Higher Portfolio



Quadratics and Polynomials

5. Quadratics and Polynomials

Section A - Revision Section

This section will help you revise previous learning which is required in this topic

- R1 I have had experience of factorising. (common factor, difference of two squares and quadratic trinomials).
- 1. Factorise fully

(a)	$98 - 8x^2$	(b)	$5s^2 - 5t^2$	(c)	$98 - 2x^2$
(d)	$75x^2 - 243$	(e)	$72 - 18x^2$	(f)	$12x - 3x^3$
(g)	$81 - x^4$	(h)	$27w - 12w^3$	(i)	$64a^4 - 4$

2. Factorise fully

- (a) $2x^2 7x + 3$ (b) $2x^2 + 11x + 12$ (c) $3x^2 + 10x + 8$ (d) $x^2 + x - 6$ (e) $6x^2 + 7x + 2$ (f) $x^2 - 3x + 2$ (g) $5x^2 + 4x - 1$ (h) $7x^2 + 16x + 4$ (i) $2x^2 + 7x - 15$
- 3. Factorise fully
 - (a) $6-x-x^2$ (b) $20+11x-3x^2$ (c) $3+x-2x^2$ (d) $15-7x-2x^2$ (e) $4-7x-2x^2$ (f) $15-2x-x^2$

4. Factorise fully

- (a) $3x^2 + 6x 24$ (b) $15x^2y + 5x$ (c) $2x^2 32$
- (d) $5x^3 45x$ (e) $18x^2 6x 12$ (f) $12x^2y + 8xy^3$
- (g) $10x^2 + 25x 15$ (h) $6x^3 + 30x^2 + 36x$ (i) $7x^2 28$
- (j) $2x^2 10x + 12$ (k) $3x^3 + 21x^2 + 30x$ (l) $6x^3 54x$

R2 I have revised solving Quadratic Equations

- 1. Solve each of these quadratic equations
 - (a) $x^2 + 7x + 12 = 0$ (b) $x^2 4 = 0$ (c) $n^2 + 3n + 2 = 0$
 - (d) $5x^2 + 15x = 0$ (e) $p^2 + 11p + 24 = 0$ (f) $12a 3a^2 = 0$
 - (g) $s^2 + 6s + 8 = 0$ (h) $r^2 25 = 0$ (i) $n^2 + 5n + 6 = 0$

2. Solve each of these quadratic equations

(a) $x^2 - 11x + 24 = 0$ (b) $4x^2 - 9 = 0$ (c) $n^2 + 3n - 10 = 0$ (d) $5x^2 + 3x = 0$ (e) $p^2 - 10p + 24 = 0$ (f) $5a^2 - 20 = 0$ (g) $2n^2 + 7n + 3 = 0$ (h) $5r^2 + 7r + 2 = 0$ (i) $3n^2 - 4n + 1 = 0$ (j) $n^2 + 8n = -15$ (k) $5r^2 - 44r + 120 = -30 + 11r$

3. Solve these equations giving your answer to 2 significant figures.

(a) $x^2 - 3x - 1 = 0$ (b) $2x^2 + 5x + 1 = 0$ (c) $5x^2 - 7x - 2 = 0$

4. Solve these equations giving your answer to 3 significant figures.

(a) $3x^2 - 10x = -2$ (b) $2x^2 = 6x - 3$ (c) $4x^2 + x = 1$

R3 I have revised using the discriminant to find the nature of the roots of a quadratic.

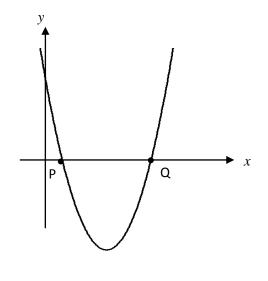
Determine the nature of the roots of each quadratic equation using the discriminant.

- (a) $x^2 + 8x + 16 = 0$ (b) $x^2 3x + 4 = 0$
- (c) $2x^2 + 3x 4 = 0$ (d) $3x^2 7x + 2 = 0$
- (e) $x^2 5x + 3 = 0$ (f) $2x^2 4x + 2 = 0$
- (g) $4 2x x^2 = 0$ (h) $3 3x + 7x^2 = 0$

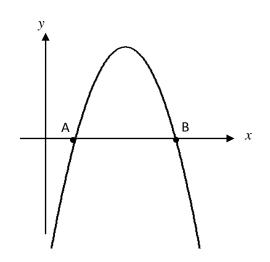
- R4 I have revised how to determine where quadratic graph cuts the axes, its turning point and its axis of symmetry.
- 1. Sketch the graph of y = (x + 3)(x 1) showing clearly where the graph cuts the axes, the axis of symmetry and the coordinates of the turning point.
- 2. Sketch the graph of $y = (x + 3)^2 + 6$ showing clearly the coordinates of the turning point and the axis of symmetry.
- 3. The graph below shows part of a parabola with equation of the form $y = (x + a)^2 + b$.

The equation of the axis of symmetry of the parabola is x = 3.

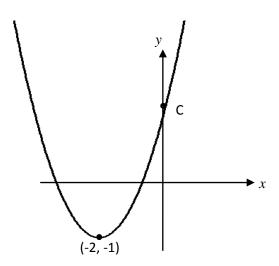
- (a) State the value of *a*.
- (b) P is the point (1, 0). State the coordinates of Q.



- 4. The graph below shows part of a parabola with equation of the form y = -(x 6)(2 x).
 - (a) State the coordinates of A and B.
 - (b) State the equation of the axis of symmetry.

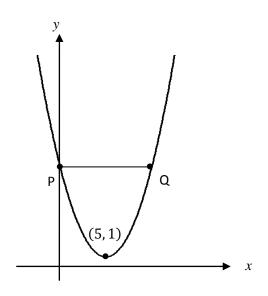


5. The graph below shows part of a parabola with equation of the form $y = (x + a)^2 + b$.



- (a) Write down the equation of the axis of symmetry.
- (b) Write down the equation of the parabola.
- (c) Find the coordinates of C.
- 6. The graph below shows part of a parabola with equation of the form $y = (x + a)^2 + b$.
 - (a) State the values of *a* and *b*.
 - (b) State the equation of the axis of symmetry.
 - (c) The line PQ is parallel to the *x*-axis.

Find the coordinates of P and Q.



R5 I have revised points of intersection of straight lines.

Find the point of intersection between the following straight lines

(1) 5x + 2y = 92x + 3y = 8(2) 3x + 5y = 225x - 2y = 16

(3)
$$5x - 3y = 12$$

 $7x - 2y = 19$
(4) $4x - 5y = -22$
 $3x + 2y = -5$

(5)
$$7x + 8y = -5$$

 $9x + 10y = -7$
(6) $6x + 5y - 9 = 0$
 $2y - 9x + 42 = 0$

(7)
$$y = 2x + 8$$

 $x - y = -5$
(8) $y = 3x - 14$
 $x - 2y = 13$

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test for Quadratics and Polynomials (Relationships and Calculus 1.1)

1. (a) (i) Show that (x - 1) is a factor of $f(x) = x^3 - 6x^2 + 11x - 6$.

(ii) Hence factorise f(x) fully.

- (b) Solve $x^3 6x^2 + 11x 6 = 0$.
- 2. (a) Show that x = 1 is a root of $x^3 + 8x^2 + 11x 20 = 0$.
 - **(b)** Hence factorise $x^3 + 8x^2 + 11x 20$ fully.
- 3. Solve the cubic equation f(x) = 0 given the following:
 - When f(x) is divided by 2x 1, the remainder is zero
 - (x + 3) is a factor of f(x)
 - the graph of y = f(x) passes through the point (2,0).
- 4. State the cubic equation f(x) given the following:
 - x = 1 is a root of f(x)
 - the graph of y = f(x) passes through the point (-4, 0)
 - when f(x) is divided by x + 5, the remainder is zero.
- 5. Find the value of k for which $2x^2 + 4x k = 0$ has equal roots.
- **6.** Find the values of k so that the graph of $y = kx^2 2x + 3$ does not cut or touch the x-axis.

Section C - Operational Skills Section

This section provides problems with the operational skills associated with Quadratics and polynomials.

O1 Given the nature of the roots of a quadratic equation, I can use the discriminant to find an unknown.

- 1. Find the value of k for which equation $2x^2 3k = 4x^2 + k^2 2k$ has equal roots. $k \neq 0$.
- 2. Find the smallest integer value of k for which

$$f(x) = (x - 2)(x^2 - 2x + k)$$
 has equal roots.

3. Find the values of *k* which ensures the following equation has equal roots

$$\frac{(x-3)^2}{x^2+3} = k.$$

4. Find two values of *p* for which the equation

 $p^2x^2 + 2(p+1)x + 4 = 0$

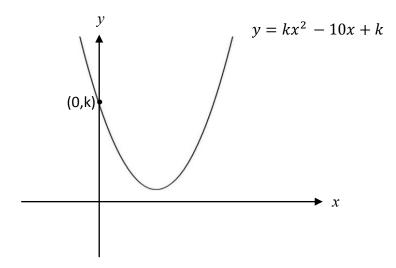
has equal roots and solve the equation for x in each case.

- 5. If the roots of the equation (x 1)(x + k) = -9 are equal, find the values of k.
- 6. Find k, if the roots of the quadratic equation

$$2x^2 + (4k+2)x + 2k^2 = 0$$

are not real.

- 7. Find the range of values of k for which $(k + 1)x^2 + 4kx + 9 = 0$ has no real roots.
- 8. Find the range of values of m for which, $2x^2 + 5mx + m = 0$, has two real and distinct roots.
- **9.** For what range of values of k does the equation $x^2 2kx + 2 = k$ have real roots.
- 10. Calculate the least positive integer value of k so that the graph of $y = kx^2 10x + k$ does not cut the x axis.



- 11. (a) Determine the nature of the roots of equation $2x^2 + 4x k = 0$ when k = 6.
 - (b) Find the value of k for which $2x^2 + 4x k = 0$ has equal roots.
- **12.** Prove that for all values of k, that the equation $x^2 2x + k^2 + 2 = 0$ has no real roots

13. Find the nature of the roots of the equation $(p-1)^2 + 3p^2 = 6p - 11$.

14. (a) Prove that the roots of the equation,

$$(9p^2 - 4qr)x^2 + 2(q + r)x - 1 = 0$$
, where $p, q, r \in Q$

are real for all values of p, q and r.

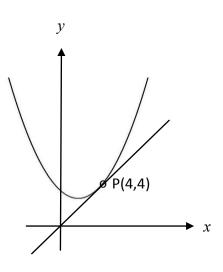
(b) Show also that if q = r the roots are rational.

O2 I can apply the condition for tangency.

- 1. The point P(4,4) lies on the parabola $y = x^2 + mx + n$
 - (a) Find a relationship between *m* and *n*.
 - (b) The tangent to the parabola at point P is the line y = x.

Find the value of m.

(c) Using your values for m and n, find the value of the discriminant of $x^2 + mx + n = 0$. What feature of the above sketch is confirmed by this value?



- 2. Show that y = 17 7x is a tangent to the parabola $y = -x^2 x + 8$ and find the point of contact.
- 3. The line y = -8x + k is a tangent to the parabola $y = 6x x^2$. Find the equation of the tangent.

- 4. (a) Show that the line y = x + 5 is a tangent to the curve with equation $y = \frac{1}{4}x^2 + 3x + 9.$
 - (b) Find the point of contact of the tangent to the curve.

O3 I can factorise a polynomial expression using the factor theorem.

- 1. Show that x = -4 is a root of $x^3 + 8x^2 + 11x 20 = 0$. Hence factorise $x^3 + 8x^2 + 11x - 20$ fully.
- 2. (a) Show that (x + 2) is a factor of $f(x) = 2x^3 + 3x^2 5x 6$.
 - (b) Hence factorise f(x) fully.
- 3. (a) Show that (x + 1) is a factor of $f(x) = 2x^3 3x^2 3x + 2$.
 - (b) Hence factorise f(x) fully.
- 4. Show that (x 2) is a factor of $f(x) = x^3 5x^2 + 2x + 8$.
 - (a) Factorise $x^3 5x^2 + 2x + 8$ fully
 - (b) Solve $x^3 + 2x = 5x^2 8$
- 5. Factorise fully
 - (a) $x^3 7x + 6$ (b) $2x^3 + 3x^2 2x 3$
 - (c) $2x^3 x^2 13x 6$ (d) $3x^3 + 8x^2 5x 6$
 - (e) $2x^4 + 6x^3 + 6x^2 + 2x$ (f) $x^5 + x^4 x 1$

O4 I can evaluate an unknown coefficient of a polynomial by applying the remainder and/or the factor theorem.

1.
$$f(x) = 2x^3 + ax^2 + bx + 4$$
.

Given that (x - 2) is a factor of f(x), and the remainder when f(x) is divided by (x - 5) is 54, find the values of *a* and *b*.

2. Find *p* if
$$(x - 4)$$
 is a factor of $x^3 - 9x^2 + px - 28$.

- 3. Given that (x + 1) is a factor of $2x^3 + 3x^2 + px 6$
 - (a) Find the value of p
 - (b) Hence or otherwise, solve $2x^3 + 3x^2 + px 6 = 0$
- 4. Find the value of k if (x+5) is a factor of $3x^4 + 15x^3 kx^2 9x + 5$
- 5. Given that (x 1) is a factor of $x^3 + x^2 (t + 1)x 4$, find the value of t.
- 6. Given that x = 3 is a root of the equation $x^4 3x^3 + px 5$, find *p*.
- 7. When $x^4 3x^3 + px 5$ is divided by (x + 3) the remainder is 16. Find the value of p.

05 I can solve polynomial equations

- 1. Solve
 - (a) $x^3 + x^2 2x = 0$ (b) $x^3 7x 6 = 0$
 - (c) $x^3 8x^2 + 9x + 18 = 0$ (d) $x^3 + x^2 9x 9 = 0$
 - (e) $x^3 + 3x^2 18x 40 = 0$ (f) $x^3 3x + 2 = 0$
- 2. The graph of $f(x) = x^2 + 6$ and the graph of $g(x) = x^3 + 3x^2 5x$ intersect at (-1, 7).

Find all the points of intersection.

3. A curve with equation $y = x^3 - x^2 - 3x + 1$ and a straight line with equation y = 2x - 4 meet at the point (1, -2).

Find the x –coordinates of the other points of contact.

O6 I can solve a polynomial equation to determine where a curve cuts the *x*-axis.

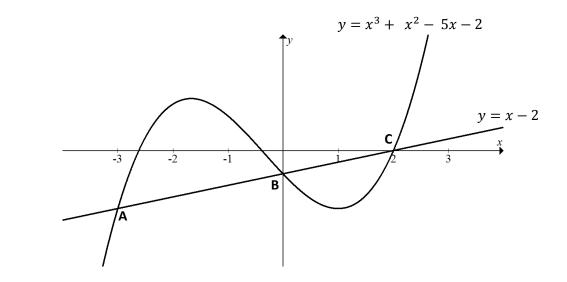
- 1. A function is defined on the set of real numbers by $f(x) = (x + 3)(x^2 + 4)$. Find where the graph of y = f(x) cuts:
 - (a) the *x* -axis;
 - (b) the y -axis.
- 2. A function is defined by the formula $g(x) = 2x^3 7x^2 17x + 10$ where x is a real number.
 - (a) Show that (x 5) is a factor of g(x), and hence factorise g(x) fully.
 - (b) Find the coordinates of the points where the curve with equation y = g(x) crosses the x and y-axes.
- **3.** A function is defined by the formula $f(x) = 5x x^3$.

Find the coordinates of the points where the graph of y = f(x) meets the x and y -axes.

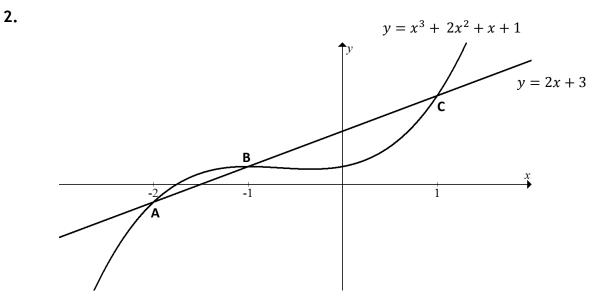
- 4. $h(x) = x^3 5x^2 + 3x + 9$
 - (a) (i) Show that (x + 1) is a factor of h(x).
 - (ii) Hence or otherwise factorise h(x) fully.
 - (b) One of the turning points of the graph of y = h(x) lies on the x-axis. Write down the coordinates of this turning point.
- 5. Find where the graph of $y = x^4 + 6x^3 12x^2 88x 96$ meets the x and y -axes.

07 I can find points of intersection by solving polynomial equations.



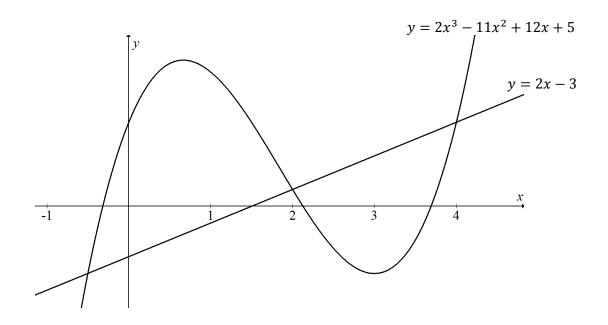


Find the coordinates of the points of intersection A, B and C where the line y = x - 2 meets the graph $y = x^3 + x^2 - 5x - 2$.

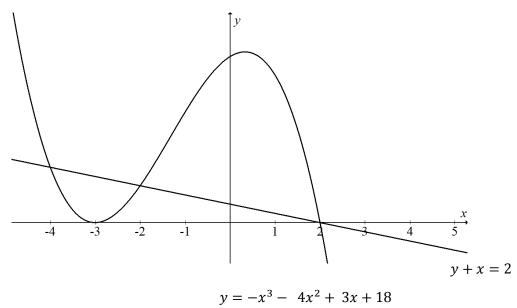


Find the coordinates of A, B and C the points of intersection between the line y = 2x + 3 and the curve $y = x^3 + 2x^2 + x + 1$.

3. (a) Fully factorise the polynomial $2x^3 - 11x^2 + 10x + 8$



- (b) Hence or otherwise find the coordinates of the points of intersection of the line y = 2x 3 and the graph $y = 2x^3 11x^2 + 12x + 5$
- 4. Find the points of intersection graph of the line y + x = 2 and the graph of $y = -x^3 4x^2 + 3x + 18$.



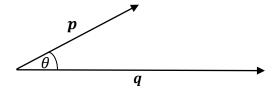
Section D - Cross Topic Exam Style Questions

Functions and Quadratics

- 1. Functions f and g are defined on the set of real numbers by
 - $f(x) = x^2 + 3$
 - g(x) = x + 4
 - (a) Find expressions for
 - (i) f(g(x))
 - (ii) g(f(x))
 - (b) Show that f(g(x)) + g(f(x)) = 0 has no real roots.

Vectors and Polynomials

2. p and q are vectors given by $p = \begin{pmatrix} k^2 \\ 3 \\ k+1 \end{pmatrix}$ and $q = \begin{pmatrix} k \\ k^2 \\ -2 \end{pmatrix}$, where k > 0.



- (a) If $p \cdot q = 1 k$, show that $k^3 + 3k^2 k 3 = 0$.
- (b) Show that (k + 3) is a factor of $k^3 + 3k^2 k 3$ and hence factorise fully.
- (c) Deduce the only possible value of k.

Vectors and Quadratics

3. P is the point
$$(1, -3, 0)$$
, Q $(1, -1, 2)$ and R $(k, -2, 0)$

(a) Express \overrightarrow{QP} and \overrightarrow{QR} in component form.

(b) Show that
$$\cos P\hat{Q}R = \frac{3}{\sqrt{2(k^2 - 2k + 6)}}$$

(c) If angle $PQR = 30^\circ$, find the possible values of k.

Answers

Section A

R1						
1.	(a)	2(7+2x)(7-2x)	(b)	5(s+t)(s-t)	(c)	2(7+x)(7-x)
	(d)	3(5x+9)(5x-9)	(e)	18(2+x)(2-x)	(f)	3x(2+x)(2-x)
	(g)	$(9+x^2)(3+x)(3-$	<i>x</i>)	(h) $3w(3+2w)(3+2$	-2w)
	(i)	$4(4a^2+1)(2a+1)($	2a —	1)		
2.	(a)	(2x-1)(x-3)	(b)	(2x+3)(x+4)	(c)	(3x+4)(x+2)
	(d)	(x+3)(x-2)	(e)	(3x+2)(2x+1)	(f)	(x-2)(x-1)
	(g)	(5x - 1)(x + 1)	(h)	(7x + 2)(x + 2)	(i)	(2x-3)(x+5)
3.	(a)	(3+x)(2-x)	(b)	(4+3x)(5-x)	(c)	(3-2x)(1+x)
	(d)	(3-2x)(5+x)	(e)	(4+x)(1-2x)	(f)	(3-x)(5+x)
4.	(a)	3(x+4)(x-2)	(b)	5x(3xy+1)	(c)	2(x+4)(x-4)
	(d)	5x(x+3)(x-3)	(e)	6(3x+2)(x-1)	(f)	$4xy(3x+2y^2)$
	(g)	5(2x-1)(x+3)	(h)	6x(x+3)(x+2)	(i)	7(x+2)(x-2)
	(j)	2(x-2)(x-3)	(k)	3x(x+2)(x+5)	(l)	6x(x+3)(x-3)
R2						
1.	(a)	x = -3, -4	(b)	x = -2, 2	(c)	n = -2, -1
	(d)	x = -3, 0	(e)	p = -3, -8	(f)	a = 0, 4
	(g)	<i>s</i> = −2, −4	(h)	r = -5, 5	(i)	n = -2, -3
2.	(a)	x = 3, 8	(b)	$x = -\frac{3}{2}, \frac{3}{2}$	(c)	n = -5, 2
	(d)	$x = -\frac{3}{5}, 0$	(e)	<i>p</i> = 4, 6	(f)	<i>a</i> = 2, −2
	(g)	$n = -3, -\frac{1}{2}$	(h)	$r = -\frac{2}{5}, -1$	(i)	$n = \frac{1}{3}, 1$
	(j)	n = -5, -3	(k)	<i>r</i> = 5, 6		
3.	(a)x	$= 3 \cdot 3, \ -0 \cdot 30$	(b)	$x = -0 \cdot 19, -2 \cdot 7$	(c)	$x = 1 \cdot 6, \ -0 \cdot 24$

4. (a)
$$x = 3 \cdot 12, 0 \cdot 214$$
 (b) $x = 2 \cdot 37, 0 \cdot 634$
(c) $x = -0.390, 0 \cdot 640$
R3
(a) 0 therefore real and equal (b) -7 therefore no real roots
(c) 41 therefore real and distinct (d) 25 therefore real and distinct
(e) 13 therefore real and distinct (f) 0 therefore real and equal
(g) 20 therefore real and distinct (h) -75 therefore no real roots
R4
1. $y = 2$
(a) $x = -2$ (b) $7 = (x + 2)^2 - 1$ (c) (0, 3)
(a) $x = -5, b = 1$ (b) $x = 5$ (c) P(0, 26), Q(10, 26)
R5
1. $x = 1, y = 2$
2. $x = 4, y = 2$
3. $x = 3, y = 1$
4. $x = -3, y = 2$
5. $x = -3, y = 2$
6. $x = 4, y = -3$
7. $x = -3, y = 2$
8. $x = 3, y = -5$

Section B

1.	(a) (i) Proof	(ii) $f(x) = (x-1)(x-2)(x-3)$		
	(b) $x = 1, x = 2$ and $x = 3$	3		
2.	(a) Proof (b) $(x - 1)(x - $	(x + 4)(x + 5)		
3.	$x = -3, x = \frac{1}{2}$ and $x = 2$			
4.	f(x) = (x - 1)(x + 4)(x + 4)	5) OR $f(x) = x^3 + 8x^2 + 11x - 20$		
5.	$k = -2$ 6. $k > \frac{1}{3}$			

Section C 01

1. k = -1

- 2. If k = 1 then there are 2 equal roots (of 1). But k = 0 also produces 2 equal roots (of 2). Therefore k = 0 is the smaller value.
- **3.** k = 0, k = 4
- 4. $p = -\frac{1}{3}$, 1 x = -6 repeated, x = -2 repeated
- **5.** k = -7, k = 5 **6.** $k < -\frac{1}{4}$ **7.** $-\frac{3}{4} < k < 3$ **8.** $m < 0, m > \frac{8}{25}$ **9.** $k \le -2, k \ge 1$ **10.** k = 6
- 11. (a) Since $b^2 4ac > 0$ (64) the roots are real and distinct. (b) k = -2
- **12.** $-4k^2 < 0$ which is true for all values of *x*.
- **13.** Since $b^2 4ac < 0$ (-8) there are no real roots.
- **14.** Since $b^2 4ac \ge 0$ for all p, q and r, the roots are always real.

1. (a) n = -12 - 4m (b) m = -7

- (c) Since $b^2 4ac < 0$ (-15), there are no real roots, curve does not cut x-axis.
- **2.** $b^2 4ac = 0$, one point of contact \therefore line is tangent to parabola. (3,-4)

3.	$k = 49. \ y = -8x + 49$					
4.	(a) $b^2 - 4ac = 0 \Rightarrow equal roots \Rightarrow only one point of contact \Rightarrow tangency$					
	(b) (-4, 1)					
03						
1.	(x+4)(x+5)(x-1)		2. $(x+2)(2$	(2x-3)(x+1)		
3.	(x + 1)(2x - 1)(x - 2))				
4.	(a) $(x-2)(x-4)(x+$	1)	(b) $x = 2, x = 4, x$	x = 2, x = 4, x = -1		
5.	(a) $(x+3)(x-1)(x-2)$ (b)		(b) $(2x+3)(x+3)$	(2x+3)(x+1)(x-1)		
	(c) $(x+2)(x-3)(2x+3)(2$	- 1)	(d) $(3x+2)(x-3)$	1)(x + 3)		
	(e) $2x(x+1)^3$		(f) $(x+1)^2 (x^2 +$	(1)(x-1)		
04						
1.	a = -10, b = 10	2.	p = 27			
3.	(a) $p = -5$	(b)	$x = -1, 1 \cdot 5, -2$			
4.	k = 2	5.	t = -3	6. $p = \frac{5}{3}$		
7.	p = 47					
05						
1.	(a) $x = -2, 0, 1$	(b)	x = -1, -2, 3	(c) $x = -1, 3, 6$		
	(d) $x = -3, -1, 3$	(e)	x = -5, -2, 4	(f) $x = -2, 1$		
2.	(-3, 15), (2, 10)	3.	$x = \pm \sqrt{5}$			
06						
1.	(a) (-3, 0)	(b)	(0, 12)			
2.	(a) proof, $(x-5)(2x-5)$	- 1)(x	+ 2) (b) (-2	, 0), $(\frac{1}{2}, 0)$, (5, 0); (0, 10)		
3.	$(0,0)(-\sqrt{5},0)(\sqrt{5},0)$					
4.	(a) (i) proof	(ii)	$(x+1)(x-3)^2$	(b) (3, 0)		
5.	(-6, 0), (-2, 0) repeate	d, (4,	0)			

~ 7

1.
$$A(-3,-5)$$
, $B(0,-2)$, $C(2,0)$
2. $A(-2,-1)$, $B(-1,1)$, $C(1,5)$
3. (a) $(x-2)(2x+1)(x-4)$
(b) $(\frac{1}{2},-4)$ (2,1) (4,5)
4. $(-4,6)$, $(-2,4)$, (2,0)
Section D
1. (a) (i) $(x+4)^2 + 3$ (ii) $x^2 + 7$
(b) Since $b^2 - 4ac < 0$ (-142) No real roots.
2. (a) Proof
(b) $(k+3)(k+1)(k-1)$ (c) $k = 1$
3. (a) $\overline{QP} = \begin{pmatrix} 0\\-2\\-2 \end{pmatrix}$, $\overline{QR} = \begin{pmatrix} k-1\\-1\\-2 \end{pmatrix}$
(b) Proof
(c) $k = 0$ and $k = 2$