# Proof by Induction (Unit 2)

**1.** Prove the following results by induction:

(i) $\forall n \in \mathbf{N}, \dfrac{1}{1.2}+\dfrac{1}{2.3}+\ldots+\dfrac{1}{n(n+1)}=\dfrac{n}{n+1};$

(ii) $\forall n \in \mathbf{N}, \sum_{r=1}^{n} r(r+1)(r+2)=\frac{1}{4}n(n+1)(n+2)(n+3);$

(iii) $\forall n \in \mathbf{N}, \sum_{r=0}^{n-1} ax^r=a\dfrac{x^n-1}{x-1}, (x \neq 1);$

(iv) $\forall n \in \mathbf{N}, \sum_{r=1}^{n} (-1)^{r-1}r^2=\frac{1}{2}(-1)^{n-1}n(n+1);$

(v) $\forall n \in \mathbf{N}, \sum_{r=1}^{n} r^5=\frac{1}{12}n^2(n+1)^2(2n^2+2n-1);$

(vi) $\forall n \in \mathbf{N}, 1.2^2+2.3^2+\ldots+n.(n+1)^2=\frac{1}{12}n(n+1)(n+2)(3n+5);$

(vii) $\forall n \in \mathbf{N}, \sum_{r=1}^{n} \dfrac{(r+1)^2}{r(r+2)}=\dfrac{n(4n^2+15n+13)}{4(n+1)(n+2)};$

(viii) $\forall n \in \mathbf{N}, \sum_{r=1}^{2n} (-1)^r r^3=n^2(4n+3);$

(ix) $\forall n \in \mathbf{N}, \sum_{r=1}^{n} \dfrac{3r+2}{r(r+1)(r+2)}(-2)^{r-1}=\dfrac{1}{2}-\dfrac{(-2)^n}{(n+1)(n+2)};$

(x) If $f_r(x)=\dfrac{x(x+1)\ldots(x+r-1)}{r!}$, then, $\forall n \in \mathbf{N}$,

$\sum_{r=1}^{n} f_r(x)=f_n(x+1)-1;$

(xi) $\forall n \in \mathbf{N}, \sum_{r=0}^{n} x^r(1+x)^{n-r}=(1+x)^{n+1}-x^{n+1};$

(xii) $\forall n \geqslant 4, 3^n > n^3;$

(xiii) $\forall n \geqslant 5, \dbinom{2n}{n} < 2^{2n-2};$

(xiv) $\forall n \geqslant 2, \sqrt[n]{n} < 2-\dfrac{1}{n};$

(xv) $\forall n \geqslant 2, \dfrac{1}{\sqrt{(2n+1)}} > \dfrac{1.3.5.\ldots.(2n-1)}{2.4.6.\ldots.(2n)} > \dfrac{1}{2\sqrt{n}}.$