7. Differentiation

Revision Section

This section will help you revise previous learning which is required in this topic.

R1 I have revised the rules of indices.

1. Simplify

(a) $x^2 \times x^5$ (b) $y^3 \times y^{-2}$ (c) $a^3 \times 5a^2$

(d) $6p^3 \times 3p^5$ (e) $5h^3 \times 2h^{-1}$ (f) $x^6 \div x^2$

(g) $\frac{a^7}{a^5}$ (h) $x^2 \div x^3$ (i) $10y^4 \div 5y^2$

2. Simplify

(a) $(x^2)^3$ (b) $(y^{-2})^4$ (c) $(z^{-2})^{-5}$

(d) $(3a^3)^2$ (e) $(2b^{-1})^5$ (f) $(5y^{-2})^3$

3. Write with positive indices

(a) y^{-5} (b) a^{-1} (c) $3x^{-4}$

(d) $\frac{1}{t^{-3}}$ (e) $\frac{5}{p^{-7}}$ (f) $\frac{2}{5p^{-7}}$

(g) $\frac{b^{-3}}{4}$ (h) $\frac{5c^{-1}}{2}$ (i) $\frac{d^{-2}}{7}$

4. Simplify

(a) $\frac{y^2 \times y^5}{y^3}$ (b) $\frac{y^3 \times y^{-2}}{y^{-6}}$ (c) $\frac{a^8}{a^2 \times a^4}$

(d) $\frac{p}{p^{-1} \times p^3}$ (e) $\frac{q^{-2} \times q^{-3}}{q^{-6}}$ (f) $\frac{5r^{-3} \times 4r^3}{2}$

(g) $\frac{f^2 \times f^{-5}}{f^{-3} \times f^4}$ (h) $\frac{s^5 \times 4s^{-5}}{2s^{-3}}$ (i) $\frac{8a^3 \times 4a^{-6}}{6a^2 \times a^{-2}}$

5. Simplify, leaving the final answer with fractional indices.

(a) $\sqrt{a}$ (b) $\sqrt[3]{b}$ (c) $\frac{1}{\sqrt[4]{c}}$

(d) $\sqrt[5]{x^3}$ (e) $\sqrt[3]{x^7}$ (f) $\frac{1}{\sqrt[4]{x^3}}$

(g) $\sqrt{x} \times \sqrt[3]{x^2}$ (h) $3m \times \sqrt[3]{m}$ (i) $\frac{\sqrt{b}}{\sqrt[3]{b}}$

(j) $\frac{4\sqrt{a^3}}{3a}$ (k) $4p \times \sqrt[3]{p^2}$ (l) $\sqrt[4]{p^3} \times \sqrt[3]{p^5}$

Differentiation

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test for Differentiation 1 (Relationships and Calculus 1.3)

1. Find $f'(x)$ given that $f(x) = \frac{3}{x^2} + 2\sqrt{x^3}$.

2. Find $f'(x)$ given that $f(x) = \frac{4}{x^2} + x\sqrt{x}$.

3. Given that $(x) = 2x^4 - 5x$, find $f'(2)$.

4. A ball is thrown vertically upwards.

After t seconds its height is h metres, where $h(t) = 1 \cdot 2 + 19 \cdot 6t + 4 \cdot 9t^2$. The velocity, $v\ ms^{-1}$, of the ball at time t is given by $v = \frac{dh}{dt}$.

Find the velocity of the ball after 1 second.

5. Differentiate the function $f(x) = 3sinx$ with respect to x.

6. Differentiate the function $f(x) = 2cosx$ with respect to x.

7. A curve has equation $y = x^3 - 3x^2 + 2x$. Find the equation of the tangent to the curve at the point where $x = 1$.

8. Find the equation of the tangent to the curve with equation $y = 3x^2 - 2x$ at the point where $x = -1$.

Differentiation

Section C – Operational Skills Section

This section provides problems with the operational skills associated with Exponentials and Logs

O1	***I can differentiate algebraic functions which can be simplified to an expression in powers of x, including terms expressed as surds.***

1. Find the derivative of the following

(a) $y = x^3$ **(b)** $y = x^5$ **(c)** $f(x) = x^4$

(d) $y = x^{12}$ **(e)** $f(x) = x^9$ **(f)** $y = x^{14}$

2. Find the derivative of the following

(a) $y = x^{-2}$ **(b)** $y = x^{-8}$ **(c)** $f(x) = x^{-4}$

(d) $y = x^{-6}$ **(e)** $f(x) = x^{-19}$ **(f)** $y = x^{-5}$

3. Find the derivative of the following

(a) $y = 2x^3$ **(b)** $y = 5x^4$ **(c)** $f(x) = 10x^7$

(d) $y = 6x^6$ **(e)** $f(x) = \frac{1}{2}x^6$ **(f)** $y = \frac{2}{3}x^9$

(g) $y = \frac{2}{5}x^{18}$ **(h)** $f(x) = \frac{2}{x^2}$ **(i)** $y = \frac{3}{x^3}$

(j) $y = \frac{25}{x^4}$ **(k)** $y = 7x^3$ **(l)** $y = \frac{3}{x^{-2}}$

(m) $f(x) = \frac{15}{x^{-3}}$ **(n)** $f(x) = 3x^{-5}$ **(o)** $y = \frac{6}{x^{-4}}$

(p) $y = \frac{14}{x^{-4}}$ **(q)** $y = 9x^4$ **(r)** $f(x) = \frac{2}{7x^{-1}}$

(s) $y = \frac{3}{3x^{-5}}$ **(t)** $f(x) = 12x^{-20}$ **(u)** $y = \frac{5}{2x^{-8}}$

4. Find the derivative of the following

(a) $y = x^{\frac{1}{2}}$ **(b)** $f(x) = x^{\frac{2}{3}}$ **(c)** $y = x^{\frac{3}{4}}$

(d) $y = x^{\frac{5}{6}}$ **(e)** $y = x^{\frac{1}{4}}$ **(f)** $f(x) = x^{\frac{2}{5}}$

(g) $f(x) = x^{\frac{1}{5}}$ **(h)** $f(x) = x^{\frac{3}{11}}$ **(i)** $y = x^{\frac{9}{13}}$

Differentiation

5. Find the derivative of the following

(a) $y = \sqrt{x}$ **(b)** $y = \sqrt{x^3}$ **(c)** $f(x) = \sqrt{x^5}$

(d) $f(x) = \sqrt[3]{x}$ **(e)** $y = \sqrt[3]{x^2}$ **(f)** $y = \sqrt[5]{x^4}$

(g) $f(x) = \frac{1}{\sqrt{x}}$ **(h)** $f(x) = \frac{1}{\sqrt[3]{x^2}}$ **(i)** $y = \frac{1}{\sqrt[3]{x^4}}$

(j) $y = \frac{1}{\sqrt[5]{x^3}}$ **(k)** $y = \frac{2}{\sqrt[3]{x^8}}$ **(l)** $f(x) = \frac{3}{\sqrt[4]{x^3}}$

(m) $y = \frac{1}{2\sqrt[3]{x^2}}$ **(n)** $y = \frac{2}{3\sqrt[4]{x^3}}$ **(o)** $f(x) = \frac{3}{5\sqrt[3]{x^7}}$

6. Find the derivative of the following

(a) $y = x^3 + 3x^2 + 5x$ **(b)** $y = 3x^5 + 2x^4 - x$ **(c)** $y = x^2 + 6x - 1$

(d) $f(x) = x^{\frac{2}{3}} + 4x^2$ **(e)** $f(x) = 3x^{\frac{1}{2}} - 2x^{-5}$ **(f)** $y = 5x^{-2} - 3x^{\frac{1}{2}}$

(g) $f(x) = \frac{1}{2\sqrt[3]{x}} + x^2$ **(h)** $y = 3x^7 - \frac{1}{5\sqrt[4]{x^3}}$ **(i)** $y = \frac{3}{5\sqrt[2]{x^5}} + 5$

(j) $y = \frac{2}{3\sqrt[4]{x^3}} + 2x^2 + x$ **(k)** $y = 5x^2 - \frac{1}{\sqrt[3]{x^2}}$ **(l)** $y = 4x^{-1} - 4x^{\frac{2}{3}}$

(m) $f(x) = 5x^3 - 6x^{-\frac{1}{2}}$ **(n)** $f(x) = 4x^2 + \frac{6}{\sqrt[3]{x}}$ **(o)** $y = x^2 - 5 - \frac{1}{x^2}$

7. Find the derivative of the following

(a) $y = (2x-3)(x+4)$ **(b)** $y = x^2(x-2)$ **(c)** $y = \frac{1}{x^2}(x^3 + 2x)$

(d) $y = \frac{1}{x}(x^2 + x)$ **(e)** $y = (\frac{1}{x} + 1)^2$ **(f)** $y = \frac{1}{\sqrt{x}}(\frac{1}{\sqrt{x}} - 1)$

8. Find the derivative of the following

(a) $y = \frac{x^2+3x+5}{x}$ **(b)** $y = \frac{2x^3+x^2+x}{x}$ **(c)** $y = \frac{x^4-6x+x^3}{x^2}$

(d) $y = \frac{x+5}{x}$ **(e)** $y = \frac{3+x^3}{x^2}$ **(f)** $y = \frac{x+2}{\sqrt{x}}$

(g) $y = \frac{(x+1)(x+2)}{x}$ **(h)** $y = \frac{(x-1)(x+3)}{x^2}$ **(i)** $y = \frac{3x^2+5x+1}{2x^2}$

Differentiation

O2 ***I am able to differentiate expressions which contain terms involving*** $\sin x$ ***and*** $\cos x$***, expressed in radians.***

1. Find the derivative of the following

(a) $y = \sin x$ **(b)** $y = \cos x$ **(c)** $f(x) = 2\sin x$

(d) $y = 5\sin x$ **(e)** $y = -2\cos x$ **(f)** $f(x) = -2\sin x$

(g) $y = -6\sin x$ **(h)** $y = \cos x + 2\sin x$ **(i)** $y = 2\sin x - \cos x$

(j) $y = 5\cos \mathrm{x} - 3\sin x$ **(k)** $y = 2\cos x + 4\sin x$ **(l)** $y = 9\sin x + \cos x$

2. Find the derivative of the following

(a) $y = \sin 2x$ **(b)** $y = \cos 2x$ **(c)** $f(x) = 2\sin 3x$

(d) $y = 2\sin 4x$ **(e)** $y = -2\cos 3x$ **(f)** $f(x) = -2\sin 2x$

(g) $y = \sin(2x + 1)$ **(h)** $y = \cos(3x - 2)$ **(i)** $f(x) = 2\sin 2x$

(j) $y = sin^2 x$ **(k)** $y = cos^3 x$ **(l)** $f(x) = \frac{1}{sin^2 x}$

(m) $f(x) = \frac{1}{cos^2 x}$ **(n)** $f(x) = \frac{1}{sin^3 x}$ **(o)** $f(x) = \frac{1}{2sin^3 x}$

(p) $y = \sin x^2$ **(q)** $y = \cos x^2$ **(r)** $y = 2\sin x^3$

O3 ***I can differentiate a composite function using the chain rule.***

1. Find the derivative of the following

(a) $y = (x + 5)^2$ **(b)** $y = (x - 8)^5$ **(c)** $y = (x + 2)^3$

(d) $y = (2x - 3)^2$ **(e)** $y = (3x - 1)^5$ **(f)** $y = (4x + 7)^4$

2. Find the derivative of the following

(a) $y = (x + 5)^{-2}$ **(b)** $y = (3x - 5)^{-3}$ **(c)** $y = (x + 2)^{-8}$

(d) $y = \frac{1}{(2x-2)^2}$ **(e)** $y = \frac{1}{(3x+3)^5}$ **(f)** $y = \frac{1}{(6x-1)^4}$

(g) $y = \sqrt{(x + 3)}$ **(h)** $y = \sqrt{(x - 2)}$ **(i)** $y = \sqrt{(x + 2)^3}$

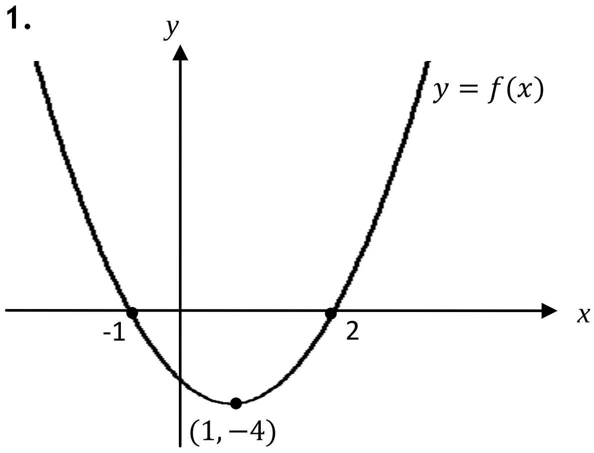
(j) $y = \frac{1}{\sqrt{(x+2)}}$ **(k)** $y = \frac{1}{\sqrt{(x-10)}}$ **(l)** $y = \frac{2}{\sqrt{(x+2)^5}}$

Differentiation

O4 ***I can sketch graphs of derivatives.***

For each graph of $y = f(x)$ sketch $y = f'(x)$.

1.



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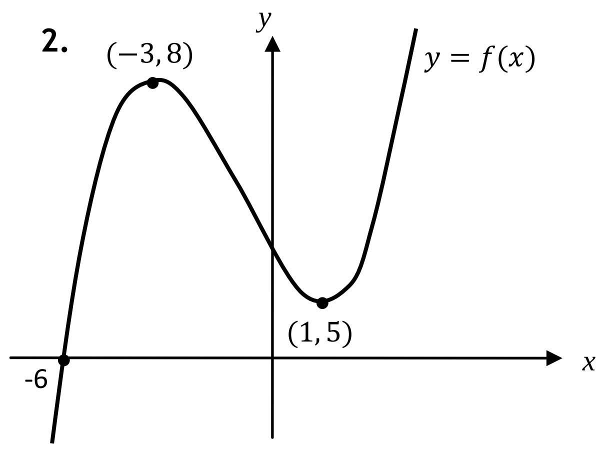

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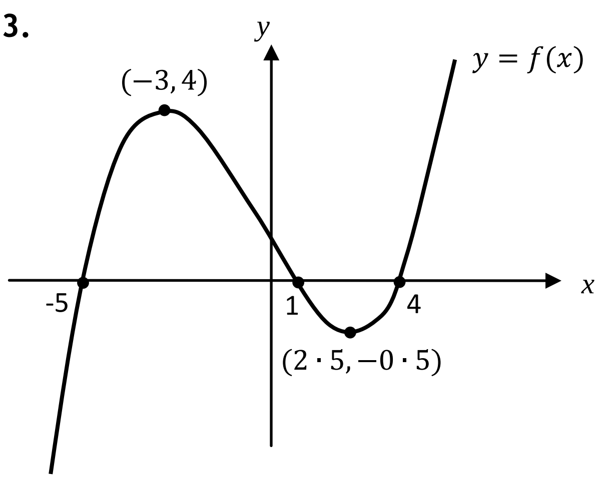

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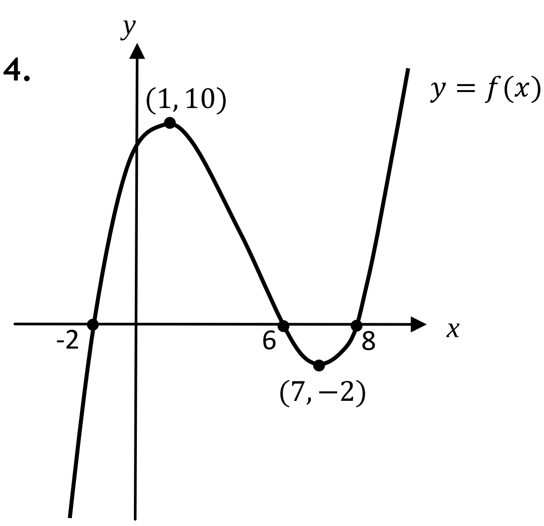

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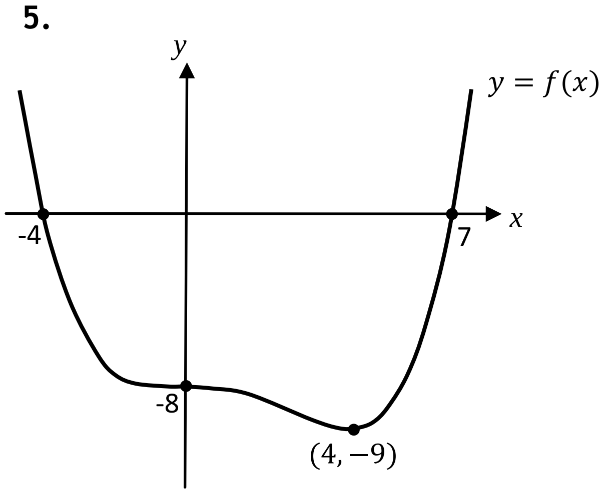

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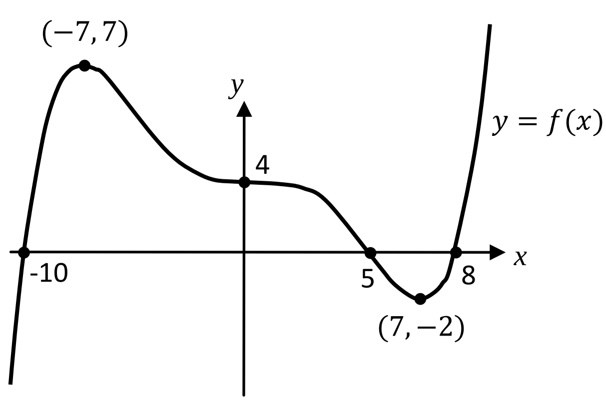

Differentiation

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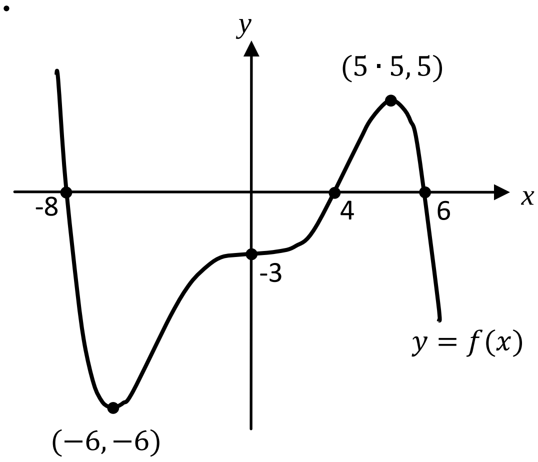

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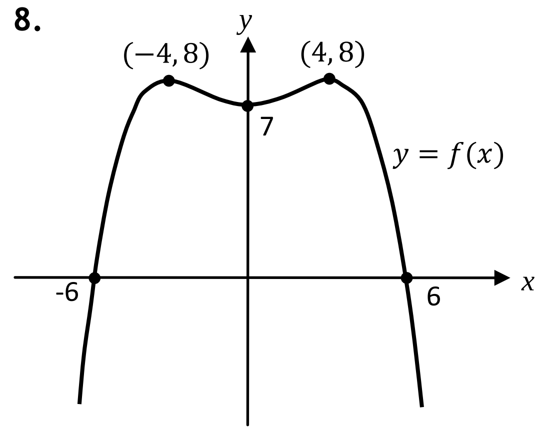


O5	***I can determine the equation of a tangent to a curve, at a given point by differentiation.***

1. Find the equation of the tangent to the curve at the given point

(a) $y = x^2$ at $(1, 4)$

(b) $y = x^2 + 2x$ at $(0, 2)$

(c) $y = x^3 + 3$ at $(2, 8)$

(d) $y = x^2 - 6x + 5$ at $x = 2$

(e) $y = \frac{x^2 + 5x + 10}{x^2}$ at $x = 1$

(f) $y = \frac{1}{\sqrt{(x+1)}}$ at $x = 3$

2. A curve has equation $y = 3x^2 - 4x$. At the point T the tangent to this curve has a gradient of 2, find the coordinates of T and hence the equation of the tangent.

3. A curve has equation $f(x) = 2x^2 + 8x - 3$. A tangent to this curve has a gradient of -4, find the equation of this tangent.

4. A tangent to the equation $y = \frac{2}{\sqrt{x}}$ has a gradient of -1, find the equation of this tangent.

5. A curve has equation $y = x^3 - 6x$. There are two tangents with a gradient 6. Find the equation of both tangents

Differentiation

O6 ***I can determine use the stationary points of a curve and state their nature.***

Find the coordinates of the Stationary points and determine their nature

1. $y = x^3 - 6x^2 + 9x$

2. $y = x^3 - 3x^2 - 9x + 12$

3. $y = x^4 - 4x^3$

4. $y = x^3 - 3x + 2$

5. $y = x^3 - 12x + 2$

6. $y = 2x^3 - 7x^2 + 4x + 1$

7. $y = 3x^4 + 16x^3$

8. $y = 8x^3 - x^2 + 11$

9. $y = \frac{x^3 + 4x + 16}{x}$

10. $y = (x - 2)(x^2 + 1)$

O7 ***I can sketch the graph of an algebraic function by determining stationary points and where it cuts the axes.***

Sketch the graph of the following functions, stating clearly where it cuts the x & y axis

1. $y = x^2 - 6x$

2. $y = x^2 + 5x + 6$

3. $y = x^2 - 5x$

4. $y = 2x^3 - 3x^2 - 27x$

5. $y = (x - 1)^2(x + 2)$

6. $y = 12 - 4x - x^2$

7. $y = x^3 - 3x^2$

8. $y = x^2(2x - 1)$

O8 ***The derivative as a rate of change***

1. If $s(t) = t^2 - 5t + 8$, what is the rate of change of s with respect to t when $t = 3$?

2. The volume of sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

What is the rate of change of V with respect to r, at $r = 2$?

3. Acceleration is defined as the rate of change of velocity.

An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos\left(2t - \frac{\pi}{2}\right)$.

(a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion.

(b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$.

4. The depth of water in a harbour, d metres, is given by the formula $d = 10\sin\left(\frac{\pi}{12}t\right)$ where t is the number of hours after midnight.

What would be the rate at which the depth of water is increasing 3 hours after midnight?

O9 ***I can determine where a function is strictly increasing/decreasing.***

1. State whether the function is increasing or decreasing

(a) $y = x^2 - 4x$ at $x = 3$

(b) $y = x^3 - 3x + 2$ at $x = 0$

(c) $y = x^2 - 10x + 4$ at $x = -2$

(d) $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 + 2$ at $x = 4$

(e) $y = 4x^2 + 5x + 7$ at $x = 1$

(f) $y = 2x^4 - 4x^2 + 12$ at $x = -2$

2. For each function state the intervals in which it is increasing AND decreasing

(a) $y = x^2 - 5x + 12$

(b) $y = 2x^2 + x + 3$

(c) $y = 8 + 2x - x^2$

(d) $y = x^3$

(e) $y = 3x - x^3$

(f) $y = x^2(3 - 2x)$

3. Show that the function $y = x^3 - x^2 + x$ is never decreasing

4. Show that the function $y = 2x^5 + 5$ is never decreasing

5. Show that the function $y = 12x^2 - 6x - 8x^3$ is never increasing

6. Show that the function $y = -x^3 - 3x^2 - 3x$ is never increasing

Differentiation

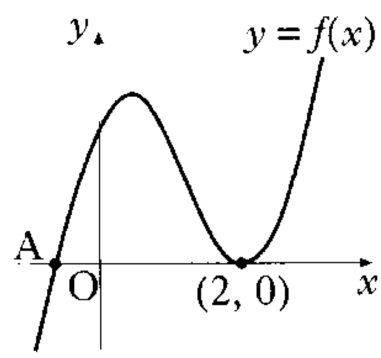
Section D - Cross Topic Questions

Differentiation and Polynomials

1. **(a)** Given that (x – 1) is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic

(b) Show that the curve with equation $y = x^4 + 4x^3 + 2x^2 - 20x + 3$ has only 1 Stationary point. Find the x coordinate and determine the nature of this point.

2. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(a) Find the x-coordinate of the maximum turning point.

(b) Factorise $2x^3 - 7x^2 + 4x + 4$.

(c) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$.

3. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(a) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully.

(b) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes.

(c) Find the greatest and least values of f in the interval $-2 \leq x \leq 2$.

Differentiation and Quadratics

4. **(a)** Write $x^2 - 10x + 27$ in the form $(x + b)^2 + c$.

(b) Hence show that the function $g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$ is always increasing.

Differentiation and Functions

5. Functions f and g are given by $\mathrm{f(x) = 3x + 1}$ and $\mathrm{g(x) = x^2 - 2}$.

(a) Find $p(x)$ where $p(x) = f(g(x))$.

(b) Find $q(x)$ where $q(x) = g(f(x))$.

(c) Solve $p'(x) = q'(x)$.

Differentiation

Answers

R1

Q1	(a) x^7	(b) y	(c) $5a^2$	(d) $18p^8$	
	(e) $10h^2$	(f) x^4	(g) a^2	(h) x^{-1}	(i) $2y^2$
Q2	(a) x^6	(b) $\frac{1}{y^8}$	(c) z^{10}	(d) $9a^6$	
	(e) $\frac{32}{b^5}$	(f) $\frac{125}{y^6}$			
Q3	(a) $\frac{1}{y^5}$	(b) $\frac{1}{a}$	(c) $\frac{3}{x^4}$	(d) t^3	
	(e) $5p^7$	(f) $\frac{2p^7}{5}$	(g) $\frac{1}{4b^3}$	(h) $\frac{5}{2c}$	(i) $\frac{1}{7d^2}$
Q4	(a) y^4	(b) y^7	(c) a^2	(d) $\frac{1}{p}$	
	(e) q	(f) 10	(g) $\frac{1}{f^4}$	(h) $2s^3$	(i) $\frac{16}{3a^3}$
Q5	(a) $a^{\frac{1}{2}}$	(b) $b^{\frac{1}{3}}$	(c) $\frac{1}{c^{\frac{1}{4}}}$ or $c^{-\frac{1}{4}}$	(d) $x^{\frac{3}{5}}$	
	(e) $x^{\frac{7}{3}}$	(f) $\frac{1}{x^{\frac{3}{4}}}$ or $x^{-\frac{3}{4}}$	(g) $x^{\frac{7}{6}}$	(h) $3m^{\frac{4}{3}}$	(i) $b^{\frac{1}{6}}$
	(j) $\frac{4a^{\frac{1}{2}}}{3}$	(k) $4p^{\frac{5}{3}}$	(l) $p^{\frac{29}{12}}$		

Section B

1.	$f(x)' = \frac{-6}{x^3} + 3\sqrt{x}$	2.	$f(x)' = \frac{-8}{x^3} + \frac{3\sqrt{x}}{2}$	3.	$f'(2) = 49$
4.	$v = 29 \cdot 4\ ms^{-1}$	5.	$f'(x) = 3cosx$		
6.	$f'(x) = -2sinx$	7.	$x + y = 1$	8.	$8x + y = -3$

Section C

O1

1.	**(a)** $3x^2$	**(b)** $5x^4$	**(c)** $4x^3$	**(d)** $12x^{11}$	**(e)** $9x^8$	**(f)** $14x^3$
2.	**(a)** $-2x^{-3}$	**(b)** $-8x^{-9}$	**(c)** $-4x^{-5}$	**(d)** $-6x^{-7}$	**(e)** $-19x^{-20}$	**(f)** $-5x^{-6}$
3.	**(a)** $6x^2$	**(b)** $20x^3$	**(c)** $70x^6$	**(d)** $36x^5$	**(e)** $3x^5$	**(f)** $6x^8$
	(g) $\frac{36}{5}x^{17}$	**(h)** $-\frac{4}{x^3}$	**(i)** $-\frac{9}{x^4}$	**(j)** $-\frac{100}{x^5}$	**(k)** $21x^2$	**(l)** $6x$
	(m) $45x^2$	**(n)** $-\frac{15}{x^6}$	**(o)** $24x^3$	**(p)** $56x^3$	**(q)** $36x^3$	**(r)** $\frac{2}{7}$
	(s) $5x^4$	**(t)** $-\frac{240}{x^{21}}$	**(u)** $20x^7$			

Differentiation

4. (a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{2}{3\sqrt[3]{x}}$ (c) $\frac{3}{4\sqrt[4]{x}}$ (d) $\frac{5}{6\sqrt[6]{x}}$ (e) $\frac{1}{4\sqrt[4]{x^3}}$ (f) $\frac{2}{5\sqrt[5]{x^3}}$

(g) $\frac{1}{5\sqrt[5]{x^4}}$ (h) $\frac{3}{11\sqrt[11]{x^8}}$ (i) $\frac{9}{13\sqrt[13]{x^4}}$

5. (a) $\frac{1}{2\sqrt{x}}$ (b) $\frac{3\sqrt{x}}{2}$ (c) $\frac{5\sqrt{x^3}}{2}$ (d) $\frac{1}{3\sqrt[3]{x^2}}$ (e) $\frac{2}{3\sqrt[3]{x}}$ (f) $\frac{4}{5\sqrt[5]{x}}$

(g) $-\frac{1}{2\sqrt{x^3}}$ (h) $-\frac{2}{3\sqrt[3]{x^5}}$ (i) $-\frac{4}{3\sqrt[3]{x^7}}$ (j) $-\frac{3}{5\sqrt[5]{x^8}}$ (k) $-\frac{16}{3\sqrt[3]{x^{11}}}$ (l) $-\frac{9}{4\sqrt[4]{x^7}}$

(m) $-\frac{1}{3\sqrt[3]{x^5}}$ (n) $-\frac{1}{2\sqrt[4]{x^7}}$ (o) $-\frac{7}{5\sqrt[3]{x^{10}}}$

6. (a) $3x^2 + 6x + 5$ (b) $15x^4 + 8x^3 - 1$ (c) $2x + 6$

(d) $\frac{2}{3\sqrt[3]{x}} + 8x$ (e) $\frac{3}{2\sqrt{x}} + \frac{10}{x^6}$ (f) $-\frac{10}{x^3} - \frac{3}{2\sqrt{x}}$

(g) $-\frac{1}{6\sqrt[3]{x^4}} + 2x$ (h) $21x^6 + \frac{3}{20\sqrt[4]{x^7}}$ (i) $-\frac{3}{2\sqrt{x^7}}$

(j) $\frac{1}{2\sqrt[4]{x^7}} + 4x + 1$ (k) $10x + \frac{2}{3\sqrt[3]{x^5}}$ (l) $-\frac{4}{x^2} - \frac{8}{3\sqrt[3]{x}}$

(m) $15x^2 + \frac{3}{\sqrt{x^3}}$ (n) $8x - \frac{2}{\sqrt[3]{x^4}}$ (o) $2x + \frac{2}{x^3}$

7. (a) $4x + 5$ (b) $3x^2 - 4x$ (c) $1 - \frac{2}{x^2}$

(d) 1 (e) $-\frac{2}{x^3} - \frac{2}{x^2}$ (f) $-\frac{1}{x^2} - \frac{1}{2\sqrt{x^3}}$

8. (a) $1 - \frac{5}{x^2}$ (b) $4x + 1$ (c) $2x + \frac{6}{x^2} + 1$

(d) $-\frac{5}{x^2}$ (e) $-\frac{6}{x^3} + 1$ (f) $\frac{1}{2\sqrt{x}} - \frac{1}{\sqrt{x^3}}$

(g) $1 - \frac{2}{x^2}$ (h) $-\frac{2}{x^2} + \frac{6}{x^3}$ (i) $-\frac{5}{2x^2} - \frac{1}{x^3}$

O2

1. (a) $\cos x$ (b) $-\sin x$ (c) $2\cos x$ (d) $5\cos x$ (e) $2\sin x$ (f) $-2\cos x$

(g) $-6\cos x$ (h) $-\sin x + 2\cos x$ (i) $2\cos x + \sin x$

(j) $-5\sin x - 3\cos x$ (k) $-2\sin x + 4\cos x$ (l) $9\cos x - \sin x$

2. (a) $2\cos 2x$ (b) $-2\sin 2x$ (c) $6\cos 3x$

(d) $8\cos 4x$ (e) $6\sin 3x$ (f) $-4\cos 2x$

(g) $2\cos(2x + 1)$ (h) $-3\sin(3x - 2)$ (i) $4\cos 2x$

(j) $2\cos x \sin x$ (k) $-3\cos^2 x \sin x$ (l) $\frac{-2\cos x}{\sin^3 x}$

(m) $\frac{2\sin x}{\cos^3 x}$ (n) $\frac{-3\cos x}{\sin^4 x}$ (o) $\frac{-3\cos x}{2\sin^4 x}$

(p) $2x\cos x^2$ (q) $-2x\sin x^2$ (r) $6x^2\cos x^3$

Differentiation

O3

1. **(a)** $2(x+5)$ **(b)** $5(x-8)^4$ **(c)** $3(x+2)^2$

(d) $4(2x-3)$ **(e)** $15(3x-1)^4$ **(f)** $16(4x+7)^3$

2. **(a)** $-2(x+5)^{-3}$ **(b)** $-9(3x-5)^{-4}$ **(c)** $-8(x+2)^{-9}$

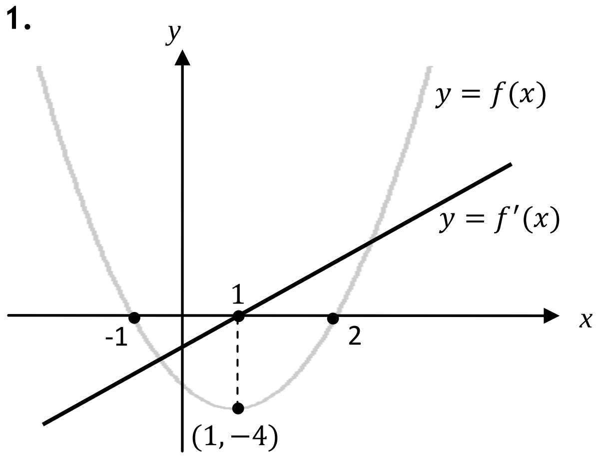
(d) $-\frac{4}{(2x-2)^3}$ **(e)** $-\frac{15}{(3x+3)^6}$ **(f)** $-\frac{24}{(6x-1)^5}$

(g) $\frac{1}{2\sqrt{x+3}}$ **(h)** $\frac{1}{2\sqrt{x-2}}$ **(i)** $\frac{3}{2\sqrt{x+2}}$

(j) $-\frac{1}{2\sqrt{(x+2)^3}}$ **(k)** $-\frac{1}{2\sqrt{(x-10)^3}}$ **(l)** $-\frac{5}{\sqrt{(x+2)^7}}$

O4

1.



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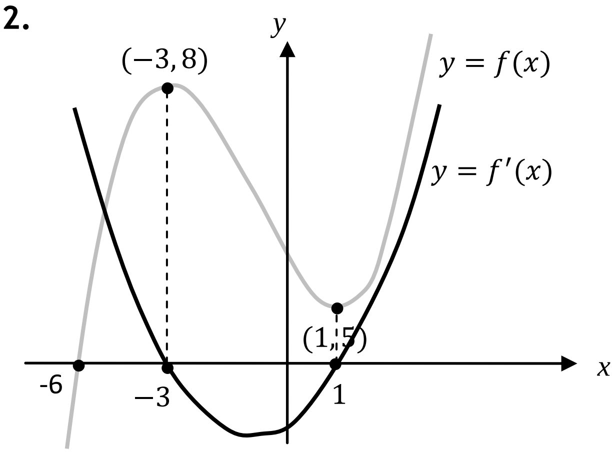

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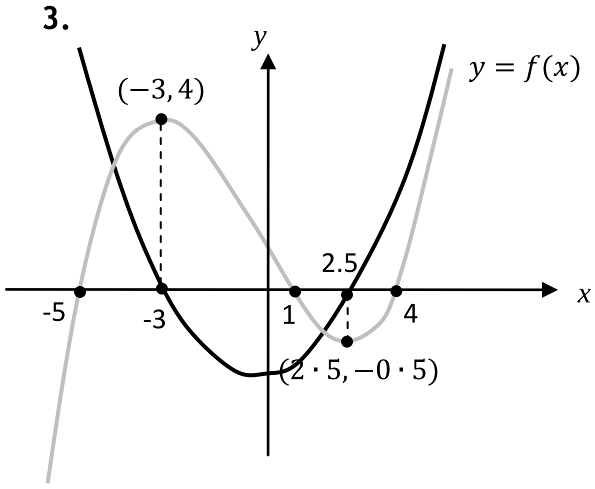

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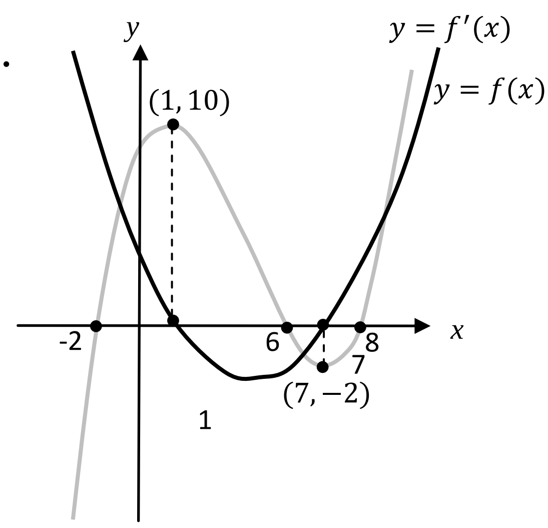

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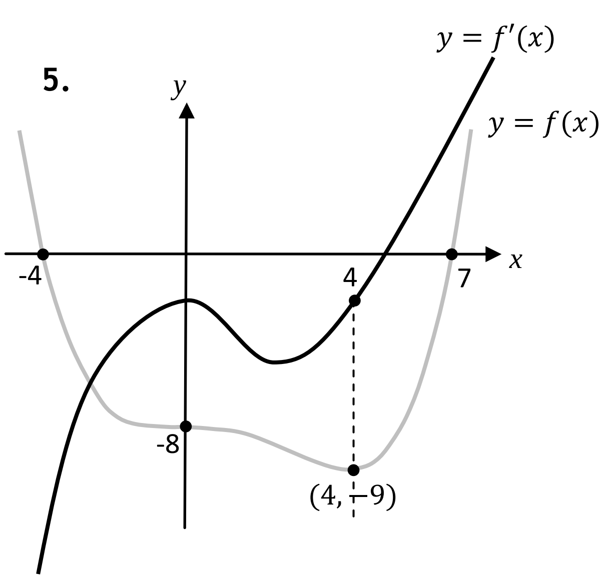

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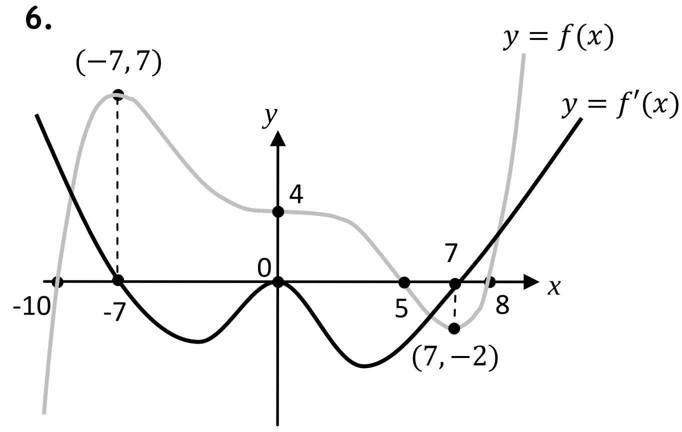

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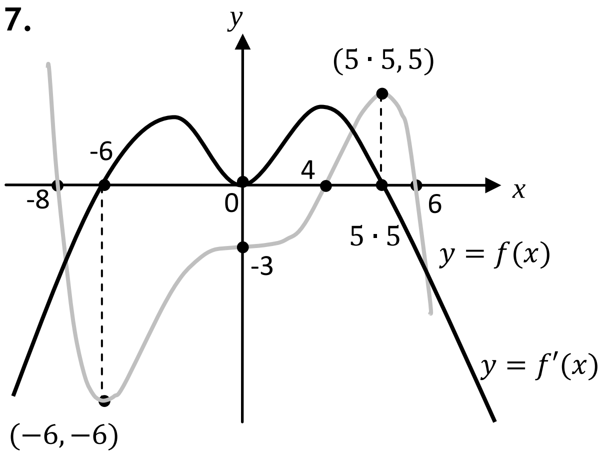

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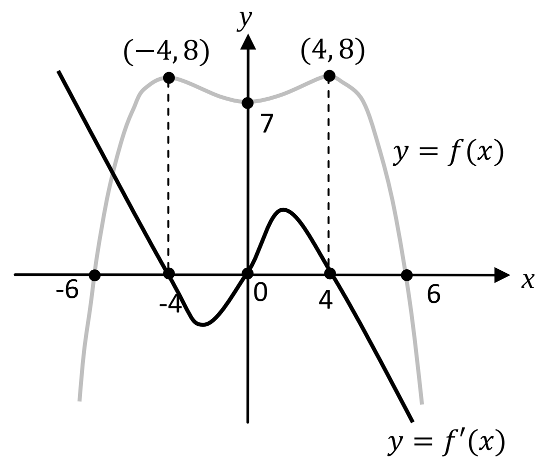

O5

1. **(a)** $2x - y + 2 = 0$ **(b)** $2x - y + 2 = 0$ **(c)** $12x - y - 16 = 0$

(d) $2x + y - 1 = 0$ **(e)** $25x + y - 41 = 0$ **(f)** $x + 16y - 5 = 0$

2. $y = 2x - 3$ **3.** $y = -4x - 21$ **4.** $y = -x + 3$

5. $y = 6x - 16$ and $y = 6x + 16$

O6

1. max t.pt @ (1, 4) and min t.pt @ (3, 0)

2. max t.pt @ (-1, 17) and min t.pt @ (3, -15)

3. falling pt of inflection @ (0, 0) and min t.pt @ (3, -27)

4. max t.pt @ (-1, 4) and min t.pt @ (1, 0)

5. max t.pt @ (-2, 18) and min t.pt @ (2, -14)

6. max t.pt @ $(\frac{1}{3}, \frac{44}{27})$ and min t.pt @ (2, -3)

7. min t.pt @ (-4, -256) and rising pt of inflection @ (0, 0)

8. max t.pt @ (0, 11) and min t.pt @ $(\frac{1}{12}, \frac{4751}{432})$

9. min t.pt @ (2, 16)

10. max t.pt @ $(\frac{1}{3}, -\frac{50}{27})$ and min t.pt @ (1, -2)

O7

1. Roots @ (0, 0) and (6, 0), min t.pt @ (3, -9)

2. Roots @ (-3, 0) and (-2, 0), y-intercept @ (0, 6), min t.pt @ $(-\frac{5}{2}, -\frac{1}{4})$

3. Roots @ (0, 0) and (5, 0), min t.pt @ $(\frac{5}{2}, -\frac{25}{4})$

4. Roots @ (-3, 0), (0, 0), $(\frac{9}{2}, 0)$, min t.pt @ $(2 \cdot 7, -55 \cdot 4)$,
max t.pt @ (-1·7, 27·4).

5. Roots @ (-2, 0) and (1, 0), max t.pt @ (-1, 4), y-intercept @ (0, 2) and min t.pt @ (1, 0)

6. Roots @ (-6, 0) and (2, 0), y-intercept @ (0, 12), max t.pt @ (-2, 16)

7. Roots @ (0, 0) and (3, 0), max t.pt @ (0, 0), y-intercept @ (0, 0) and min t.pt @ (2, -4)

8. Roots @ (0, 0) and ($\frac{1}{2}$, 0), max t.pt @ (0, 0), y-intercept @ (0, 0) and min t.pt @ ($\frac{1}{3}$, $-\frac{1}{27}$)

O8

1. $S'(3) = 1$ **2.** $V'(2) = 16\pi$

3.(a) $a(t) = -16\sin\left(2t - \frac{\pi}{4}\right)$ **(b)** $v'(t) = a(t) = 6 \cdot 5 > 0$ $\therefore$ velocity increasing

4. $d'(t) = \frac{5\pi}{6}\cos\frac{\pi}{12}t$,rate 3 hours after midnight $= d'(3) = 1 \cdot 85 = \frac{5\pi\sqrt{2}}{12}$

O9

1. (a) Increasing (b) Decreasing

(c) Decreasing (d) Increasing

(e) Increasing (f) Decreasing

2. (a) Decreasing $x < \frac{5}{2}$ and Increasing $x > \frac{5}{2}$

(b) Decreasing $x < -\frac{1}{4}$ and Increasing $x > -\frac{1}{4}$

(c) Increasing $x < 1$ and Decreasing $x > 1$

(d) Decreasing $x < 0$ and Increasing $x > 0$

(e) Decreasing $x < -1$ and $x > 1$; Increasing $-1 < x < 1$

(f) Decreasing $x < 0$ and $x > 1$; Increasing $0 < x < 1$

3. $\frac{dy}{dx} = 3\left(x - \frac{1}{3}\right)^2 + \frac{2}{3} > 0$ for all x $\therefore$ never decreasing

4. $\frac{dy}{dx} = 10x^4 \geq 0$ for all x $\therefore$ never decreasing

5. $\frac{dy}{dx} = -6(2x-1)^2 \leq 0$ for all x $\therefore$ never increasing

6. $\frac{dy}{dx} = -3(x+1)^2 \leq 0$ for all x $\therefore$ never increasing

Section D

1. (a) $(x-1)(x^2+4x+5)$

(b) $\frac{dy}{dx} = 4(x-1)(x^2+4x+5) = 0$ has only one s.pt. when $x = 1$

since $x^2 + 4x + 5 = 0$ yields no real roots [$b^2 - 4ac < 0$ (-4)].

Min t.p. @ (1, -10)

2. (a) $x = \frac{1}{2}$ (b) $(2x+1)(x-2)^2$ (c) $A(-\frac{1}{2}, 0)$; $x < -\frac{1}{2}$

Differentiation

3. **(a)** proof ; $f(x) = (x-3)(2x-3)(x+1)$

(b) Roots @ (3, 0), ($\frac{3}{2}$, 0), (-1, 0)   y-intercept @ (0, 9)

(c) Max pt @ (0, 9), min tp @ $(\frac{7}{3}, -\frac{100}{27})$

End points $f(-2) = -35$ and $f(2) = -3$

Therefore least value of f in the interval is -35 when $x = -2$ and greatest value is 9 when $x = 0$.

4. **(a)** $(x-5)^2 + 2$

(b) $g'(x) = (x-5)^2 + 2$ which is ≥ 2 for all x. Since $g'(x) > 0$ for all x, $g(x)$ is always increasing.

5. **(a)** $p(x) = 3x^2 - 5$

(b) $q(x) = 9x^2 + 6x - 1$

(c) $p'(x) = 6x$

$q'(x) = 18x + 6$

$p'(x) = q'(x)$ when $x = -\frac{1}{2}$