EF4. Vectors

Section A - Revision Section

This section will help you revise previous learning which is required in this topic.

R1 I have revised National 5 vectors and 3D coordinate.

1. If vector $\boldsymbol{a} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, find the resultant vector of:

(a) $\boldsymbol{a} + \boldsymbol{b}$ (b) $\boldsymbol{a} - \boldsymbol{b}$ (c) $3\boldsymbol{a} + \boldsymbol{b}$

(d) $\boldsymbol{a} - 2\boldsymbol{b}$ (e) $5\boldsymbol{a} - 3\boldsymbol{b}$ (f) $2\boldsymbol{a} + 4\boldsymbol{b}$

2. If vector $\boldsymbol{a} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and vector $\boldsymbol{b} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$, find the resultant vector of

(a) $\boldsymbol{a} + \boldsymbol{b}$ (b) $\boldsymbol{a} - \boldsymbol{b}$ (c) $2\boldsymbol{a} + 3\boldsymbol{b}$

(d) $5\boldsymbol{a} - \boldsymbol{b}$ (e) $3\boldsymbol{a} - 2\boldsymbol{b}$ (f) $\boldsymbol{a} + 4\boldsymbol{b}$

3. If $\boldsymbol{p} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix}$, find:

(a) $|\boldsymbol{p}|$ (b) $|\boldsymbol{q}|$ (c) $|\boldsymbol{p} + \boldsymbol{q}|$

(d) $|\boldsymbol{p} - \boldsymbol{q}|$ (e) $|3\boldsymbol{p} - \boldsymbol{q}|$ (f) $|2\boldsymbol{p} + 3\boldsymbol{q}|$

4. Three vectors are defined as $\overrightarrow{AB} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$ and $\overrightarrow{EF} = \begin{pmatrix} 1 \\ 1 \\ 5 \end{pmatrix}$, find:

(a) $|\overrightarrow{AB}|$ (b) $|\overrightarrow{CD}|$ (c) $|\overrightarrow{EF}|$

Vectors

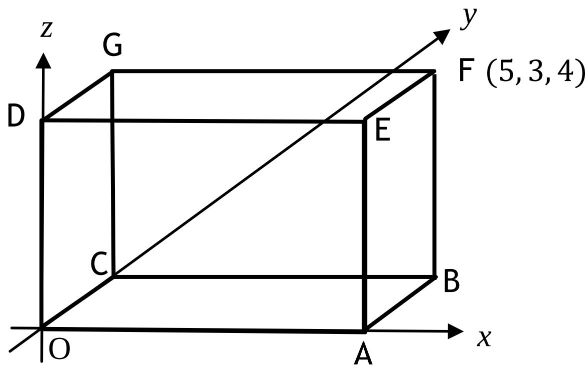
5. Three points A, B and C have the coordinates $(2, 5, 3)$, $(-1, 3, 0)$ and $(1, 4, 2)$ respectively. Find the vectors

(a) $\overrightarrow{OA}$ **(b)** $\overrightarrow{OB}$ **(c)** $\overrightarrow{OC}$

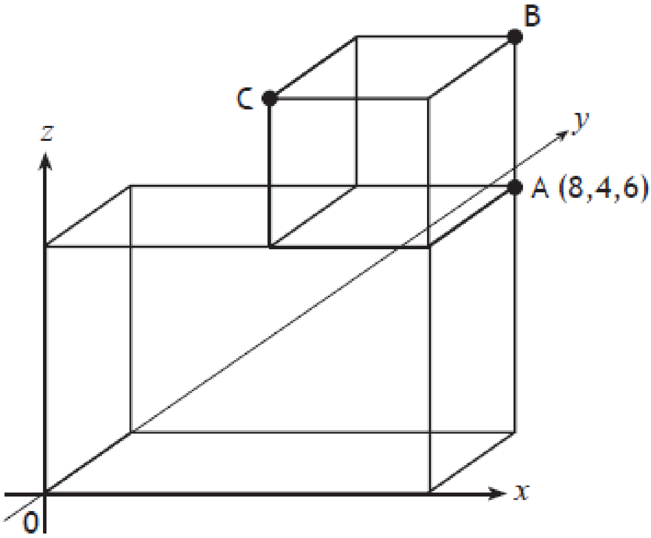
(d) $\overrightarrow{AB}$ **(e)** $\overrightarrow{BC}$ **(f)** $\overrightarrow{AC}$

6. The diagram shows the cuboid OABCDEFG. O is the origin and OA, OC and OD are aligned with the x, y and z axes respectively. The point F has coordinates $(5, 3, 4)$.

List the coordinates of the other six vertices.



7.

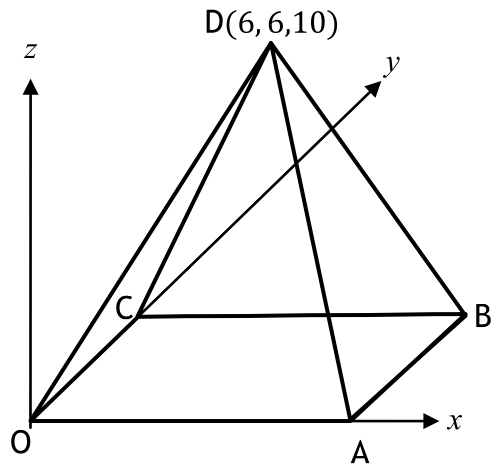


The diagram shows a cube placed on top of a cuboid, relative to the coordinate axes. A is the point $(8, 4, 6)$.

Write down the coordinates of B and C.

8. The diagram shows the square based pyramid DOABC. O is the origin with OA and OC aligned with the x and y axes respectively. The point D has coordinates $(6, 6, 10)$.

Write down the coordinates of the points A, B and C.

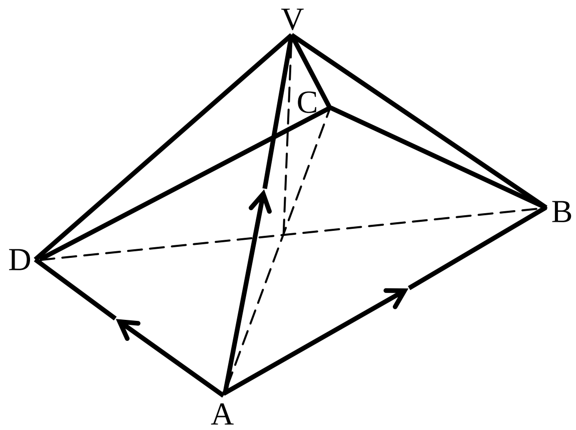


Vectors

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test (Expressions and Functions 1.4)

1. VABCD is a pyramid with rectangular base ABCD.



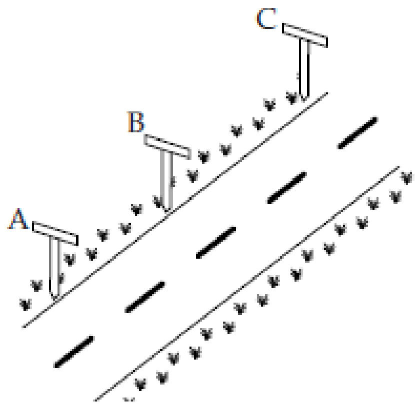
The vectors $\overrightarrow{AB}, \overrightarrow{AD}$ and $\overrightarrow{AV}$ are given by

$$\overrightarrow{AB} = \begin{pmatrix} 8 \\ 2 \\ 2 \end{pmatrix}; \quad \overrightarrow{AD} = \begin{pmatrix} -2 \\ 10 \\ -2 \end{pmatrix} \text{ and } \overrightarrow{AV} = \begin{pmatrix} 1 \\ 7 \\ 7 \end{pmatrix}.$$

Express $\overrightarrow{CV}$ in component form.

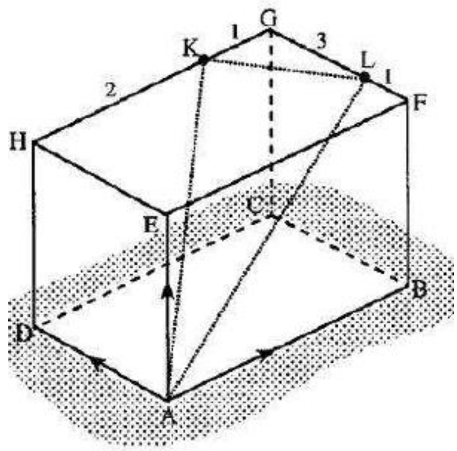
2. Road makers look along the tops of a set of T-rods to ensure that straight sections of road are being created.

Relative to suitable axes the top left corners of the T-rods are the points A (-8, -10, -2), B (-2, -1, 1) and C(6, 11, 5).



Determine whether or not the section of road ABC has been built in a straight line.

Vectors



3. ABCDEFGH is a cuboid.

K lies two thirds of the way along HG.
(i.e. HK:KG = 2:1).

L Lies one quarter of the way along FG.
(i.e. FL:LG = 1:3).

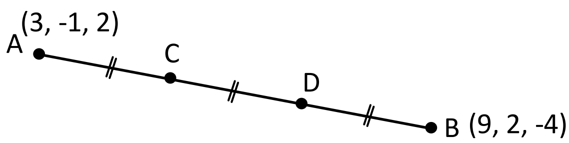
$\overrightarrow{AB}, \overrightarrow{AD}$ and $\overrightarrow{AE}$ can be represented by the vectors

$\begin{pmatrix}3\\6\\3\end{pmatrix}, \begin{pmatrix}-8\\4\\4\end{pmatrix}$ and $\begin{pmatrix}1\\-3\\5\end{pmatrix}$ respectively.

(a) Calculate the components of $\overrightarrow{AK}$.

(b) Calculate the components of $\overrightarrow{AL}$.

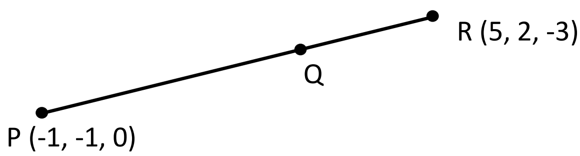
4. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, -1, 2) and (9, 2, -4).



(a) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(b) Find the coordinates of C and D.

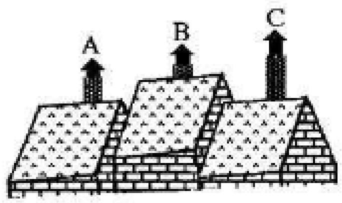
5. The point Q divides the line joining P (-1, -1, 0) to R (5, 2, -3) in the ratio 2:1.



Find the coordinates of Q.

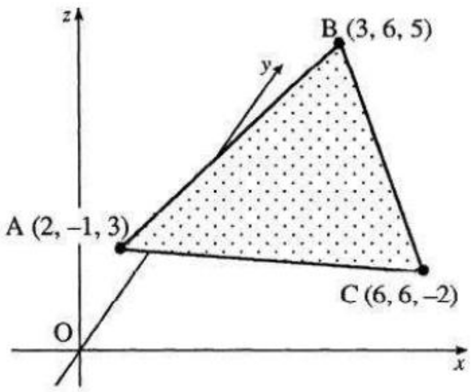
Vectors

6. Relative to a suitable set of axes, the tops of three chimneys have coordinates given by A(1, 3, 2), B (2, -1, 4) and C (4, -9, 8).



Show that A, B and C are collinear.

7. A triangle ABC has vertices A (2, -1, 3), B (3, 6, 5) and C (6, 6, -2).



(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$.

(b) Calculate the size of angle BAC.

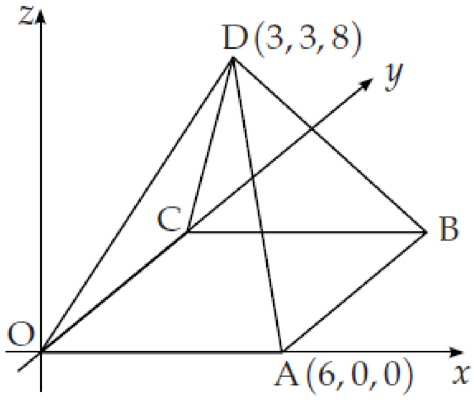
Vectors

8. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

The coordinates of A and D are (6, 0, 0) and (3, 3, 8).

C lies on the y-axis.



(a) Write down the coordinates of B.

(b) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$.

(c) Calculate the size of angle ADB.

Vectors

Section C – Operational Skills Section

This section provides problems with the operational skills associated with Exponentials and Logs

O1 I can express and manipulate vectors in the form $\boldsymbol{ai} + \boldsymbol{bj} + \boldsymbol{ck}$.

1. Write the following vectors, given in unit vector form, in component form.

(a) $\boldsymbol{a} = 2\boldsymbol{i} + 3\boldsymbol{j} + \boldsymbol{k}$ **(b)** $\boldsymbol{b} = 4\boldsymbol{i} + 2\boldsymbol{j}$ **(c)** $\boldsymbol{c} = \boldsymbol{i} - 6\boldsymbol{j} - 4\boldsymbol{k}$

2. Write the following vectors, given in component form, in unit vector form.

(a) $\boldsymbol{p} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ **(b)** $\boldsymbol{q} = \begin{pmatrix} 6 \\ -2 \\ 7 \end{pmatrix}$ **(c)** $\boldsymbol{r} = \begin{pmatrix} 1 \\ -4 \\ 0 \end{pmatrix}$

3. Two vectors are defined, in unit vector form, as $\boldsymbol{p} = 3\boldsymbol{i} - \boldsymbol{k}$ and $\boldsymbol{q} = \boldsymbol{i} - \boldsymbol{2j} + \boldsymbol{3k}$.

(a) Express $\boldsymbol{p} + 2\boldsymbol{q}$ **in unit vector form.**

(b) Express $3\boldsymbol{p} - 4\boldsymbol{q}$ **in unit vector form.**

(c) Find $|\boldsymbol{p} + 2\boldsymbol{q}\ |$**.**

(d) Find $|3\boldsymbol{p} - 4\boldsymbol{q}|$**.**

Vectors

O2	***I can calculate the scalar product and know that perpendicular vectors have a scalar product of zero.***

1. Find the scalar product of each of the pairs of vectors below and state clearly which pairs are perpendicular.

(a) $\overrightarrow{AB} = \begin{pmatrix} 1 \\ -3 \\ 5 \end{pmatrix}$ and $\overrightarrow{CD} = \begin{pmatrix} 2 \\ -2 \\ 3 \end{pmatrix}$.

(b) $\boldsymbol{p} = \begin{pmatrix} -6 \\ 1 \\ 2 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$.

(c) $\boldsymbol{a} = 3\boldsymbol{i} - \boldsymbol{4j} + \boldsymbol{2k}$ and $\boldsymbol{b} = -\boldsymbol{i} + \boldsymbol{3j} + \boldsymbol{k}$

2. If $\left|\overrightarrow{AB}\right| = 3$ and $\left|\overrightarrow{AC}\right| = 4$ and $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are inclined at an angle of $60°$, find the scalar product $\overrightarrow{AB} \cdot \overrightarrow{AC}$.

3. If $|\boldsymbol{a}| = \frac{\sqrt{2}}{3}$ and $|\boldsymbol{b}| = \frac{3}{4}$ and $\boldsymbol{p}$ and $\boldsymbol{q}$ are inclined at an angle of $45°$, find the scalar product $\boldsymbol{p} \cdot \boldsymbol{q}$.

O3	***I can determine whether or not coordinates are collinear, using the appropriate language, and can apply my knowledge of vectors to divide lines in a given ratio.***

1. The point Q divides the line joining P$(-1, -1, 3)$ and R$(5, -1, -3)$ in the ratio 5:1. Find the coordinates of Q.

2. The point B divides the line joining A$(1, -2, 4)$ and C$(-11, 13, -8)$ in the ratio 1:2. Find the coordinates of B.

Vectors

3. John is producing a 3D design on his computer.

Relative to suitable axes 3 points in his design have coordinates P$(-3, 4, 7)$, Q$(-1, 8, 3)$ and R$(0, 10, 1)$.

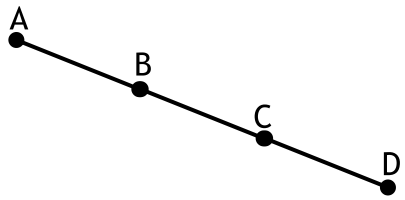
(a) Show that P, Q and R are collinear.

(b) Find the coordinates of S such that $\overrightarrow{PS} = 4\overrightarrow{PQ}$.

4. A and B are the points $(0, -2, 3)$ and $(3, 0, 2)$ respectively.

B and C are the points of trisection of AD, that is AB = BC = CD.

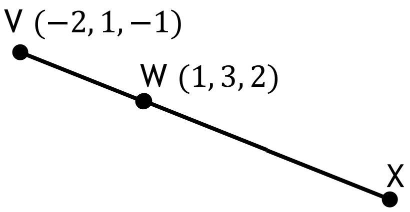
Find the coordinates of D.



5. The points V, W and X are shown on the line opposite.

V, W and X are collinear points such that WX = 2VW.

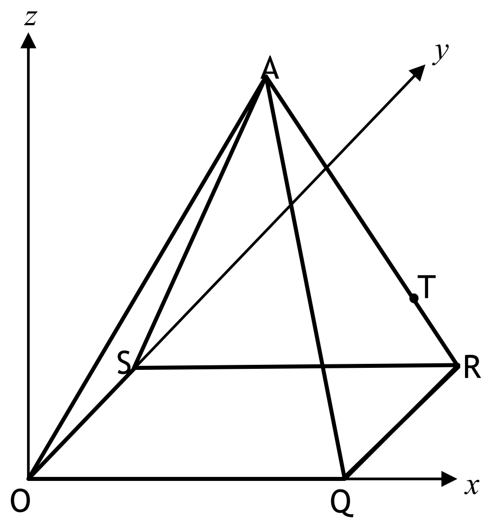
Find the coordinates of X.



6. AOQRS is a pyramid. Q is the point $(16, 0, 0)$, R is $(16, 8, 0)$ and A is $(8, 4, 12)$. T divides RA in the ratio 1:3.

(a) Find the coordinates of the point T.

(b) Express $\overrightarrow{QT}$ in component form.



Vectors

O4 I can apply knowledge of vectors to find an angle in three dimensions.

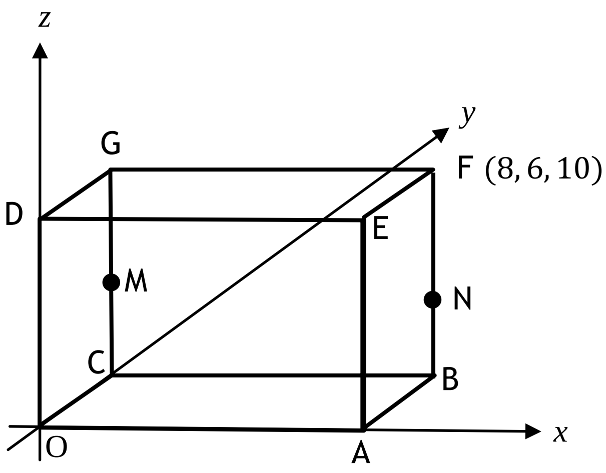
1. Three planes, Tango (T), Delta (D) and Bravo (B) are being tracked by radar. Relative to a suitable origin, the positions of the three planes are T$(23, 0, 8)$, D$(-12, 0, 9)$ and B$(28, -15, 7)$

(a) Express the vectors $\overrightarrow{BT}$ and $\overrightarrow{BD}$ in component form.

(b) Find the size of angle TBD.

2. The diagram shows a cuboid OABCDEFG with the lines OA, OC and OD lying on the axes.

The point F has coordinates $(8, 6, 10)$, M is the midpoint of CG and N divides BF in the ratio 2:3.

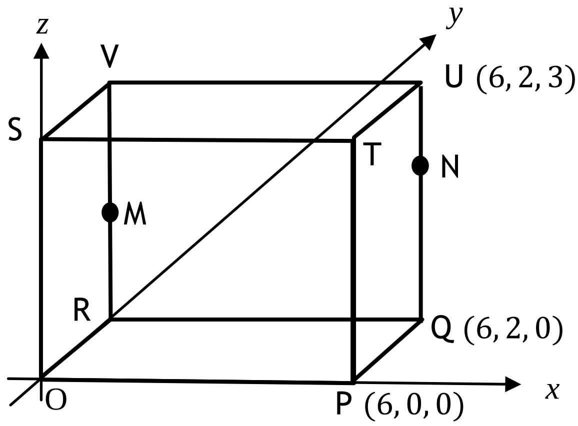


(a) State the coordinates of A, M and N.

(b) Determine the components of the vectors $\overrightarrow{MA}$ and $\overrightarrow{MN}$.

(c) Find the size of angle AMN.

3. In the diagram OPQRSTUV is a cuboid. M is the midpoint of VR and N is the point on UQ such that $\text{UN} = \frac{1}{3}\text{UQ}$.



(a) State the coordinates of T, M and N.

(b) Determine the components of the vectors $\overrightarrow{TM}$ and $\overrightarrow{TN}$.

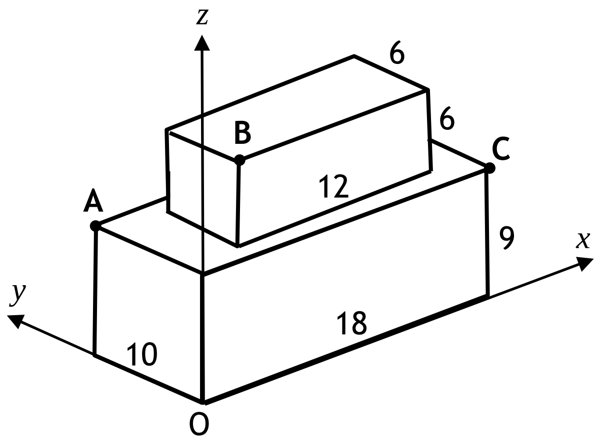
(c) Find the size of angle MTN.

4. A cuboid measuring 12cm by 6cm by 6cm is placed centrally on top of another cuboid measuring 18cm by 10cm by 9cm.

Coordinate axes are taken as shown.

(a) The point A has coordinates $(0, 10, 9)$ and the point C has coordinates $(18, 0, 9)$. Write down the coordinates of B.

(b) Find the size of angle ABC.



O5 I know the properties of the scalar product and their uses.

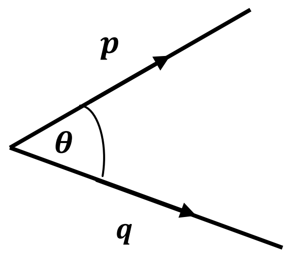
1. Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are defined by $\boldsymbol{p} = -3\boldsymbol{i} - 12\boldsymbol{k}$ and $\boldsymbol{q} = 8\boldsymbol{i} + 7\boldsymbol{j} - 2\boldsymbol{k}$.

Determine whether or not $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular to each other.

2. For what value of p are the vectors $\boldsymbol{a} = \begin{pmatrix} p \\ -2 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 3 \\ 14 \\ 2p \end{pmatrix}$ perpendicular?

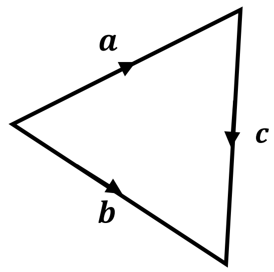
3. The diagram shows vectors $\boldsymbol{p}$ and $\boldsymbol{q}$.

If $|\boldsymbol{p}| = 3$, $|\boldsymbol{q}| = 4$ and $\boldsymbol{p}.(\boldsymbol{p} + \boldsymbol{q}) = 15$, find the size of the acute angle θ between $\boldsymbol{p}$ and $\boldsymbol{q}$.



Vectors

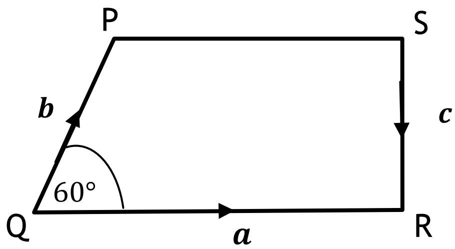
4. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ form an equilateral triangle of length 3 units.



(a) Find the scalar product $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$.

(b) What does this tells us about the vectors $\boldsymbol{a}$ and $\boldsymbol{b}+\boldsymbol{c}$.

5. The vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$ are shown on the diagram. Angle PQR = $60°$.



It is also given that $|\boldsymbol{a}| = 3$ and $|\boldsymbol{b}| = 2$.

(a) **Evaluate $\boldsymbol{a}.(\boldsymbol{b}+\boldsymbol{c})$ and $\boldsymbol{c}.(\boldsymbol{a}-\boldsymbol{b})$.**

(b) **Find $|\boldsymbol{b}+\boldsymbol{c}|$ and $|\boldsymbol{a}-\boldsymbol{b}|$.**

Vectors

Section D - Cross Topic Exam Style Questions

The examples given below do not fit here.

Need to develop questions which combine

Vectors and logs,

vectors and trig and

vectors and functions

Vectors

Section A

R1

1. **(a)** $\begin{pmatrix}5\\5\end{pmatrix}$ **(b)** $\begin{pmatrix}-1\\-3\end{pmatrix}$ **(c)** $\begin{pmatrix}9\\7\end{pmatrix}$ **(d)** $\begin{pmatrix}-4\\-7\end{pmatrix}$ **(e)** $\begin{pmatrix}1\\-7\end{pmatrix}$ **(f)** $\begin{pmatrix}16\\18\end{pmatrix}$

2. **(a)** $\begin{pmatrix}5\\4\\3\end{pmatrix}$ **(b)** $\begin{pmatrix}1\\-4\\-1\end{pmatrix}$ **(c)** $\begin{pmatrix}12\\12\\8\end{pmatrix}$ **(d)** $\begin{pmatrix}13\\-4\\3\end{pmatrix}$ **(e)** $\begin{pmatrix}5\\-8\\-1\end{pmatrix}$ **(f)** $\begin{pmatrix}11\\16\\9\end{pmatrix}$

3. **(a)** $\sqrt{14}$ **(b)** $\sqrt{10}$ **(c)** $\sqrt{26}$ **(d)** $\sqrt{22}$ **(e)** $\sqrt{130}$ **(f)** $\sqrt{158}$

4. **(a)** $\sqrt{13}$ **(b)** 3 **(c)** $\sqrt{27}$

5. **(a)** $\begin{pmatrix}2\\5\\3\end{pmatrix}$ **(b)** $\begin{pmatrix}-1\\3\\0\end{pmatrix}$ **(c)** $\begin{pmatrix}1\\4\\2\end{pmatrix}$ **(d)** $\begin{pmatrix}-3\\-2\\-3\end{pmatrix}$ **(e)** $\begin{pmatrix}2\\1\\2\end{pmatrix}$ **(f)** $\begin{pmatrix}-1\\-1\\-1\end{pmatrix}$

6. $A(12, 0, 0)$, $B(12, 12, 0)$, $C(0, 12, 0)$

Section B

1. $\overrightarrow{CV} = \begin{pmatrix}-5\\-5\\7\end{pmatrix}$

2. The section of the road is straight as they are collinear.

3. (a) $\overrightarrow{AK} = \begin{pmatrix}-5\\5\\11\end{pmatrix}$ (b) $\overrightarrow{AL} = \begin{pmatrix}2\\4\\9\end{pmatrix}$

4. (a) $\overrightarrow{AB} = \begin{pmatrix}6\\3\\-6\end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix}2\\1\\-2\end{pmatrix}$ (b) C (5, 0, 0) and D (7, 1, -2)

5. Q (3, 1, -2)

6. Proof [since $\overrightarrow{AC} = 3\overrightarrow{AB}$ and with point A in common then A, B and C are collinear *or equivalent*]

7. (a) $\overrightarrow{AB} = \begin{pmatrix}1\\7\\2\end{pmatrix}$ $\overrightarrow{AC} = \begin{pmatrix}4\\7\\-5\end{pmatrix}$ (b) $B\hat{A}C = 51 \cdot 9°$

8. (a) B(6, 0, 0) (b) $\overrightarrow{DA} = \begin{pmatrix}3\\-3\\-8\end{pmatrix}$ $\overrightarrow{DB} = \begin{pmatrix}3\\3\\-8\end{pmatrix}$ (c) $A\hat{D}B = 38 \cdot 7°$

Vectors

O1

1. **(a)** $\begin{pmatrix}2\\3\\1\end{pmatrix}$ **(b)** $\begin{pmatrix}4\\2\\0\end{pmatrix}$ **(c)** $\begin{pmatrix}1\\-6\\-4\end{pmatrix}$

2. **(a)** $\boldsymbol{i}+2\boldsymbol{j}+3\boldsymbol{k}$ **(b)** $6\boldsymbol{i}-2\boldsymbol{j}+7\boldsymbol{k}$ **(c)** $\boldsymbol{i}-4\boldsymbol{j}$

3. **(a)** $5\boldsymbol{i}-4\boldsymbol{j}+5\boldsymbol{k}$ **(b)** $5\boldsymbol{i}+8\boldsymbol{j}-15\boldsymbol{k}$ **(c)** $\sqrt{66}$ **(d)** $\sqrt{314}$

Vectors

O2

1. **(a)** 23 **(b)** 0 (perpendicular) **(c)** -13

2. 6

3. $\frac{1}{4}$

O3

1. $Q(4,-1,-2)$ **2.** $Q(-3,3,0)$

3. **(a)** $\overrightarrow{QR} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$, and $\overrightarrow{PQ} = \begin{pmatrix} 2 \\ 4 \\ -4 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}$ with conclusion

(b) $S(5,20,-9)$

4. $D(9,4,0)$ **5.** $X(7,7,8)$ **6.** **(a)** $T(14,7,3)$ **(b)** $\overrightarrow{QT} = \begin{pmatrix} -2 \\ 7 \\ 3 \end{pmatrix}$

O4

1. **(a)** $\overrightarrow{BT} = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$ and $\overrightarrow{BD} = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$ **(b)** $50 \cdot 9°$

2. **(a)** $A(8,0,0)$, $M(0,6,5)$, $N(8,6,4)$ **(b)** $\overrightarrow{MA} = \begin{pmatrix} 8 \\ -6 \\ -5 \end{pmatrix}$ and $\overrightarrow{MN} = \begin{pmatrix} 8 \\ 0 \\ -1 \end{pmatrix}$

(c) $40 \cdot 0°$

3. **(a)** $T(6,0,3)$, $M(0,2,1 \cdot 5)$, $N(6,2,2)$ **(b)** $\overrightarrow{TM} = \begin{pmatrix} -6 \\ 2 \\ -1 \cdot 5 \end{pmatrix}$ and $\overrightarrow{TN} = \begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$

(c) $67 \cdot 8°$

4. **(a)** $B(3,2,15)$ **(b)** $98 \cdot 5°$

O5

Vectors

1. $\boldsymbol{p} \cdot \boldsymbol{q} = 0$ therefore $\boldsymbol{p}$ and $\boldsymbol{q}$ are perpendicular.

2. $p = 4$

3. $\theta = 60°$

4. **(a)** $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c}) = \boldsymbol{0}$ **(b)** $\boldsymbol{a}$ is perpendicular to $\boldsymbol{b} + \boldsymbol{c}$

5. **(a)** $\boldsymbol{a}.(\boldsymbol{b} + \boldsymbol{c}) = 3, \boldsymbol{c}.(\boldsymbol{a} - \boldsymbol{b}) = -3$ **(b)** $|\boldsymbol{b} + \boldsymbol{c}| = 1, |\boldsymbol{a} - \boldsymbol{b}| = \sqrt{7}$.

Vectors

Cross Topic Questions

O6

Vectors and Polynomials

1. **(a)** Proof **(b)** $(k+3)(k+1)(k-1)$ **(c)** $k=1$ as $k>0$

Vectors and Quadratics

1. **(a)** $\overrightarrow{QP}=\begin{pmatrix}0\\-2\\-2\end{pmatrix}$ and $\overrightarrow{QR}=\begin{pmatrix}k-1\\-1\\-2\end{pmatrix}$ **(b)** Proof **(c)** $k=0,\ 2$