

MATHEMATICS



Relationships and Calculus

Unit Assessment Practice

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$

represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$

represents a circle centre (a, b) and radius r .

Scalar Product: $a \cdot b = |a||b|\cos\theta$, where θ is the angle between a and b

or $a \cdot b = a_1b_1 + a_2b_2 + a_3b_3$ where $a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

Trigonometric formulae:

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + c$

1. a) A function f is defined by the formula $f(x) = x^3 - 3x^2 - 6x + 8$ where x is a real number.
- i) Show that $x - 1$ is a factor of $f(x)$.
 - ii) Hence factorise $f(x)$ fully.
 - iii) Solve $f(x) = 0$.
- b) A function f is defined by the formula $f(x) = x^3 - 4x^2 + x + 6$ where x is a real number.
- i) Show that $x - 3$ is a factor of $f(x)$.
 - ii) Hence factorise $f(x)$ fully.
 - iii) Solve $f(x) = 0$.
- c) A function f is defined by the formula $f(x) = x^3 - 2x^2 - 11x + 12$ where x is a real number.
- i) Show that $x - 1$ is a factor of $f(x)$.
 - ii) Hence factorise $f(x)$ fully.
 - iii) Solve $f(x) = 0$.
- d) A function f is defined by the formula $f(x) = x^3 + 9x^2 + 24x + 16$ where x is a real number.
- i) Show that $x + 4$ is a factor of $f(x)$.
 - ii) Hence factorise $f(x)$ fully.
 - iii) Solve $f(x) = 0$.

2. a) The function $f(x) = kx^2 + 3x + 3$ has equal roots
What is the range of values for k ?
- b) The function $f(x) = kx^2 + 2x - 5$ has real distinct roots.
What is the range of values for k ?
- c) The function $f(x) = kx^2 - 8x + 2$ has no real roots
What is the range of values for k ?
- d) The function $f(x) = kx^2 - 2x + 7$ has equal roots.
What is the range of values for k ?
3. a) Solve $2\cos 2x = \sqrt{3}$, for $0 \leq x \leq 180$
- b) Solve $4\sin 2x = 2$, for $0 \leq x \leq 180$
- c) Solve $\sqrt{2}\cos 2x = 1$, for $0 \leq x \leq 180$
- d) Solve $3\sin 2x = 3$, for $0 \leq x \leq 180$

4. a) Solve $2\sin 2t - \sin t = 0$, for $0 \leq x \leq 180$
- b) Solve $3\sin 2x + \sin x = 0$, for $0 \leq x \leq 180$
- c) Solve $4\sin 2\alpha - \sin \alpha = 0$, for $0 \leq x \leq 180$
- d) Solve $5\sin 2x - \sin x = 0$, for $0 \leq x \leq 180$
5. a) Find $f'(x)$, given that $f(x) = 5\sqrt{x} - \frac{2}{x^3}$, $x > 0$.
- b) Find $f'(x)$, given that $f(x) = 2\sqrt{x} + 3x^{-4}$, $x > 0$.
- c) Find $f'(x)$, given that $f(x) = 2x^{\frac{1}{2}} - \frac{3}{x^5}$, $x > 0$.
- d) Find $f'(x)$, given that $f(x) = 6\sqrt{x} - \frac{5}{x^6}$, $x > 0$.
6. a) Differentiate the function $f(x) = 4\cos x$ with respect to x .
- b) Differentiate the function $f(x) = 7\sin x$ with respect to x .
- c) Differentiate the function $f(x) = -2\cos x$ with respect to x .
- d) Differentiate the function $f(x) = 3\cos x$ with respect to x .

7. a) A curve has equation $y = 3x^2 + 2x + 2$, find the equation of the tangent to the curve at $x = -1$.
- b) A curve has equation $y = 5x^2 - 3x + 2$, find the equation of the tangent to the curve at $x = 2$.
- c) A curve has equation $y = 4x^2 + 2x - 1$, find the equation of the tangent to the curve at $x = -2$.
- d) A curve has equation $y = 3x^2 - 2x + 5$, find the equation of the tangent to the curve at $x = 1$.
8. a) Find $\int (5x^{\frac{3}{2}} + \frac{1}{x^3}) dx, x \neq 0$.
- b) Find $\int (2x^{\frac{1}{2}} - \frac{1}{x^5}) dx, x \neq 0$.
- c) Find $\int (4x^{\frac{2}{3}} + \frac{1}{x^2}) dx, x \neq 0$.
- d) Find $\int (5x^{\frac{1}{4}} - \frac{1}{x^7}) dx, x \neq 0$.
9. a) $f'(x) = (x + 3)^{-7}$, find $f(x), x \neq -3$.
- b) $f'(x) = (x - 1)^{-6}$, find $f(x), x \neq 1$.
- c) $f'(x) = (x + 4)^{-3}$, find $f(x), x \neq -4$.
- d) $f'(x) = (x - 9)^{-2}$, find $f(x), x \neq 9$.

- 10. a)** Find $\int 3 \cos \theta \, d\theta$
- b)** Find $\int 2 \sin \theta \, d\theta$
- c)** Find $\int -6 \cos \theta \, d\theta$
- d)** Find $\int 4 \cos \theta \, d\theta$

- 11. a)** $\int_1^3 (4x - x^2) \, dx$
- b)** $\int_1^2 (2x + x^3) \, dx$
- c)** $\int_1^2 (6x - x^3) \, dx$
- d)** $\int_1^4 (8x + x^2) \, dx$

Answers

1. a) i) remainder = 0 ii) $(x - 1)(x + 2)(x - 4)$
 iii) $x = 1, x = -2, x = 4$
- b) i) remainder = 0 ii) $(x - 3)(x - 2)(x + 1)$
 iii) $x = 3, x = 2, x = -1$
- c) i) remainder = 0 ii) $(x - 1)(x - 4)(x + 3)$
 iii) $x = 1, x = 4, x = -3$
- d) i) remainder = 0 ii) $(x + 4)(x + 4)(x + 1)$
 iii) $x = -4, x = -4, x = -1$

2. a) $k = \frac{3}{4}$ b) $k > -\frac{1}{5}$ c) $k > 8$ (d) $k = \frac{1}{7}$

3. a) $x = 15^\circ$ and 165° b) $x = 15^\circ$ and 75°
c) $x = 22.5^\circ$ and 157.5° d) $x = 45^\circ$

4. a) $t = 0^\circ, 75.5^\circ, 180^\circ$ b) $t = 0^\circ, 99.6^\circ, 180^\circ$
c) $t = 0^\circ, 82.8^\circ, 180^\circ$ d) $t = 0^\circ, 84.3^\circ, 180^\circ$

5. a) $f'(x) = \frac{5}{2}x^{-\frac{1}{2}} + 6x^{-4}$ b) $f'(x) = x^{-\frac{1}{2}} - 12x^{-5}$
c) $f'(x) = x^{-\frac{1}{2}} + 15x^{-6}$ d) $f'(x) = 3x^{-\frac{1}{2}} + 30x^{-7}$

6. a) $f'(x) = -4\sin x$ b) $f'(x) = 7\cos x$
c) $f'(x) = 2\sin x$ d) $f'(x) = -3\sin x$

7. a) $y - 3 = -4(x + 1)$ b) $y - 16 = 17(x - 2)$
c) $y - 11 = -14(x + 2)$ d) $y - 6 = 4(x - 1)$

8. a) $2x^{\frac{5}{2}} - \frac{1}{2}x^{-2} + c$ b) $\frac{4}{3}x^{\frac{3}{2}} + \frac{1}{4}x^{-4} + c$
c) $\frac{12}{5}x^{\frac{5}{3}} - x^{-1} + c$ d) $4x^{\frac{5}{4}} + \frac{1}{6}x^{-6} + c$

9. a) $f'(x) = -\frac{1}{6}(x + 3)^{-6} + c$ b) $f'(x) = -\frac{1}{5}(x - 1)^{-5} + c$
c) $f'(x) = -\frac{1}{2}(x + 4)^{-2} + c$ d) $f'(x) = -(x - 9)^{-1} + c$

10. a) $3\sin\theta + c$ b) $-2\cos\theta + c$ c) $-6\sin\theta + c$ d) $4\sin\theta + c$

11. a) $\frac{22}{3}$ b) $\frac{27}{4}$ c) $\frac{21}{4}$ d) 81