EF2. Trigonometry 1

Section A - Revision Section

This section will help you revise previous learning which is required in this topic.

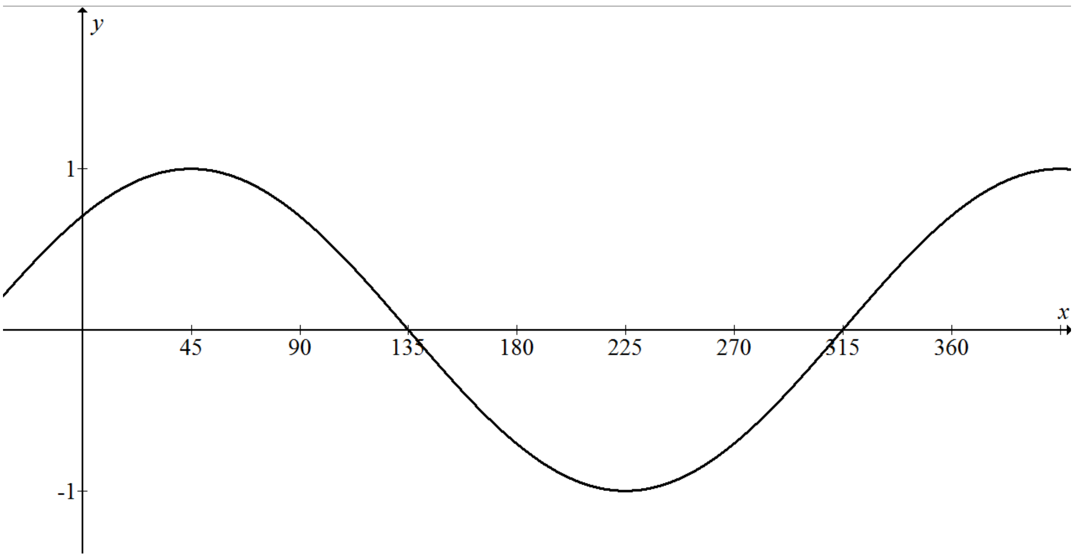
R1 Trig Graphs and Equations from National 5

1. Sketch the graphs of

(a) $y = -2\sin x°, 0 \leq x \leq 360.$ (b) $y = 3\cos x° - 1, 0 \leq x \leq 360.$

(c) $y = \tan x°, 0 \leq x \leq 360.$

2. Part of the graph of $y = \cos(x - a)°$ is shown.



Write down the value of a.

3. Solve the equations

(a) $5\tan x° - 6 = 2,$ $0 \leq x < 360.$

(b) $4\cos x° + 3 = 0,$ $0 \leq x \leq 360.$

(c) $7\sin x° + 1 = -5,$ $0 \leq x \leq 360.$

Trigonometry 1

4. If $\sin x° = \frac{4}{5}$ and $\cos x° = \frac{3}{5}$, calculate the value of $\tan x°$.

5. Simplify $\dfrac{\cos^3 x°}{1-\sin^2 x°}$.

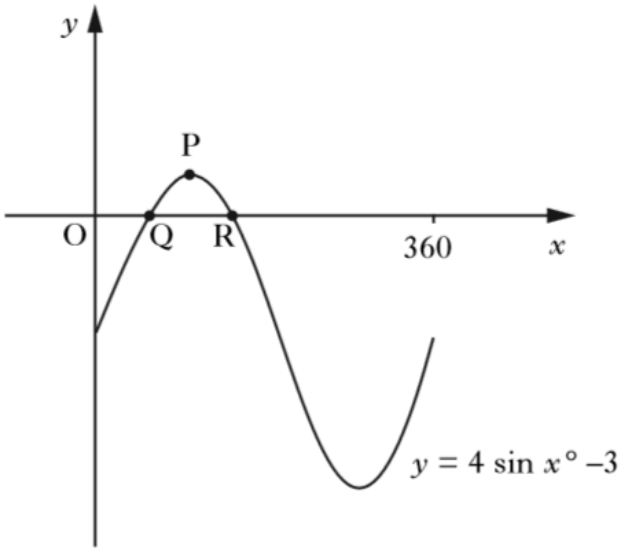
6. Part of a graph of $y = 4\sin x° - 3$ is shown.

The graph cuts the x-axis at Q and R.

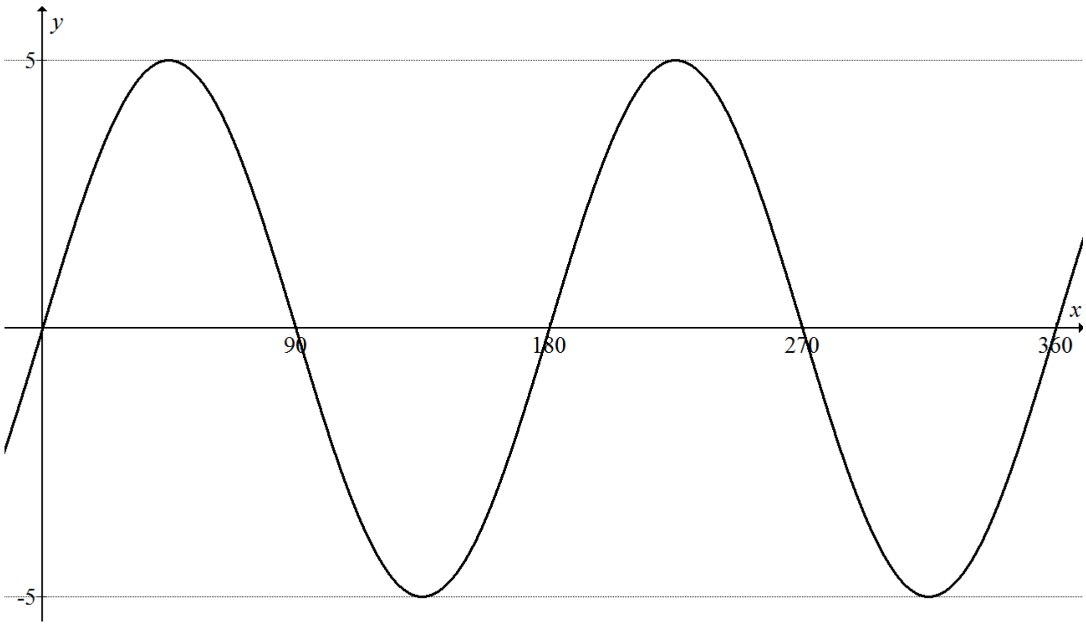
P is the maximum turning point.

(a) Write down the coordinates of P.

(b) Calculate the x-coordinates of Q and R.



7. Part of the graph of $y = a\sin bx°$ is shown.



Write down the values of a and b.

Trigonometry 1

Section B - Assessment Standard Section

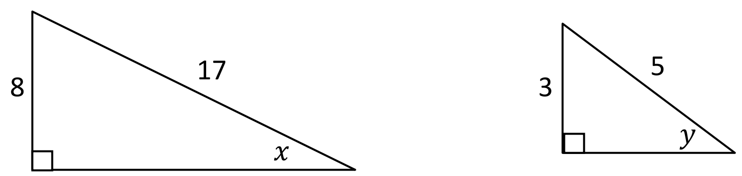
This section will help you practise for your Assessment Standard Test (Expressions and Functions 1.2)

1. A and B are acute angles such that $\sin A = \frac{7}{25}$, $\cos A = \frac{24}{25}$, $\sin B = \frac{12}{13}$ and $\cos B = \frac{5}{13}$.

   Find the exact value of $\cos (A + B)$.

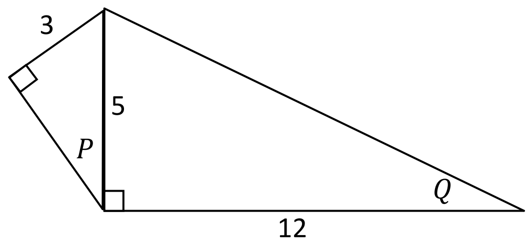
2. The diagram shows two right-angled triangles.

   Find the exact value of $\sin (x + y)$.



3. The diagram shows two right-angled triangles.

   Find the exact value of $\cos(P - Q)$.



4. Show that $\sin x \cos x \tan x \equiv 1 - cos^2 x$.

5. Show that $1 - \cos 2x \equiv \tan x \sin 2x$.

6. Show that $\sin 4B \equiv 4 \sin B \cos B(1 - 2\sin^2 B)$.

Trigonometry 1

7. Express $4\cos x° + \sin x°$ in the form $k\sin(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

8. Express $\sin x° - 3\cos x°$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \leq a < 360$.

9. $f(x) = 2\cos x° - 3\sin x°$.

   Express $f(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

10. Express $f(t) = \cos 30t° + \sqrt{3}\sin 30t°$ in the form $k\cos(30t - a)°$ where $k > 0$ and $0 \leq a < 360$.

Trigonometry 1

Section C – Operational Skills Section

This section provides problems with the operational skills associated with Trigonometry 1.

O1	***I can convert radians to degrees and vice versa.***

1. Convert the following angles from degrees to radians, giving you answer as an exact value.

(a) $30°$ **(b)** $45°$ **(c)** $60°$

(d) $90°$ **(e)** $180°$ **(f)** $360°$

(g) $150°$ **(h)** $240°$ **(i)** $315°$

2. Convert the following angles from degrees to radians, giving you answer to 3 significant figures.

(a) $37°$ **(b)** $142°$ **(c)** $307°$

3. Convert the following angles from radians to degrees.

(a) π radians **(b)** 2π radians **(c)** $\frac{\pi}{3}$ radians

(d) $\frac{\pi}{2}$ radians **(e)** $\frac{3\pi}{2}$ radians **(f)** $\frac{2\pi}{3}$ radians

(g) $\frac{5\pi}{3}$ radians **(h)** $\frac{\pi}{4}$ radians **(i)** $\frac{7\pi}{6}$ radians

4. Convert the following angles from radians to degrees, giving you answer to 3 significant figures.

(a) 1 radian **(b)** $1 \cdot 4$ radians **(c)** 3 radians

Trigonometry 1

O2 I can use and apply exact values.

1. Write down the exact value of

(a) $\sin 30°$ **(b)** $\sin 60°$ **(c)** $\sin 45°$

(d) $\sin 135°$ **(e)** $\sin 270°$ **(f)** $\sin 240°$

2. Write down the exact value of

(a) $\cos 30°$ **(b)** $\cos 60°$ **(c)** $\cos 45°$

(d) $\cos 120°$ **(e)** $\cos 180°$ **(f)** $\cos 210°$

3. Write down the exact value of

(a) $\tan 30°$ **(b)** $\tan 60°$ **(c)** $\tan 45°$

(d) $\tan 150°$ **(e)** $\tan 90°$ **(f)** $\tan 315°$

4. Write down the exact value of

(a) $\sin\frac{\pi}{6}$ **(b)** $\cos\frac{\pi}{4}$ **(c)** $\tan\frac{\pi}{3}$

(d) $\cos 2\pi$ **(e)** $\tan 2\pi$ **(f)** $\sin\pi$

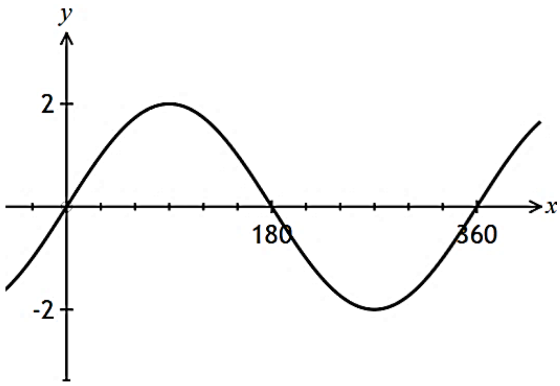
(g) $\tan\frac{5\pi}{4}$ **(h)** $\sin\frac{11\pi}{6}$ **(i)** $\cos\frac{7\pi}{6}$

Trigonometry 1

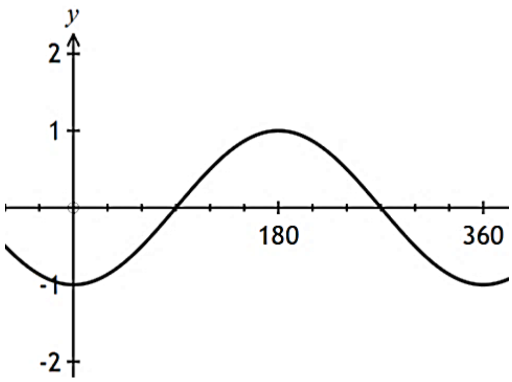
O3	***I can sketch or identify a basic trig graph under the transformations kf(x), f(x) + k, f(kx), f(x + k), -f(x) or a combination of these.***

1. Write down the equation of each of the graphs

(a)



(b)



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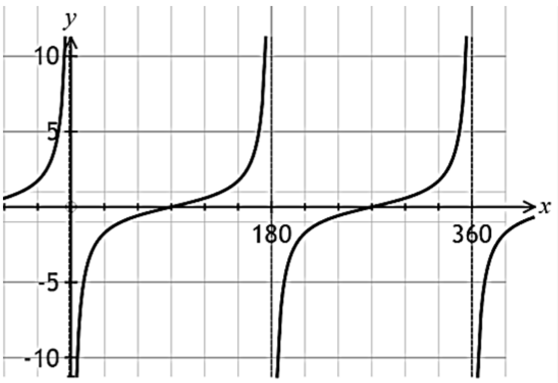

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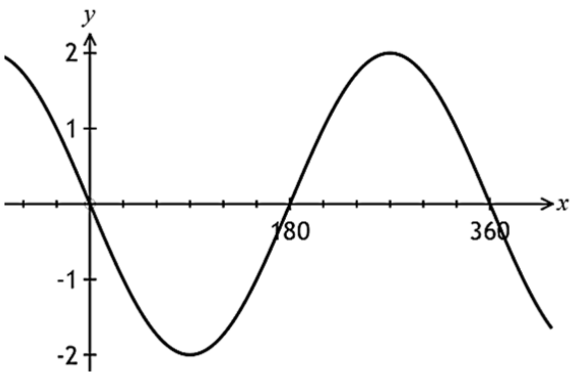

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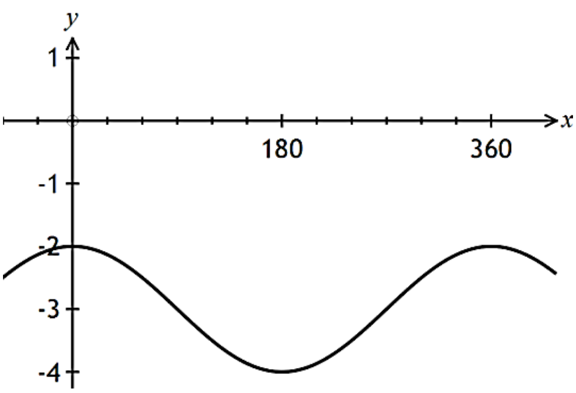

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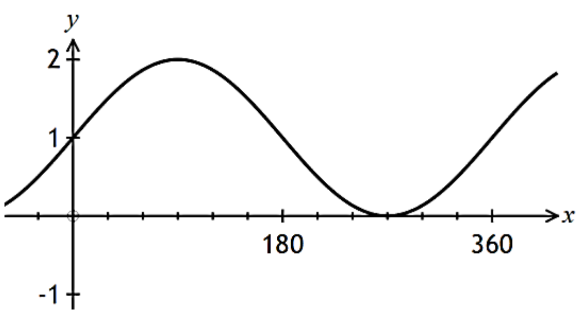


Trigonometry 1

2. Sketch each graph showing clearly the coordinates of the maximum and minimum values and where each graph cuts the axes.

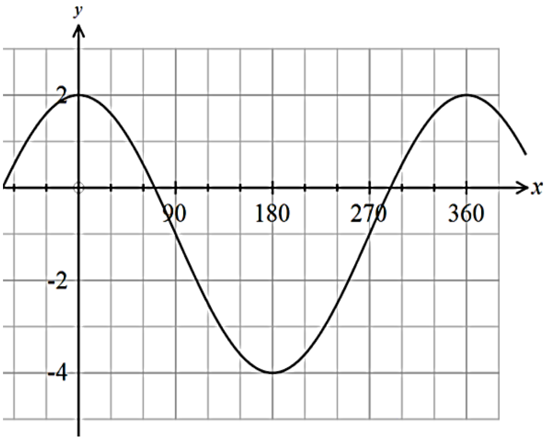
(a) $y = \sin x° + 1$ for $0 \leq x \leq 360$

(b) $y = -5\cos x$ for $0 \leq x \leq 2\pi$

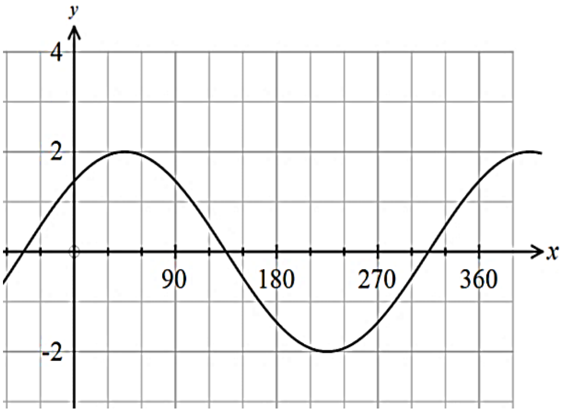
(c) $y = \tan(x - 45)°$ for $0 \leq x \leq 360$

3. Write down the equation of each of the graphs

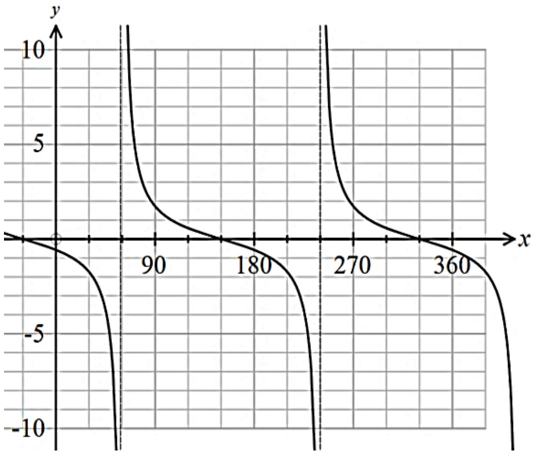
(a)



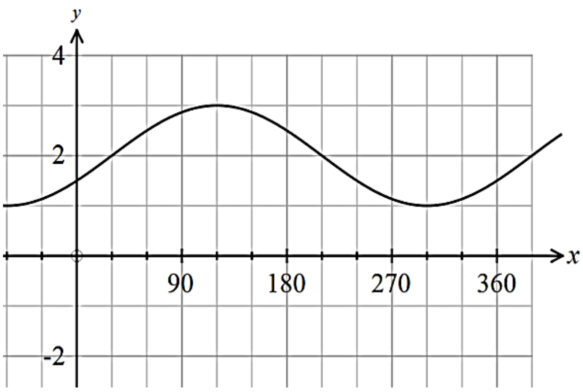
(b)



(c)



(d)



Trigonometry 1

4. Sketch each graph showing clearly the coordinates of the maximum and minimum values and where each graph cuts the axes.

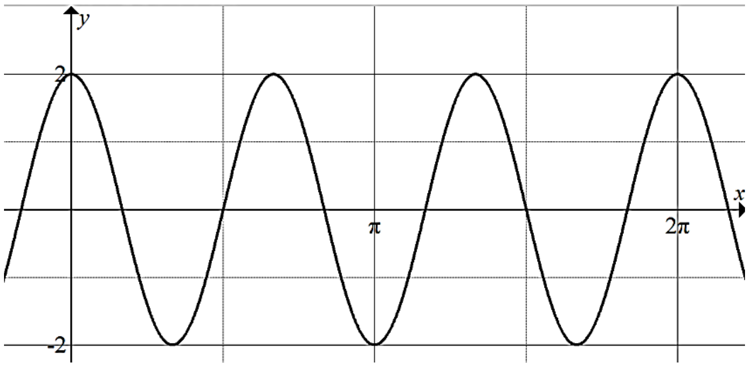
(a) $y = 4\cos 2x°$ for $0 \le x \le 360$

(b) $y = \sin\left(x - \frac{\pi}{6}\right) + 2$ for $0 \le x \le 2\pi$

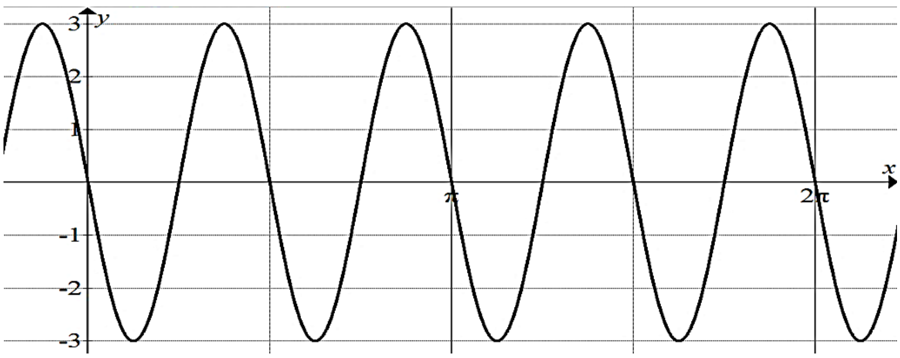
(c) $y = \cos 2x - 1$ for $0 \le x \le 2\pi$

5. Write down the equation of each of the graphs

(a)



(b)



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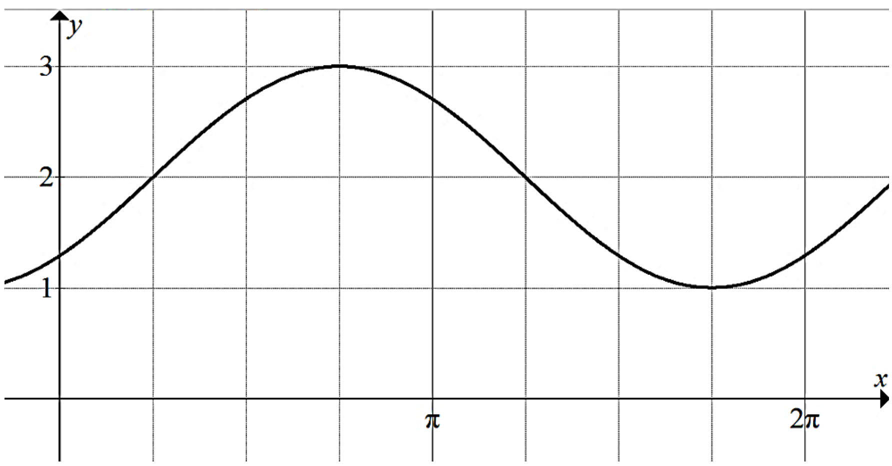


Trigonometry 1

O4	***I can use the addition and double angle formulae.***

1. Expand and use exact values to simplify

(a) $\sin(x - 60)°$ **(b)** $\cos\left(x - \frac{\pi}{4}\right)$

(c) $\sin(x + \pi)$ **(d)** $\cos\left(x + \frac{\pi}{3}\right)$

2. Use an appropriate substitution then expand to find the exact values of

(a) $\sin 15°$ **(b)** $\cos 105°$

3. Given that $\sin x° = \frac{3}{5}$ and $\cos x° = \frac{4}{5}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\sin 3x°$ (Hint $3x = 2x + x$)

4. Given that $\sin x° = \frac{1}{\sqrt{5}}$ and $\cos x° = \frac{2}{\sqrt{5}}$, find the exact values of:

(a) $\sin 2x°$

(b) $\cos 2x°$

(c) $\cos 3x°$

5. Given $\tan 2x = \frac{3}{4}$, $0 < x < \frac{\pi}{4}$, find the exact value of $\cos x$.

Trigonometry 1

O5	***I can convert $acosx + bsinx$ to $kcos(x \pm \alpha)$ or $ksin(x \pm \alpha)$, where a is in any quadrant $k > 0$.***

1. A function g is defined as $g(x) = 3\cos x° + \sin x°$.

Express $g(x)$ in the form $k\sin(x + \alpha)°$ where $k > 0$ and $0 \leq \alpha < 360$.

2. Express $3\sin x - 4\cos x$ in the form $a\sin(x - b)$ where $a > 0$ and $0 \leq b < 2\pi$.

3. Express $\sin x - \sqrt{3}\cos x$ in the form $k\cos(x + a)$ where $k > 0$ and $0 \leq a < 2\pi$.

4. A function f is defined as $f(x) = 2\cos x° - \sin x°$.

Express $f(x)$ in the form $k\sin(x - a)°$ where $k > 0$ and $0 \leq a < 360$.

5. A function f is defined as $f(x) = \sqrt{3}\cos 2x° + \sin 2x°$.

Express $f(x)$ in the form $k\cos(2x - a)°$ where $k > 0$ and $0 \leq a < 180$.

6. A function f is defined as $f(x) = \sqrt{5}\cos 3x° - 2\sin 3x°$.

Express $f(x)$ in the form $k\sin(3x + \alpha)°$ where $k > 0$ and $0 \leq \alpha < 360$.

7. A function f is defined as $f(x) = \sqrt{7}\cos 2x° - 3\sin 2x°$.

Express $f(x)$ in the form $k\cos(2x - a)°$ where $k > 0$ and $0 \leq a < 360$.

Trigonometry 1

O6	***I have experience of using wave functions to find the maximum and minimum values.***

1. **(a)** Express $\sin x - \cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a < 2\pi$.

(b) Hence state the maximum and minimum values of $\sin x - \cos x$ and determine the values of x, in the interval $0 \leq x < 2\pi$, at which these maximum and minimum values occur.

2. **(a)** Express $12\sin x + 5\cos x$ in the form $k\sin(x + a)$ where $k > 0$ and $0 \leq a < 2\pi$.

(b) Hence state the maximum value of $4 + 12\sin x + 5\cos x$ and determine the value of x, in the interval $0 \leq x < 2\pi$, at which the maximum occurs.

3. A function f is defined as $f(x) = 4\cos x° + 3\sin x°$.

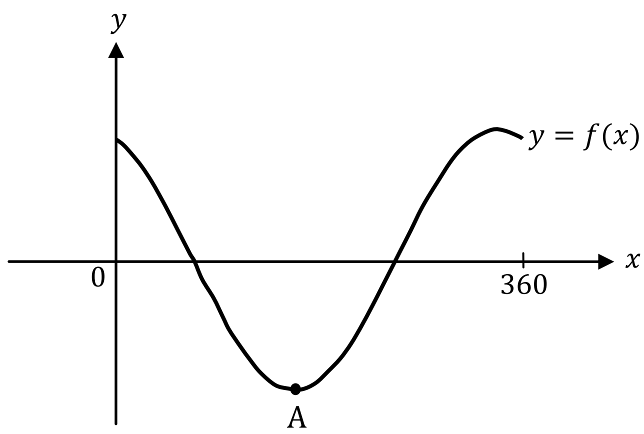
Find the maximum and minimum values of $f(x)$ and the values of, in the range $0 \leq x < 360$, at which the maximum and minimum values occur.

4. A function f is defined as $f(x) = 5\cos x° - 2\sin x°$.

(a) Express $f(x)$ in the form $k\cos(x + a)°$ where $k > 0$ and $0 \leq a < 360$.

(b) Part of the graph of $y = f(x)$ is shown in the diagram.

Find the coordinates of the minimum turning point A.

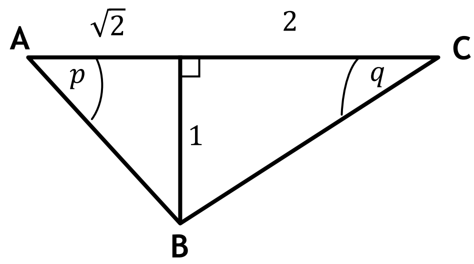


Trigonometry 1

O7	***I can apply Trig Formulae to Mathematical Problems (excluding where trig equations have to be solved but including exact values).***

1. If $\cos 2x = -\frac{31}{49}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$.

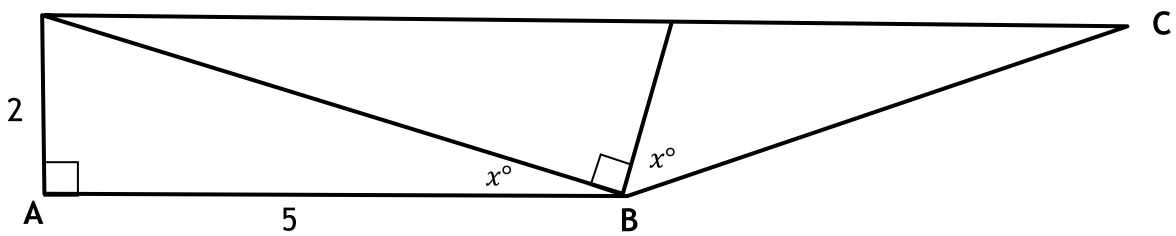
2. In triangle ABC, show that:



(a) The exact value of $\sin 2p = \frac{2\sqrt{2}}{3}$

(b) The exact value of $\cos(p+q) = \frac{2\sqrt{2}-1}{\sqrt{15}}$

3. For the shape shown, find the exact value of $\cos(A\hat{B}C)°$



4. It is given that $\cos a = \frac{3}{5}$ and $\sin b = \frac{2}{3}$.

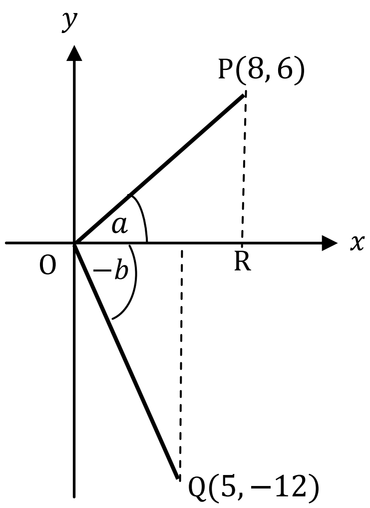
(a) Find the exact value of $\sin(a+b)$ and $\cos(a+b)$.

(b) Hence find the exact value of $\tan(a+b)$.

Trigonometry 1

5. On the coordinate diagram shown, P is the point $(8, 6)$ and Q is the point $(5, -12)$.

Angle POR = a and angle ROQ = $-b$.



(a) Find the exact value of $\sin(a - b)$.

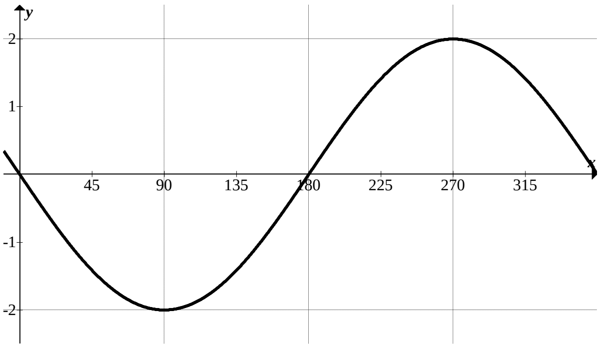
(b) Find the exact value of $\cos 2a$.

Trigonometry 1

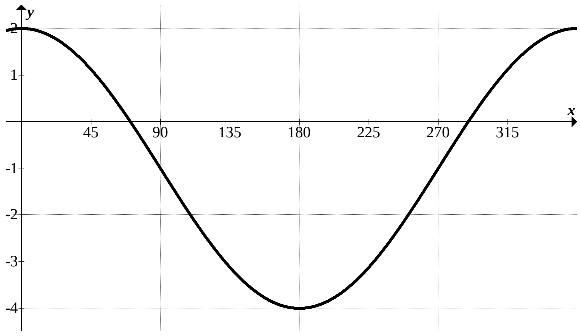
Answers

Section A

1.(a)



(b)



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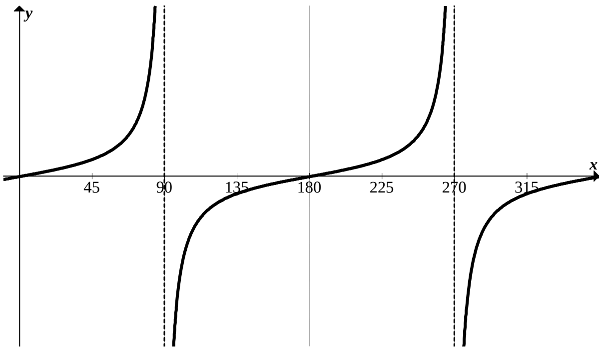


2. $a = 45$ **3.(a)** $x = 58, 238$ **(b)** $x = 189, 221$ **(c)** $x = 239, 301$

4. $\frac{4}{3}$ **5.** $\cos x°$

6(a) $P(90, 1)$ **(b)** $Q(48 \cdot 6, 0), R(131, 0)$ **7.** $a = 5, b = 2$

Section B

1. $\frac{36}{325}$ **2.** $\frac{77}{85}$ **3.** $\frac{63}{65}$ **4.** Proof **5.** Proof

6. Proof **7.** $\sqrt{17}\sin(x + 76)^\circ$ **8.** $\sqrt{10}\sin(x - 72)^\circ$

9. $\sqrt{13}cos\,(x + 56)^\circ$ **10.** $2cos\,(30t - 60)^\circ$

Trigonometry 1

O1

1. (a) $\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{3}$
   (d) $\frac{\pi}{2}$ (e) π (f) 2π
   (g) $\frac{5\pi}{6}$ (h) $\frac{4\pi}{3}$ (i) $\frac{7\pi}{4}$
2. (a) $0 \cdot 646$ (b) $2 \cdot 48$ (c) $5 \cdot 36$
3. (a) 180 (b) 360 (c) 60
   (d) 90 (e) 270 (f) 120
   (g) 300 (h) 45 (i) 210
4. (a) $57 \cdot 3$ (b) $80 \cdot 2$ (c) 172

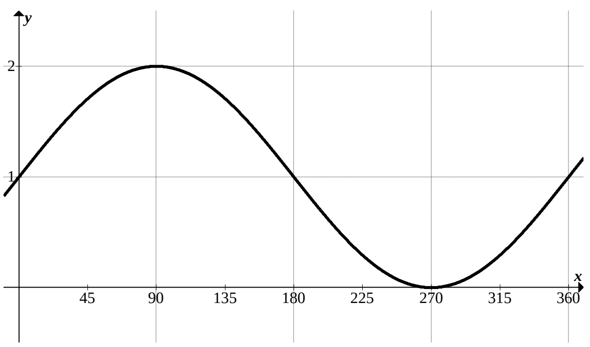
O2

1. (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{2}}$ (e) -1 (f) $-\frac{\sqrt{3}}{2}$
2. (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{1}{2}$ (c) $\frac{1}{\sqrt{2}}$ (d) $-\frac{1}{2}$ (e) -1 (f) $-\frac{\sqrt{3}}{2}$
3. (a) $\frac{1}{\sqrt{3}}$ (b) $\sqrt{3}$ (c) 1 (d) $-\frac{1}{\sqrt{3}}$ (e) Undefined (f) -1
4. (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{3}$ (d) 1 (e) 0 (f) 0
   (g) 1 (h) $-\frac{1}{2}$ (i) $-\frac{\sqrt{3}}{2}$

O3

1. (a) $y = 2\sin x°$ (b) $y = -\cos x°$ (c) $y = \tan(x - 90)°$
   (d) $y = -2\sin x°$ (e) $y = \cos x° - 3$ (f) $y = \sin x° + 1$

2.(a)



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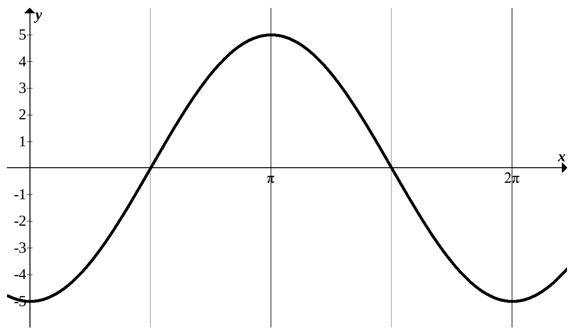

Trigonometry 1

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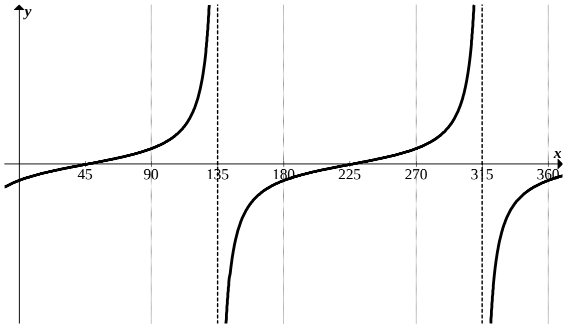
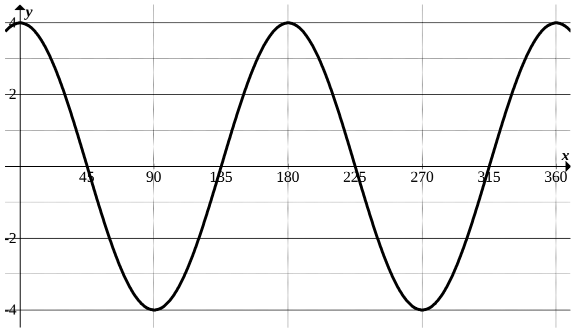


3. **(a)** $y = 3\cos x° - 1$

(b) $y = 2\sin(x + 45)°$ or $y = 2\cos(x - 45)°$

(c) $y = -\tan(x + 30)°$ **(d)** $y = \sin(x - 30)° + 2$

4.(a)



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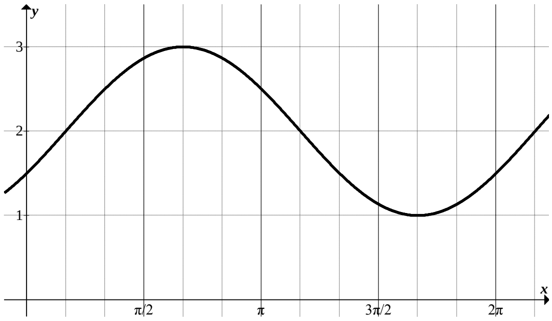

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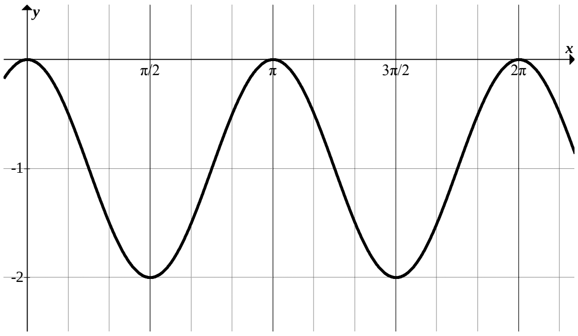

O4

1. **(a)** $\frac{1}{2}\sin x° - \frac{\sqrt{3}}{2}\cos x°$ **(b)** $\frac{1}{\sqrt{2}}\cos x + \frac{1}{\sqrt{2}}\sin x$

(d) $-\sin x$ **(c)** $\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\sin x$

2. **(a)** $\frac{\sqrt{3}-1}{2\sqrt{2}}$ **(b)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$

Trigonometry 1

3. **(a)** $\frac{24}{25}$ **(b)** $\frac{7}{25}$ **(c)** $\frac{117}{125}$

4. **(a)** $\frac{4}{5}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{2}{5\sqrt{5}}$

5. $\frac{3}{\sqrt{10}}$

O5

1. $\sqrt{10}\sin(x + 71.6)°$ **2.** $5\sin(x - 0\cdot93)$ **3.** $2\cos\left(x + \frac{7\pi}{6}\right)$

4. $\sqrt{5}\sin(x - 243\cdot4)°$ **5.** $2\cos(2x - 30)°$ **6.** $3\sin(3x + 138\cdot2)°$

7. $4\cos(2x - 311\cdot4)°$

O6

1. **(a)** $\sqrt{2}\sin\left(x - \frac{\pi}{4}\right)$ **(b)** min $-\sqrt{2}$ at $x = \frac{7\pi}{4}$, max $\sqrt{2}$ at $x = \frac{3\pi}{4}$

2. **(a)** $13\sin(x + 0.395)$ **(b)** max 17 at $x = 1.18$

3. min -5 at $x = 216\cdot9$, max 5 at $x = 36\cdot9$

4. **(a)** $\sqrt{29}\cos(x + 21.8)°$ **(b)** $(158\cdot2, -\sqrt{29})$