Higher Portfolio

Integration

8. Integration

Section A - Revision Section - There is no revision section for this topic.

Section B - Assessment Standard Section

This section will help you practise for your Assessment Standard Test for Integration 1 (Relationships and Calculus 1.3)

1. Find $\int \frac{1}{3x^4} dx$, where $x \neq 0$.

2. Find $\int (4\sqrt{x} + \frac{1}{x^3}) \, dx$, where $x > 0$.

3. $f'(x) = 6x^2 - 2x$, find $f(x)$.

4. $g'(x) = (2x + 3)^{-2}$, find $g(x)$, $x \neq \frac{-3}{2}$.

5. Find $\int 5sin\theta \, d\theta$.

6. Find $\int 8cos\theta \, d\theta$.

7. Find the value of $\int_1^2 (3x^2 + 4)dx$.

8. Find the value of $\int_1^2 (4 - x^2)dx$.

Integration

Section C – Operational Skills Section

This section provides problems with the operational skills associated with Exponentials and Logs.

O1	*I can integrate an algebraic function*

1. **Carry out the following integrations**

(a) $\int \sqrt{x}\ dx$

(b) $\int \sqrt{x^3}\ dx$

(c) $\int \sqrt{x^5}\,dx$

(d) $\int \sqrt[3]{x}\ dx$

(e) $\int \sqrt[3]{x^2}\ dx$

(f) $\int \sqrt[5]{x^4}\ dx$

(g) $\int \frac{1}{\sqrt{x}}dx$

(h) $\int \frac{1}{\sqrt[3]{x^2}}dx$

(i) $\int \frac{1}{\sqrt[3]{x^4}}dx$

(j) $\int \frac{1}{\sqrt[5]{x^3}}dx$

(k) $\int \frac{2}{\sqrt[3]{x^8}}dx$

(l) $\int \frac{3}{\sqrt[4]{x^3}}dx$

(m) $\int \frac{1}{2\sqrt[3]{x^2}}dx$

(n) $\int \frac{2}{3\sqrt[4]{x^3}}dx$

(o) $\int \frac{3}{5\sqrt[3]{x^7}}dx$

2. **Carry out the following integrations**

(a) $\int x^3 + 3x^2 + 5x\ dx$

(b) $\int 3x^5 + 2x^4 - x\ dx$

(c) $\int x^2 + 6x - 1\ dx$

(d) $\int x^{\frac{2}{3}} + 4x^2\ dx$

(e) $\int 3x^{\frac{1}{2}} - 2x^{-5}\ dx$

(f) $\int 5x^{-2} - 3x^{\frac{1}{2}}\ dx$

(g) $\int \frac{1}{2\sqrt[3]{x}} + x^2\ dx$

(h) $\int 3x^7 - \frac{1}{5\sqrt[4]{x^3}}dx$

(i) $\int \frac{3}{5\sqrt[2]{x^5}} + 5\ dx$

(j) $\int \frac{2}{3\sqrt[4]{x^3}} + 2x^2 + x\ dx$

(k) $\int 5x^2 - \frac{1}{\sqrt[3]{x^2}}dx$

(l) $\int 4x^{-2} - 4x^{\frac{2}{3}}\ dx$

(m) $\int 5x^3 - 6x^{-\frac{1}{2}}\ dx$

(n) $\int 4x^2 + \frac{6}{\sqrt[3]{x}}dx$

(o) $\int x^2 - 5 - \frac{1}{x^2}dx$

Integration

1. Find

(a) $\int (x + 2)^8 \ dx$

(b) $\int (2x + 4)^3 \ dx$

(c) $\int (5x + 7)^4 \ dx$

(d) $\int (2x - 1)^5 \ dx$

(e) $\int 6(5 - 4x)^6 \ dx$

(f) $\int (10 - x)^{-10} \ dx$

(g) $\int 3(4x + 1)^{-3} \ dx$

(h) $\int 2(5x - 9)^{-5} \ dx$

(i) $\int (3 - 7x)^{-4} \ dx$

(j) $\int (x - 1)^{\frac{1}{2}} \ dx$

(k) $\int (2x - 1)^{\frac{1}{3}} \ dx$

(l) $\int (2x - 1)^{\frac{1}{4}} \ dx$

(m) $\int (2x - 2)^{\frac{1}{2}} \ dx$

(n) $\int (3x + 4)^{\frac{2}{3}} \ dx$

(o) $\int (7 + 3x)^{\frac{3}{4}} \ dx$

2. Find

(a) $\int \frac{1}{(5x+3)^5} \ dx$

(b) $\int \frac{dx}{(3x-2)^4}$

(c) $\int \frac{3}{(4-2x)^6} \ dx$

(d) $\int \frac{2 \ dx}{(x-2)^3}$

(e) $\int \frac{3 \ dx}{(4x+2)^4}$

(f) $\int \frac{1}{(5x-2)^{\frac{1}{2}}} \ dx$

3. Find

(a) $\int \sqrt{4x + 2} \ dx$

(b) $\int 6\sqrt{3x + 1} \ dx$

(c) $\int \sqrt{9 - 5x} \ dx$

(d) $\int \sqrt[3]{2x - 3} \ dx$

(e) $\int \sqrt[3]{6x - 2} \ dx$

(f) $\int \sqrt[4]{2x + 4} \ dx$

(g) $\int \frac{1}{\sqrt{(3x-4)}} \ dx$

(h) $\int \frac{dx}{\sqrt{(x+8)}}$

(i) $\int \frac{2dx}{\sqrt{(2x-5)}}$

Find

1. $\int \sin^2 x \ dx$

2. $\int \cos^2 x \ dx$

3. $\int 2\sin^2 x \ dx$

4. $\int 2\cos^2 x \ dx$

Integration

04   I can evaluate the definite integral of a function.

1. Find

(a) $\int_0^1 (x^2 - 3x + 4)\ dx$

(b) $\int_0^1 (4x^2 + 3x)\ dx$

(c) $\int_0^1 (x^3 + 2x^2 - 1)\ dx$

(d) $\int_0^2 (2x - 1)(x + 2)\ dx$

(e) $\int_{-1}^1 2x^2\,(2x + 1)\ dx$

(f) $\int_{-2}^1 (2x^3 - x^2 + 3x)\ dx$

2. Find

(a) $\int_{-1}^1 (5x^3 - 2x)\ dx$

(b) $\int_{-1}^1 (3x^2 - 4x + 2)\ dx$

(c) $\int_{-1}^1 (3x + 2)(x - 2)\ dx$

(d) $\int_0^2 (3x^2 + 8x - 5)\ dx$

(e) $\int_{-2}^0 (x - 3)^2\ dx$

(f) $\int_{-1}^0 (x^2 - 2x + 7)\ dx$

(g) $\int_0^3 x(x - 2)(x - 3)\ dx$

(h) $\int_{-2}^2 (x + 2)(x - 2)\ dx$

(i) $\int_1^4 (x - 1)(x - 2)\ dx$

3. Evaluate

(a) $\int_0^\pi \cos 2x\ dx$

(b) $\int_0^{\pi/2} \cos 2x\ dx$

(c) $\int_0^\pi \sin 2x\ dx$

(d) $\int_0^{\pi/4} \sin 2x\ dx$

(e) $\int_0^{\pi/3} \cos 3x\ dx$

(f) $\int_0^{2\pi} \cos \frac{1}{2} x\ dx$

4. Evaluate

(a) $\int_0^\pi (\sin t + \cos t)\ dt$

(b) $\int_0^{\pi/4} \sin 4t + \cos 4t\ dt$

(c) $\int_0^{\pi/4} \cos\left(2t + \frac{\pi}{2}\right)\ dt$

(d) $\int_{\pi/6}^{\pi/4} \sin\left(2t - \frac{\pi}{3}\right)\ dt$

Integration

5. Evaluate

(a) $\int_0^{1/2}(x^3 + 12x^2 + 7)\ dx$

(b) $\int_{-1}^{1/2}(3x^2 - 4x)\ dx$

(c) $\int_0^{2/3}(9x^2 + 8)\ dx$

(d) $\int_{-1/2}^{1}(9x^2 + 2x - 1)\ dx$

(e) $\int_0^{\sqrt{3}}(2x + 4)\ dx$

(f) $\int_1^{\sqrt{3}}(10 - 2x)\ dx$

O5	I can evaluate one of the limits of a definite integral given the value of the definite integral.

1. Find a, when $a > 0$

(a) $\int_0^a(2x + 2)dx = 8$

(b) $\int_0^a x^2\ dx = \frac{64}{3}$

2. Given that, $\int_0^a 3x^{1/2}\ dx = 16$, calculate the value of a.

3. Find a for $0 \le t \le 2\pi$ given:

(a) $\int_0^a \cos t\ dt = 1$

(b) $\int_0^a \sin t\ dt = 2$

4. Given that $\int_0^a 5\sin 3x\ dx = \frac{10}{3}$, $0 \le a \le \pi$, calculate the value of a.

5. Determine p, given that $\int_1^p x^{1/2}\ dx = 42$

6. Given that $\int_0^k \frac{1}{(4-3x)^2}\ dx = \frac{1}{36}$, find k.

Integration

1. Given the gradient $\frac{dy}{dx}$ of the curve at the point (x, y) and a point on the curve, find the equation of each curve:

   a) $\frac{dy}{dx} = 3x^2 - 6x + 1$ (3,4)

   b) $\frac{dy}{dx} = 4x^3 - 6x^2$ (1,9)

2. Find the solution to the following differential equations:

   a) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^3}$ and $y = 0$ when $x = 1$

   b) $\frac{dy}{du} = \frac{u^2 + 1}{u^2}$ and $y = 4$ when $u = 2$

3. A curve has gradient given by $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$. The curve passes through the point $(9,10)$. Find the equation of the curve.

4. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

   If $f'(x) = \sin 3x$, express y in terms of x.

5. A curve for which $\frac{dy}{dx} = 3 \sin 2x$ passes through the point $\left(\frac{5}{12}\pi, \sqrt{3}\right)$.

6. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find and expression for the velocity v where $a = \frac{dv}{dt}$.

7. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1,9)$. Express y in terms of x.

Integration

Integration and the wave function

1. **(a)** The expression $3 \sin x - 5 \cos x$ can be written in the form $R \sin(x + a)$ where $R > 0$ and $0 \leq a \leq 2\pi$.

    Calculate the values of R and a.

    (b) Hence find the value of t, where $0 \leq t \leq 2$, for which

    $$\int_0^t (3 \cos x + 5 \sin x) \, dx = 3$$

Integration and Functions

2. **(a)** The functions f and g are defined as $f(x) = 3x + 2$ and $g(x) = x^5$.

    Find $g(f(x))$.

    (b) If $p'(x) = g(f(x))$, and $p(-\frac{1}{3}) = 1$, find $p(x)$.

Integration and Rates of Change

3. The rate of change of the temperature, $T°C$, of a mug of coffee is given by

    $$\frac{dT}{dt} = \frac{1}{25}t - k, 0 \leq t \leq 50$$

    With t, the time elapsed in minutes since pouring and k a constant.

    Initially the temperature of the coffee is $100°C$.

    After 10 minutes it is $82°C$.

    Express T in terms of t.

Integration

Answers

Section B

1. $-\frac{1}{9x^3} + c$ **2.** $\frac{8\sqrt{x^3}}{3} - \frac{1}{2x^2} + c$

3. $f(x) = 2x^3 - x^2 + c$ **4.** $g(x) = \frac{1}{-2(2x+3)} + c$ **5.** $-5\cos\theta + c$

6. $8\sin\theta + c$ **7.** 11 units   **8.** $1\frac{2}{3}$ units.

Section C

O1

1. (a) $\frac{2}{3}x^{\frac{3}{2}} + c$ **(b)** $\frac{2}{5}x^{\frac{5}{2}} + c$ **(c)** $\frac{2}{3}x^{\frac{7}{2}} + c$

(d) $\frac{7}{4}x^{\frac{4}{3}} + c$ **(e)** $\frac{3}{5}x^{\frac{5}{3}} + c$ **(f)** $\frac{5}{9}x^{\frac{9}{5}} + c$

(g) $2x^{\frac{1}{2}} + c$ **(h)** $3x^{\frac{1}{3}} + c$ **(i)** $-\frac{3}{x^{\frac{1}{3}}} + c$

(j) $\frac{5}{2}x^{\frac{2}{5}} + c$ **(k)** $-\frac{6}{5x^{\frac{5}{3}}} + c$ **(l)** $2x^{\frac{1}{2}} + c$

(m) $\frac{2}{3}x^{\frac{1}{3}} + c$ **(n)** $\frac{8}{3}x^{\frac{1}{4}} + c$ **(o)** $-\frac{9}{20x^{\frac{4}{3}}} + c$

2. (a) $\frac{x^4}{4} + x^3 + \frac{5}{2}x^2 + c$ **(b)** $\frac{x^6}{2} + \frac{2}{5}x^5 + \frac{x^2}{2} + c$

(c) $\frac{x^3}{3} + 3x^2 - x + c$ **(d)** $\frac{3}{5}x^{\frac{5}{3}} + \frac{4}{3}x^3 + c$

(e) $2x^{\frac{3}{2}} + \frac{1}{2x^4} + c$ **(f)** $-\frac{5}{x} - 2x^{\frac{3}{2}} + c$

(g) $\frac{3}{4}x^{\frac{2}{3}} + \frac{1}{3}x^3 + c$ **(h)** $\frac{3}{8}x^8 - \frac{4}{5}x^{\frac{1}{4}} + c$

(i) $-\frac{6}{15\sqrt{x^3}} + 5x + c$ **(j)** $\frac{8}{3}x^{\frac{1}{4}} + \frac{2}{3}x^3 + \frac{1}{2}x^2 + c$

(k) $\frac{5}{3}x^3 - 3x^{\frac{1}{3}} + c$ **(l)** $-\frac{4}{x} - \frac{12}{5}x^{\frac{5}{3}} + c$

(m) $\frac{5}{4}x^4 - 12x^{\frac{1}{2}} + c$ **(n)** $\frac{4}{3}x^3 + 9x^{\frac{2}{3}} + c$

(o) $\frac{x^3}{3} - 5x - \frac{1}{x} + c$

Integration

1. (a) $\frac{1}{9}(x+9)^9 + c$ **(b)** $\frac{1}{8}(2x+4)^4 + c$ **(c)** $\frac{1}{25}(5x+7)^5 + c$

(d) $\frac{1}{12}(2x-1)^6 + c$ **(e)** $-\frac{6}{28}(5-4x)^7 + c$ **(f)** $\frac{1}{9}(10-x)^{-9} + c$

(g) $-\frac{3}{8}(4x+1)^{-2} + c$ **(h)** $-\frac{1}{10}(5x-9)^{-4} + c$ **(i)** $\frac{1}{21}(3-7x)^{-3} + c$

(j) $\frac{2}{3}(x-1)^{\frac{3}{2}} + c$ **(k)** $\frac{3}{8}(2x-1)^{\frac{4}{3}} + c$ **(l)** $\frac{2}{5}(2x-1)^{\frac{5}{4}} + c$

(m) $\frac{1}{3}(2x-2)^{\frac{3}{2}} + c$ **(n)** $\frac{1}{5}(3x+4)^{\frac{5}{3}} + c$ **(o)** $\frac{4}{21}(7+3x)^{\frac{7}{4}} + c$

2. (a) $-\frac{1}{20}(5x+3)^{-4} + c$ **(b)** $-\frac{1}{9}(3x-2)^{-3} + c$ **(c)** $\frac{3}{10}(4-2x)^{-5} + c$

(d) $-1(x-2)^{-2} + c$ **(e)** $-\frac{1}{4}(4x+2)^{-3} + c$ **(f)** $\frac{2}{5}(5x-2)^{\frac{1}{2}} + c$

3. (a) $\frac{1}{6}(4x+2)^{\frac{3}{2}} + c$ **(b)** $\frac{4}{3}(3x+1)^{\frac{3}{2}} + c$ **(c)** $-\frac{2}{15}(9-5x)^{\frac{3}{2}} + c$

(d) $\frac{3}{8}(2x-3)^{\frac{4}{3}} + c$ **(e)** $\frac{1}{8}(6x-2)^{\frac{4}{3}} + c$ **(f)** $\frac{2}{5}(2x+4)^{\frac{5}{4}} + c$

(g) $\frac{2}{3}(3x-4)^{\frac{1}{2}} + c$ **(h)** $2(x+8)^{\frac{1}{2}} + c$ **(i)** $2(2x-5)^{\frac{1}{2}} + c$

1. $\frac{1}{2}(x - \frac{1}{2}\sin 2x + c)$ **2.** $\frac{1}{2}(x + \frac{1}{2}\sin 2x + c)$

3. $x - \frac{1}{2}\sin 2x + c$ **4.** $x + \frac{1}{2}\sin 2x + c$

1. (a) $\frac{17}{6}$ **(b)** $\frac{17}{6}$ **(c)** $-\frac{1}{12}$ **(d)** $\frac{22}{3}$ **(e)** $\frac{4}{3}$ **(f)** -3

2. (a) 0   **(b)** 6   **(c)** -6   **(d)** 4   **(e)** $\frac{98}{3}$ **(f)** $\frac{25}{3}$

(g) $\frac{9}{4}$ **(h)** $-\frac{32}{3}$ **(i)** $\frac{9}{2}$

3. (a) 0   **(b)** 0   **(c)** 0   **(d)** $\frac{1}{2}$ **(e)** 0   **(f)** 0

4. (a) 2   **(b)** $\frac{1}{2}$ **(c)** $-\frac{1}{2}$ **(d)** $\frac{2-\sqrt{3}}{4}$

Integration

5. (a) $\frac{257}{64}$ **(b)** $\frac{21}{8}$ **(c)** $\frac{56}{9}$ **(d)** $\frac{21}{8}$ **(e)** $3+4\sqrt{3}$

(f) $10\sqrt{3} - 12$

O5

1(a) $\cancel{a = -4}, a = 2$ **1(b)** $a = 4$ **2.** $a = 4$ **3(a)** $a = \frac{\pi}{2}$

3(b) $a = \pi$ **4.** $a = \frac{\pi}{3}, \pi$ **5.** $p = 16$ **6.** $k = \frac{1}{3}$

O6

1(a) $y = x^3 - 3x^2 + x + 1$ **1(b)** $y = x^4 - 2x^3 + 10$

2(a) $y = x^4 - \frac{1}{x^2}$ **2(b)** $y = u - \frac{1}{u} + \frac{5}{2}$ **3.** $y = 2\sqrt{x} + 4$

4. $f(x) = \frac{7}{6} - \frac{1}{3}\cos 3x$ **5.** $y = \frac{\sqrt{3}}{4} - \frac{3}{2}\cos 2x$

6. $V(t) = \frac{32}{3} - \frac{4}{3}(4 - t)^{\frac{3}{2}}$ **7.** $y = 2x^2 - 2x^3 + 5$

Section D

1. **(a)** $\sqrt{34}\sin(x + 5 \cdot 25)$ **(b)** $t = 0 \cdot 68$ radians

2. **(a)** $(3x + 2)^5$ **(b)** $p(x) = \frac{(3x+2)^6}{18} + \frac{17}{18}$

3. $T = \frac{1}{50}t^2 - 2t + 100$