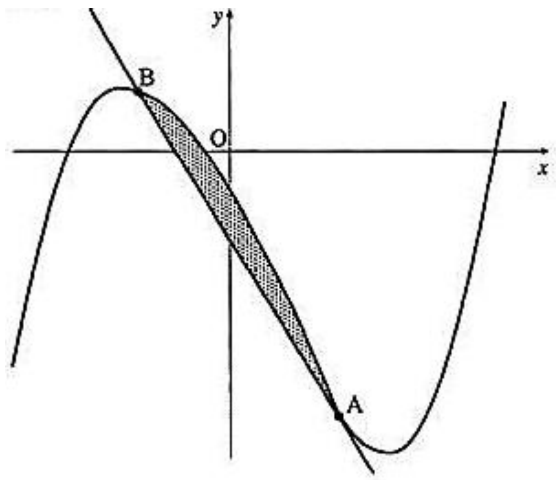


# Optimisation and Areas Between Curves

**Integration and Differentiation**

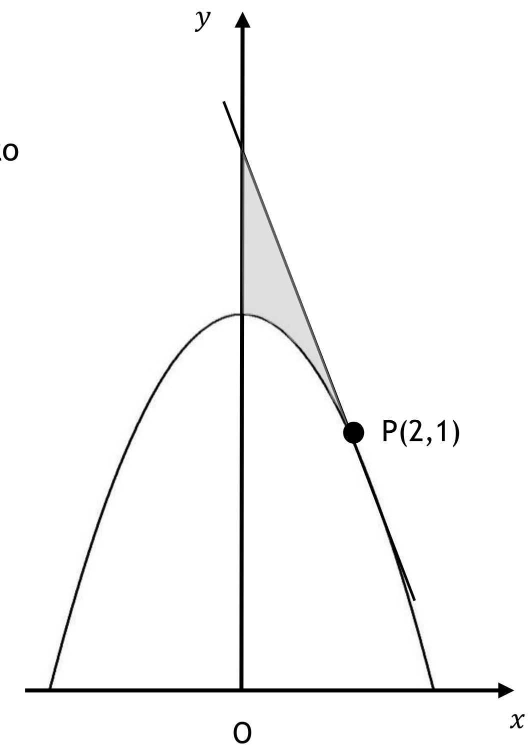
**9.** In the diagram below a winding river has been modelled by the curve $\boldsymbol{y = x^3 - x^2 - 6x - 2}$ and the road has been modelled by a straight line AB. The road is a tangent to the river at the point A$(\mathbf{1},\ \mathbf{-8})$.

**(a)** Find the equation of the tangent at A hence find the coordinates of B.




**(b)** Find the area of the shaded part which represents the land bounded by the river and the road.

**10.** **(a)** Find the equation of the tangent to the parabola $y = 5 - x^2$ at P(2,1)




**(b)** Calculate the area of the shaded region bounded by the tangent, the parabola and the y axis.

# Optimisation and Areas Between Curves

## Answers

### Section A

**R1**

**1.**

**(a)** $3x^2$ **(b)** $5x^4$ **(c)** $4x^3$ **(d)** $-6x^{-7}$ **(e)** $-19x^{-20}$ **(f)** $-5x^{-6}$

**(g)** $36x^5$ **(h)** $3x^5$ **(i)** $6x^8$ **(j)** $\frac{36}{5}x^{17}$ **(k)** $-\frac{4}{x^3}$ **(l)** $-\frac{9}{x^4}$

**(m)** $45x^2$ **(n)** $-\frac{15}{x^6}$ **(o)** $24x^3$ **(p)** $56x^3$ **(q)** $36x^3$ **(r)** $\frac{2}{7}$

**(s)** $\frac{1}{2\sqrt{x}}$ **(t)** $\frac{2}{3\sqrt[3]{x}}$ **(u)** $\frac{3}{4\sqrt[4]{x}}$ **(v)** $\frac{1}{2\sqrt{x}}$ **(w)** $\frac{5\sqrt{x^3}}{2}$ **(x)** $\frac{1}{3\sqrt[3]{x^2}}$

**(y)** $\frac{4}{5\sqrt[5]{x}}$ **(z)** $-\frac{1}{2\sqrt{x^3}}$ **(aa)** $-\frac{2}{3\sqrt[3]{x^5}}$ **(bb)** $-\frac{16}{3\sqrt[3]{x^{11}}}$

**(cc)** $-\frac{1}{3\sqrt[3]{x^5}}$ **(dd)** $-\frac{1}{2\sqrt[4]{x^7}}$

**2.**

**(a)** $15x^4 + 8x^3 - 1$ **(b)** $\frac{2}{3\sqrt[3]{x}} + 8x$ **(c)** $-\frac{10}{x^3} - \frac{3}{2\sqrt{x}}$

**(d)** $21x^6 - \frac{3}{20\sqrt[4]{x^7}}$ **(e)** $-\frac{3}{2\sqrt{x^7}}$ **(f)** $-\frac{1}{2\sqrt[4]{x^7}} + 2x + 1$

**(g)** $-\frac{4}{x^2} - \frac{8}{3\sqrt[3]{x}}$ **(h)** $15x^2 + \frac{3}{\sqrt{x^3}}$ **(i)** $2x + \frac{2}{x^3}$

**3.**

**(a)** $2x - 1$ **(b)** $2x + 4$ **(c)** $3x^2 - 4x$

**(d)** $4x - 12$ **(e)** $-\frac{2}{x^3} - \frac{2}{x^2}$ **(f)** $-\frac{1}{4\sqrt[4]{x^5}} + \frac{1}{2\sqrt{x^3}}$

**4.**

**(a)** $-\frac{5}{x^2}$ **(b)** $2x + \frac{6}{x^2} + 1$ **(c)** $\frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{x^3}}$

**(d)** $1 - \frac{2}{x^2}$ **(e)** $-\frac{2}{x^2} + \frac{6}{x^3}$ **(f)** $-\frac{5}{2x^2} - \frac{1}{x^3}$

**R2**

**(1)** $\cos x$ **(2)** $-\sin x$ **(3)** $5\cos x$

**(4)** $2\sin x$ **(5)** $-6\cos x$ **(6)** $9\cos x - 7\sin x$

**(7)** $-2\sin x + 4\cos x$ **(8)** $-3\sin x - 5\cos x$ **(9)** $3\cos x + 5\sin x$

# Optimisation and Areas Between Curves

**R3**

**(1)** $5(x-8)^4$ **(2)** $3(x+2)^2$ **(3)** $15(3x-1)^4$

**(4)** $-8(x+2)^{-9}$ **(5)** $-9(3x-5)^{-4}$ **(6)** $-\frac{15}{(3x+3)^6}$

**(7)** $\frac{1}{2\sqrt{x-2}}$ **(8)** $\frac{3\sqrt{(x+2)}}{2}$ **(9)** $-\frac{5}{\sqrt{(x+2)^7}}$

**R4**

1. (a) $\frac{17}{6}$ (b) $\frac{17}{6}$ (c) $-\frac{1}{12}$ (d) $\frac{22}{3}$ (e) $\frac{4}{3}$

   (f) $-15$

2. (a) $0$ (b) $6$ (c) $-6$ (d) $14$ (e) $\frac{98}{3}$

   (f) $\frac{25}{3}$ (g) $\frac{9}{4}$ (h) $-\frac{32}{3}$ (i) $\frac{9}{2}$

**R5**

1. (a) $0$ (b) $0$ (c) $0$ (d) $\frac{1}{2}$ (e) $0$

   (f) $0$ (g) $2$ (h) $\frac{1}{2}$ (i) $-\frac{1}{2}$ (j) $-\frac{1}{4}(\sqrt{3}-2)$

2. (a) $\frac{257}{64}$ (b) $\frac{21}{8}$ (c) $\frac{56}{9}$ (d) $\frac{21}{8}$ **(e)** $3+4\sqrt{3}$

   **(f)** $5\sqrt{3}-\frac{39}{4}$

**R6**

1. **(a)** $\frac{(x+2)^2}{9}+C$ **(b)** $\frac{(2x+4)^4}{8}+C$ **(c)** $25+C$

   **(d)** $\frac{(2x-1)^6}{12}+C$ **(e)** $\frac{-3(5-4x)^7}{14}+C$ **(f)** $\frac{(10-x)^{-9}}{9}+C$

   **(g)** $\frac{-3(4x+1)^{-2}}{8}+C$ **(h)** $\frac{-(5x-9)^{-4}}{10}+C$ **(i)** $\frac{(3-7x)^{-3}}{21}+C$

   **(j)** $\frac{2(x-1)^{\frac{3}{2}}}{3}+C$ **(k)** $\frac{3(2x-1)^{\frac{4}{3}}}{8}+C$ **(l)** $\frac{2(2x-1)^{\frac{5}{4}}}{5}+C$

# Optimisation and Areas Between Curves

**(m)** $\frac{(4x-2)^{\frac{3}{2}}}{3}+C$ **(n)** $\frac{(3x+4)^{\frac{5}{3}}}{5}+C$

**2.** **(a)** $\frac{-1}{20(5x+3)^4}+C$ **(b)** $\frac{-1}{9(3x-2)^3}+C$ **(c)** $\frac{3}{10(4-2x)^5}+C$

**(d)** $\frac{-1}{(x-2)^2}+C$ **(e)** $\frac{-1}{4(4x+2)^3}+C$ **(f)** $\frac{2}{5}\sqrt{(5x-2)}+C$

**(g)** $\frac{1}{6}(4x+2)^{\frac{3}{2}}+C$ **(h)** $\frac{2}{5}(2x+4)^{\frac{5}{4}}+C$ **(i)** $\frac{2}{3}\sqrt{(3x-4)}+C$

**(i)** $2\sqrt{(2x-5)}+C$

**R7**

**1.** **(a)** $\frac{1}{2}x-\frac{1}{4}\sin 2x+C$ **(b)** $\frac{1}{2}x+\frac{1}{4}\sin 2x+C$

**(c)** $x-\frac{1}{2}\sin 2x+C$ **(d)** $x+\frac{1}{2}\sin 2x+C$

**Section B**

**1.** $x=10$ **2.** $x=\frac{2}{\sqrt{3}}$ **3.** $Area=6\frac{3}{4}$

**4.** $Area=10\frac{2}{3}$ **5.** $Area=36$ **6.** $Area=4\frac{1}{2}$

**Section C**

**O1**

**1. (a)** 2 secs **(b)** 20 metres

**2. (a)** proof **(b)** $x=6cm, h=3cm$

3. (a) proof (b) $\frac{10}{\sqrt{3}}$ cm 4. (a) proof (b) $x=80$

5. (a) proof (b) $x=4$m, $length=8$m

6. (a) proof (b) 30 square metres

**O2**

# Optimisation and Areas Between Curves

1. (a) $\frac{125}{6}$ square units (b) 18 square units

2. (a) $P(-3,0), Q(3,0)$ (b) 72 square units

3. (a) $A(-2,0), B(5,0)$ (b) $101 \cdot 75$ square units

4. $\frac{\sqrt{3}}{4}$ square units 5. (a) $h = 3$ (b) $k = 4 \cdot 5$

## O3

| | | | | | |
|---|---|---|---|---|---|
| **1.** | 36 square units | **2.** | $20\frac{5}{6}$ square units | **3.** | $42\frac{2}{3}$ square units |
| **4.** | $20\frac{5}{6}$ square units | **5a** | $Proof$ | **5b.** | 9 square units |
| **6a** | $A(1,3), B(3,3)$ | **6b** | $1\frac{1}{3}$ square units | **7a.** | $p = 1, q = 2$ |
| **7b.** | $2\frac{1}{2}$ square units | **8.** | $B$ | **9.** | $21\frac{1}{12}$ square units |
| **10.** | $26\frac{2}{3}$ square units | **11.** | $\frac{2}{3}$ square units | **12.** | $40\frac{1}{2}$ square units |
| **13.** | $Proof$ | **14a.** | $y = 4x - x^2$ | 14b. | $Proof$ |

**Section D**

**1.** **(a)** $m = 3, n = 2$ **(b)** $(0 \cdot 6, 1 \cdot 3)$ $(2 \cdot 6, 1 \cdot 3)$ **(c)** $12 \cdot 4$

**2.** C

**3.** **(a)** $x = 0, \pi$ and $2\pi$ **(b)** 4

**4.** **(a)** $x = 0$, and $\frac{\pi}{3}$ **(b)** $\frac{1}{4}$

**5.** **(a)** $R = \sqrt{34}$, and $a = 5 \cdot 25$ **(b)** $t = 0 \cdot 6$

**6.** $\frac{128}{3}$

**7.** **(a)** $A(-3,-10), C(3,2)$ **(b)** $\frac{148}{3}$

**8.** **(a)i** Proof ii $(x-4)(x-2)(x+1)$ iii $x = -1, 2$ $and$ $4.$

**(b)** $\frac{32}{3}$

**9.** **(a)** $y = -5x - 3$ **(b)** $\frac{4}{3}$

# Optimisation and Areas Between Curves

**10.** **(a)** $y = -4x + 9$ **(b)** $\frac{8}{3}$